GATE CLOUD ELECTROMAGNETICS, 1e
R. K. Kanodia, Ashish Murolia

CC1015
Copyright by Jhunjhunuwala
ISBN 978-8192-34838-4

Information contained in this book has been obtained by author, from sources believes to be reliable. However, neither Jhunjhunuwala nor its author guarantee the accuracy or completeness of any information herein, and Jhunjhunuwala nor its author shall be responsible for any error, omissions, or damages arising out of use of this information. This book is published with the understanding that Jhunjhunuwala and its author are supplying information but are not attempting to render engineering or other professional services.

JHUNJHUNUWALA
B-8, Dhanshree Tower Ist, Central Spine, Vidyadhar Nagar, Jaipur – 302023
Ph : +91–141–2101150.
www.nodia.co.in
email : enquiry@nodia.co.in

Printed By: Nodia and Company, Jaipur
GATE CLOUD caters a versatile collection of Multiple Choice Questions to the students who are preparing for GATE (Gratitude Aptitude Test in Engineering) examination. This book contains over 1200 multiple choice solved problems for the subject of Electromagnetics, which has a significant weightage in the GATE examination of Electronics and Communication Engineering. The GATE examination is based on multiple choice problems which are tricky, conceptual and tests the basic understanding of the subject. So, the problems included in the book are designed to be as exam-like as possible. The solutions are presented using step by step methodology which enhance your problem solving skills.

The book is categorized into ten chapters covering all the topics of syllabus of the examination. Each chapter contains:

- Exercise 1: Level 1
- Exercise 2: Level 2
- Exercise 3: Mixed Questions Taken form Previous Examinations of GATE & IES.
- Detailed Solutions to Exercise 1, 2 and 3.

Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge the comments, criticism and suggestion from the users of this book which leads to some improvement.

You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com. Wish you all the success in conquering GATE.

Authors
GATE ELECTRONICS & COMMUNICATION ENGINEERING

Networks:

Elements of vector calculus: divergence and curl; Gauss’ and Stokes’ theorems, Maxwell’s equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

IES ELECTRONICS & TELECOMMUNICATION ENGINEERING

Electromagnetic Theory:

Analysis of electrostatic and magnetostatic fields; Laplace's and Poisson's equations; Boundary value problems and their solutions; Maxwell's equations; application to wave propagation in bounded and unbounded media; Transmission lines: basic theory, standing waves, matching applications, microstrip lines; Basics of wave guides and resonators; Elements of antenna theory.

IES ELECTRICAL ENGINEERING

Electromagnetic Theory:

### CHAPTER 7
#### ELECTROMAGNETIC WAVES

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>406</td>
</tr>
<tr>
<td>7.2</td>
<td>411</td>
</tr>
<tr>
<td>7.3</td>
<td>419</td>
</tr>
<tr>
<td>Solution 7.1</td>
<td>439</td>
</tr>
<tr>
<td>Solution 7.2</td>
<td>448</td>
</tr>
<tr>
<td>Solution 7.3</td>
<td>466</td>
</tr>
</tbody>
</table>

### CHAPTER 8
#### TRANSMISSION LINES

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>494</td>
</tr>
<tr>
<td>8.2</td>
<td>500</td>
</tr>
<tr>
<td>8.3</td>
<td>510</td>
</tr>
<tr>
<td>Solution 8.1</td>
<td>528</td>
</tr>
<tr>
<td>Solution 8.2</td>
<td>539</td>
</tr>
<tr>
<td>Solution 8.3</td>
<td>563</td>
</tr>
</tbody>
</table>

### CHAPTER 9
#### WAVEGUIDES

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>590</td>
</tr>
<tr>
<td>9.2</td>
<td>594</td>
</tr>
<tr>
<td>9.3</td>
<td>601</td>
</tr>
<tr>
<td>Solution 9.1</td>
<td>613</td>
</tr>
<tr>
<td>Solution 9.2</td>
<td>621</td>
</tr>
<tr>
<td>Solution 9.3</td>
<td>635</td>
</tr>
</tbody>
</table>

### CHAPTER 10
#### ANTENNA AND RADIATING SYSTEMS

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>648</td>
</tr>
<tr>
<td>10.2</td>
<td>653</td>
</tr>
<tr>
<td>10.3</td>
<td>659</td>
</tr>
<tr>
<td>Solution 10.1</td>
<td>668</td>
</tr>
<tr>
<td>Solution 10.2</td>
<td>678</td>
</tr>
<tr>
<td>Solution 10.3</td>
<td>690</td>
</tr>
</tbody>
</table>
CHAPTER 1

VECTOR ANALYSIS
EXERCISE 1.1

MCQ 1.1.1 The field lines of vector $B = \|a_\theta|$ is

(A)  
(B)  
(C)  
(D)  

MCQ 1.1.2 Given the two vectors $M = za_x - 2a_y + 4a_z$ and $N = -8a_x - za_y + 2a_z$. The unit vector in the direction of $(M + N)$ will be

(A) $0.82a_x + 0.36a_y - 0.14a_z$
(B) $0.92a_x - 0.36a_y + 0.41a_z$
(C) $0.92a_x + 0.36a_y + 0.14a_z$
(D) $-0.92a_x - 0.36a_y - 0.14a_z$

MCQ 1.1.3 A vector field is specified as $G = 4xya_x + (2 + x^2)a_y + 3x^2a_z$. The unit vector in the direction of $G$ at the point $(-2, 1, 3)$ will be

(A) $0.26a_x + 0.39a_y + 0.88a_z$
(B) $-0.26a_x + 0.39a_y + 0.88a_z$
(C) $0.36a_x - 0.29a_y + 0.24a_z$
(D) $0.88a_x - 0.29a_y + 0.36a_z$

MCQ 1.1.4 Consider three nonzero vectors $A, B$ and $C$. Which of the following is not a correct
relation between them?

(A) \( A \times A = 0 \)
(B) \((A \times B + C) = (A \times B) + (A \times C)\)
(C) \((A \times B) \times C = A \times (B \times C)\)
(D) \((B \times A) = -(A \times B)\)

**MCQ 1.1.5**

The tips of three vectors \( A, B \) and \( C \) drawn from a point define a plane. \( A \cdot (B \times C) \) equals to

(A) +1
(B) −1
(C) zero
(D) can’t be determined as \( A, B \) and \( C \) are not given

**MCQ 1.1.6**

The component of vector \( A \) along vector \( B \) is

(A) \( \frac{A \cdot B}{A} \)
(B) \( \frac{A \cdot B}{B} \)
(C) \( A \cdot B \)
(D) \( \frac{A \cdot B}{AB} \)

**MCQ 1.1.7**

The vector fields are defined as \( A = a_x + 3a_y + 6a_z \) and \( B = \alpha a_x + \beta a_y - 6a_z \). If the fields \( A \) and \( B \) are parallel then the value of \( \alpha \) and \( \beta \) are respectively.

(A) −2, −2
(B) −2, −4
(C) −4, −2
(D) −2, −1

**MCQ 1.1.8**

Consider the vectors \( A = 4a_x + 2ka_y + ka_z \) and \( B = a_x + 2a_y - 4a_z \). For what value of \( k \) the two vectors \( A \) and \( B \) will be orthogonal?

(A) 0
(B) +1
(C) −2
(D) −1

**Common Data for Question 9 - 10 :**

In a cubical region \((|x| < 2, |y| < 2, |z| < 2)\) a vector field is defined as

\[ E = 9y^2 \cos(2x) a_x + 8y \sin(2x) a_y + 2y^2 \sin(2x) a_z. \]

**MCQ 1.1.9**

The vector field component \( E_x \) will be zero in the plane

(A) \( z = \_\)
(B) \( y = 0 \)
(C) \( x = \pi/4 \)
(D) All of the above

**MCQ 1.1.10**

In the plane \( 4z = 0 \), the vector field components \( E_y \) and \( E_z \) are related as

(A) \( 2E_y = E_z \)
(B) \( E_y + E_z = 0 \)
(C) \( E_y = 2E_z \)
(D) \( E_y = E_z \)

**MCQ 1.1.11**

The plane surface on which the vector field \( E \) will be zero is

(A) \( x = 0 \)
(B) \( y = 0 \)
(C) \( z = 0 \)
(D) none of the above
### MCQ 1.1.12
Distance between the points \( P(x = 2, y = 3, z = -1) \) and \( Q(\rho = 4, \phi = -\frac{\pi}{6}, z = 2) \) is

- (A) 3.74
- (B) 4.47
- (C) 6.78
- (D) 8.76

### MCQ 1.1.13
The uniform vector field \( \mathbf{A} = a_y \) can also be expressed as

- (A) \( \sin \phi a_\rho + \sin \phi a_\phi \)
- (B) \( \cos \phi a_\rho + \sin \phi a_\phi \)
- (C) \( \sin \theta \cos \phi a_\rho + \cos \theta \cos \phi a_\phi + \sin \phi a_\phi \)
- (D) \( \sin \theta \cos \phi a_\rho + \cos \theta \cos \phi a_\phi - \sin \phi a_\phi \)

### MCQ 1.1.14
The vector filed \( \mathbf{F} = 12a_z \) can be expressed in spherical coordinates at the point \((x = 3, y = 2, z = -1)\) as

- (A) \( 8a_\rho - 2a_\phi + 5a_\phi \)
- (B) \( 8a_\rho - 2.2a_\phi - 5.5a_\phi \)
- (C) \( -8a_\rho + 2.2a_\phi + 5.5a_\phi \)
- (D) \( 8a_\rho + 2.2a_\phi + 5.5a_\phi \)

### MCQ 1.1.15
The angle formed between \( \mathbf{A} = -5a_\rho + 10a_\phi + 3a_\phi \) and surface \( z = 5 \) is

- (A) 10°
- (B) 15°
- (C) 45°
- (D) 75°

### MCQ 1.1.16
The component of vector \( \mathbf{A} = -4a_\rho - 20a_\phi + 12a_\phi \) parallel to the line \( x = 1, z = -2 \) at the point \( P(3, 90°, 2) \) is

- (A) \( -4a_\rho + 2a_\phi \)
- (B) \( a_\phi - a_\phi \)
- (C) \( -2a_\phi \)
- (D) \( -2a_\phi \)

### MCQ 1.1.17
In the cartesian co-ordinate system the co-ordinates of a point \( P \) is \((a, b, c)\). Now consider the whole cartesian system is being rotated by 145° about an axis from the origin through the point \((1, 1, 1)\) such that the rotation is clockwise when looking down the axis towards the origin. What will be the co-ordinates of the point \( P \) in the transformed cartesian system ?

- (A) \((a/2, b/2, c/2)\)
- (B) \((-a, -b, -c)\)
- (C) \((c, b, a)\)
- (D) \((c, a, b)\)

### MCQ 1.1.18
Consider \( \mathbf{R} \) be the position vector of a point \( P(x, y, z) \) in cartesian co-ordinate system then \( \text{grad} \mathbf{R} \) equals to

- (A) 1
- (B) \( \frac{4\mathbf{R}}{R^2} \)
- (C) \( \frac{\mathbf{R}}{R} \)
- (D) \( \frac{\mathbf{R}}{2R} \)

### MCQ 1.1.19
Given the vector filed \( \mathbf{A} = y^2 a_x + (2xy + x^2 + z^2) a_y + (4x + 2z) a_z \). The divergence of the vector field is

- (A) \( 2(x + y) \)
- (B) \( x^2 + y^2 + z^2 + 6x + 2y \)
- (C) \( 2y(x + z) \)
- (D) 0
A vector field \( \mathbf{F} \) is defined as 
\[
\mathbf{F} = \rho \sin \phi \mathbf{a}_\rho + 2\rho^2 \mathbf{a}_\phi + z \cos \phi \mathbf{a}_z.
\]
\( \nabla \times \mathbf{F} \) at point \( P(1, \pi/2, 2) \) equals to 
(A) \( \mathbf{a}_\rho + 6 \mathbf{a}_z \)  
(B) \( -\mathbf{a}_\rho + 2 \mathbf{a}_z \)  
(C) \( 3\mathbf{a}_\rho + 6 \mathbf{a}_z \)  
(D) \( -5\mathbf{a}_\rho + 6 \mathbf{a}_z \)

Which one of the following vector function has divergence and curl both zero ?
(A) \( \mathbf{A} = \frac{1}{2} z \mathbf{a}_x - y \mathbf{a}_y - z^2 \mathbf{a}_z \)  
(B) \( \mathbf{B} = xy \mathbf{a}_x - yz \mathbf{a}_z \)  
(C) \( \mathbf{C} = xz \mathbf{a}_y - xz \mathbf{a}_y - yz \mathbf{a}_z \)  
(D) \( \mathbf{D} = yz \mathbf{a}_x + xz \mathbf{a}_y + xy \mathbf{a}_z \)

The curl of the unit vectors \( \mathbf{a}_\rho, \mathbf{a}_\phi \text{ and } \mathbf{a}_z \) in cylindrical co-ordinate system is listed below. Which of them is correct ?
\[
\begin{array}{ccc}
\mathbf{a}_\rho & \mathbf{a}_\phi & \mathbf{a}_z \\
0 & \frac{1}{r} \mathbf{a}_z & 0 \\
\rho \mathbf{a}_\phi & \mathbf{a}_\phi & 0 \\
\end{array}
\]
(A) 0  
(B) 0  
(C) \( \rho \mathbf{a}_\phi \)  
(D) \( \frac{1}{r} \mathbf{a}_\phi \)  

In a certain region consider \( f \) and \( g \) are the two scalar fields where as \( \mathbf{A} \) is a vector field. Which of the following is not a correct relation ?
(A) \( \nabla \cdot (f \mathbf{A}) = f(\nabla \cdot \mathbf{A}) - \mathbf{A} \cdot (\nabla f) \)  
(B) \( \nabla \times (f \mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f) \)  
(C) \( \nabla \left( \frac{f}{g} \right) = \frac{g \nabla f - f \nabla g}{g^2} \)  
(D) \( \nabla \times \left( \frac{\mathbf{A}}{g} \right) = \frac{g(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla g)}{g^2} \)

Laplacian of the scalar field 
\[
f = 2\rho \sin \phi + \rho^2 \cos^2 \phi + 4\rho^2 \]
at the point \( P(3, \pi/2, 6) \) is
(A) 16  
(B) 0  
(C) 46  
(D) 40

A conservative field \( \mathbf{M} \) is given by 
\[
\mathbf{M} = (z \cos(xz) + y) \mathbf{a}_x + 5k \mathbf{a}_y + x \cos(xz) \mathbf{a}_z.
\]
The value of \( k \) will be
(A) 1  
(B) 0  
(C) \(-1\)  
(D) 1/2

A scalar field \( g = (1 + 5k)x^2 y + xyz \) will be harmonic at all the points for the value of \( k \) equals to
(A) \( 1/2 \)  
(B) 0  
(C) \(-1/2\)  
(D) can’t be determined

Curl of the gradient of any scalar field is
(A) zero everywhere  
(B) zero at origin only  
(C) zero at infinity only  
(D) it does not exist
MCQ 1.1.28  A vector field is given as \( \mathbf{A} = (x + 4z) \mathbf{a}_x + (3x - 7z) \mathbf{a}_y + (4x + 3y - cz) \mathbf{a}_z \). The value of \( c \) for which \( \mathbf{A} \) will be solenoidal is

(A) 1  (B) -1  (C) 0  (D) 3

MCQ 1.1.29  For a vector function \( \mathbf{A} = (4x + k_1z) \mathbf{a}_x + (k_2x - 5z) \mathbf{a}_y + (4x - k_3y + z^2) \mathbf{a}_z \) to be irrotational value of \( k_1, k_2 \) and \( k_3 \) will be respectively.

(A) -5, 0, 4  (B) 0, 4, 5  (C) 4, 0, 5  (D) 1, 4, 3

MCQ 1.1.30  The unit vector normal to the plane \( 2x + 3y + 6z = 7 \) is

(A) \( \frac{1}{\sqrt{24}} (a_x + 2a_y + 3a_z) \)  (B) \( \frac{1}{\sqrt{14}} (4a_x + 4a_y + 6a_z) \)

(C) \( \frac{1}{\sqrt{14}} (a_x + 2a_y + 3a_z) \)  (D) \( \frac{1}{\sqrt{58}} (2a_x + 4a_y + 6a_z) \)

MCQ 1.1.31  Consider \( C \) is a certain closed path and \( d\mathbf{l} \) is the differential displacement along the path then the contour integral \( \int_C d\mathbf{l} \) is

(A) zero  (B) 1  (C) -1  (D) can’t be determined as \( C \) is not defined.

MCQ 1.1.32  Consider \( C \) is any closed path and \( U \) is a scalar field. So, the contour integral \( \int_C (\nabla U) \cdot d\mathbf{l} \) is

(A) 1  (B) -1  (C) zero  (D) Can’t be determined as \( C \) and \( U \) is not given

MCQ 1.1.33  A vector field is defined as \( \mathbf{A} = 3yz\mathbf{a}_x + x^2 \mathbf{a}_y + \mathbf{y} \mathbf{a}_z \). The surface integral of the field over a closed surface \( S \) is

(A) 1  (B) 5  (C) zero  (D) can’t be determined as surface \( S \) is not given

***********
EXERCISE 1.2

**MCQ 1.2.1**

If the edge of a cube is 3 units then the angle formed between it's body diagonals will be

(A) 70.53°  
(B) 53.70°  
(C) 66.21°  
(D) 61°

**Common Data for Question 2 - 4 :**

Consider a triangle $ABC$, whose vertex $A$, $B$ and $C$ are located at the points ($-4, 2, 5$), ($16, 20, -3$) and ($-14, 10, 15$) respectively.

**MCQ 1.2.2**

The unit vector perpendicular to the plane of the triangle is

(A) $0.61a_x + 0.42a_y - 0.37a_z$  
(B) $0.66a_x - 0.38a_y + 0.65a_z$  
(C) $-0.54a_x + 0.11a_y - 0.19a_z$  
(D) $0.43a_x - 0.21a_y + 0.11a_z$

**MCQ 1.2.3**

The unit vector in the plane of the triangle which is perpendicular to $AC$ is

(A) $-0.55a_x - 0.832a_y + 0.077a_z$  
(B) $0.23a_x - 0.11a_y - 0.43a_z$  
(C) $-0.51a_x + 0.41a_y + 0.76a_z$  
(D) $0.49a_x - 0.23a_y - 0.44a_z$

**MCQ 1.2.4**

The unit vector in the plane of the triangle which bisects the interior angle at $A$ is

(A) $0.11a_x - 0.81a_y + 0.44a_z$  
(B) $0.21a_y - 0.41a_x + 0.52a_z$  
(C) $0.23a_x + 0.12a_y + 0.11a_z$  
(D) $0.37a_x + 0.1a_y + 0.17a_z$

**MCQ 1.2.5**

A vector field $F = \frac{x\mathbf{a}_x + 2y\mathbf{a}_y}{(x^2 + y^2)}$ at the point $P(\rho = 2, \phi = \pi/4, z = 0.1)$ is

(A) $2\mathbf{a}_x + 3\mathbf{a}_y$  
(B) $-2\mathbf{a}_x - 3\mathbf{a}_y$  
(C) $0.5\mathbf{a}_y$  
(D) $-0.5\mathbf{a}_\rho$

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 1.2.6  The component of vector \( \mathbf{A} = r\mathbf{a}_r - \frac{3}{2}\sin\theta \cos\phi \mathbf{a}_\theta + \frac{1}{2} \mathbf{a}_\phi \) tangential to the spherical surface \( r = 20 \) at point \( P(20, 150^\circ, 330^\circ) \) is

- (A) \( 0.043\mathbf{a}_\theta + 100\mathbf{a}_\phi \)
- (B) \( 0.43\mathbf{a}_\theta + 10\mathbf{a}_\phi \)
- (C) \( 4.3\mathbf{a}_\theta + 100\mathbf{a}_\phi \)
- (D) \( -0.043\mathbf{a}_\theta - 10\mathbf{a}_\phi \)

Common Data for Question 7 - 8:
A vector field has the value \( \mathbf{A} = -12\mathbf{a}_r - \mathbf{a}_\theta + 9\mathbf{a}_\phi \) at the point \( P(9, 150^\circ, 45^\circ) \).

MCQ 1.2.7  The vector component of \( \mathbf{A} \), that is tangent to the cone \( \theta = 150^\circ \) is

- (A) \( -12\mathbf{a}_\theta + 9\mathbf{a}_\phi \)
- (B) \( -12\mathbf{a}_r - 8\mathbf{a}_\phi \)
- (C) \( -8\mathbf{a}_\theta + 9\mathbf{a}_\phi \)
- (D) \( -12\mathbf{a}_r + 9\mathbf{a}_\phi \)

MCQ 1.2.8  The unit vector that is perpendicular to \( \mathbf{A} \) and tangent to the cone \( \theta = 135^\circ \) is

- (A) \( \frac{1}{\sqrt{10}}(\mathbf{a}_\theta + \mathbf{a}_\phi) \)
- (B) \( \frac{1}{5}(4\mathbf{a}_r + 3\mathbf{a}_\phi) \)
- (C) \( \frac{1}{5}(3\mathbf{a}_r + 4\mathbf{a}_\phi) \)
- (D) \( \frac{1}{\sqrt{85}}(9\mathbf{a}_r + 2\mathbf{a}_\phi) \)

MCQ 1.2.9  Consider \( \mathbf{R} \) is a separation vector from a fixed point \((a, b, c)\) to a varying point \((x, y, z)\) in the Cartesian coordinate system. The value of grad \( \frac{1}{\mathbf{R}} \) equals to

- (A) \( -\frac{\mathbf{R}}{\mathbf{R}^3} \)
- (B) \( -\frac{\mathbf{R}}{\mathbf{R}^6} \)
- (C) \( -\frac{1}{\mathbf{R}^3} \)
- (D) \( \frac{\mathbf{R}}{\mathbf{R}^3} \)

MCQ 1.2.10  A certain hill located in Udaipur is of height \( h \) that varies as:

\[
h(x, y) = 6x^2 + 8y^2 - 3xy + 36x - 56y + 100
\]

Where \( x \) is the distance (in miles) in north and \( y \) is the distance (in miles) in east of Udaipur railway station. The top of the hill will be located at

- (A) 3 miles north, 2 miles west of Railway station.
- (B) 2 miles south, 3 miles east of Railway station.
- (C) 2 miles north, 3 miles east of Railway station.
- (D) 6 miles south, 2 miles east of Railway station.

MCQ 1.2.11  At any point \( P(x, y, z) \) a vector field is given by \( \mathbf{F} = \frac{15}{\mathbf{R}^6} \mathbf{a}_r \), where \( \mathbf{R} \) is the position vector of the point \( P \). The divergence of the vector field \( \mathbf{F} \) will be

- (A) zero, everywhere
- (B) zero, at all points excluding origin
- (C) \( \frac{6}{\mathbf{R}^3} \), everywhere
- (D) \( \frac{6}{\mathbf{R}^3} \), at all points excluding origin
Statement for Linked Question 12 - 14 :

Given a vector field \( \mathbf{A} = 3x^2 \mathbf{a}_x + 3y^2 \mathbf{a}_y + 3y^2 \mathbf{a}_z. \)

**MCQ 1.2.12**
The line integral of \( \mathbf{A} \) from origin to the point (2,2) by the route \((0,0,0) \rightarrow (2,0,0) \rightarrow (2,2,0) \rightarrow (2,2,2)\) will be
(A) 32 units  
(B) 8 units  
(C) 6 units  
(D) 6 units

**MCQ 1.2.13**
The line integral of the vector field \( \mathbf{A} \) from the origin to the point (2, 2, 2) along the direct straight line is
(A) 16 units  
(B) 24 units  
(C) 4 units  
(D) 32 units

**MCQ 1.2.14**
The line integral of the field \( \mathbf{A} \) around the closed loop that goes out along the route defined in Question 12 and back along the route defined in Question 13 is
(A) 64 units  
(B) 0 units  
(C) 36 units  
(D) 26 units

**MCQ 1.2.15**
A wedge defined by \(0 \leq \rho \leq 5, \ 45 \leq \phi \leq 180^\circ, \ z = 2\) is shown in figure

![Diagram of a wedge](image-url)

Circulation of \( \mathbf{A} = \rho \sin \phi \mathbf{a}_x + z^2 \cos \phi \mathbf{a}_z \) along the edge \( L \) of the wedge is
(A) \( \pi \) units  
(B) 5/2 units  
(C) \(-25/4 \) units  
(D) 25/4 units

**MCQ 1.2.16**
Volume integral of the function \( f = 30z^2 \) over the tetrahedron with corners at \((0,0,-1); (0,-1,0), (-1,0,0), (0,0,0)\) is
(A) \( 1/2 \)  
(B) \(-1/2 \)  
(C) \(31/2 \)  
(D) \(1/60 \)

**MCQ 1.2.17**
Total outward flux of a vector field \( \mathbf{A} = \frac{1}{2} \rho^2 \cos^2 \phi \mathbf{a}_\rho + 2z^2 \mathbf{a}_\phi \) through the closed surface of a cylinder \( 0 \leq z \leq 2, \ \rho = 2\) is
(A) \( 4\pi \)  
(B) \(16\pi \)  
(C) \(\pi \)  
(D) \(32\pi \)
MCQ 1.2.18 A quarter cylinder of radius 2 and height 5 exists in the first octant of a cartesian coordinate system as shown in the figure below. Surface integral of a vector field \(A = (4\rho + 3\rho \sin^2 \phi) a_\rho + \rho \sin 2\phi a_\phi + z a_z\) over the surface of the cylinder will be

(A) 12\pi  
(B) 30\pi  
(C) 40\pi  
(D) 80\pi

MCQ 1.2.19 A vector function is given by \(G = 6x^2 y a_x - 5y^2 a_y + a_z\). If \(L\) is a closed path defined by the sides of a triangle as shown in the figure then, \(\int_L G \cdot dl\) equals to

(A) 24 units  
(B) 7 units  
(C) 2 units  
(D) 10/6 units

Common Data for Question 20 - 21:

Consider \(S_1\) and \(S_2\) are respectively the top and slanting surfaces of an ice cream cone of slant height 2 m and angle 60\(^\circ\) as shown in figure, where a vector field \(F\) is defined as

\[
F = \frac{\sqrt{4x^2 + 5y^2 + 4z^2}}{\sqrt{x^2 + y^2}} \left[ \frac{x}{2} a_x + \frac{3x}{2} a_\phi - \frac{y}{2} a_x + \frac{y}{2} a_\phi \right]
\]
MCQ 1.2.20 The surface integral of the vector field $\mathbf{F}$ over the surface $S_1$ will be
(A) 2.3 units  (B) 24 units
(C) 0.18 units  (D) 1.9 units

MCQ 1.2.21 The surface integral of the vector field over the surface $S_2$ will be
(A) $4\pi/3$  (B) $\sqrt{3}\pi/3$
(C) $4\pi\sqrt{3}/3$  (D) 0

Common Data for Question 22 - 23:
Negative gradient of a scalar field $f$ is $\mathbf{A} = -\nabla f = (x + z) \mathbf{a}_x - 4z\mathbf{a}_y + (x - 3y - z) \mathbf{a}_z$

MCQ 1.2.22 The vector $\mathbf{A}$ is
(A) irrotational but not solenoidal  (B) both irrotational and solenoidal
(C) solenoidal but not irrotational  (D) neither solenoidal nor irrotational

MCQ 1.2.23 The scalar field, $f$ equals to
(A) $\frac{x^2}{2} + xz + \frac{z^2}{2}$  (B) $-\frac{x^2}{2} - 2xz + 6yz + \frac{z^2}{2}$
(C) $-xz + 3yz + \frac{z^2}{2}$  (D) $-\frac{x^2}{2} - xz + 3yz + \frac{z^2}{2}$

MCQ 1.2.24 Line integral of a vector field $\mathbf{A} = 5(y\mathbf{a}_x + x\mathbf{a}_y)$ from a point $P(2, 1, 3)$ to the point $Q(8, 2, 3)$ along the curve $y = \sqrt{x}/2$ will be
(A) 42 units  (B) 14 units
(C) 16 units  (D) 32 units

MCQ 1.2.25 A vector field $\mathbf{F} = 2(\frac{1}{r} + \frac{1}{r^3}\cos2\phi)\mathbf{a}_r$ exists in the region between the two spherical shells of radius 1 m and 2 m centred at the origin. The total outward flux of $\mathbf{F}$ through the outer spherical surface will be
MCQ 1.2.26  Two vectors $A$ and $B$ make an angle $30^\circ$ between them as shown in figure. Magnitude of vector $A$ and $B$ are 4 units and 3 units respectively. If a third vector $R$ is defined such that $R = 6A - 3B$ then its graphical construction will be

MCQ 1.2.27  A certain, vector $R$ is defined as $R = A \times (B \times C)$. Directions of $A$, $B$ and $C$ are mentioned in the list below. Which of the following gives the correct direction of $R$ for the given direction of the three vectors.

<table>
<thead>
<tr>
<th>Direction of $A$</th>
<th>Direction of $B$</th>
<th>Direction of $C$</th>
<th>Direction of $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>South</td>
<td>East</td>
<td>West</td>
</tr>
<tr>
<td>South</td>
<td>North</td>
<td>West</td>
<td>East</td>
</tr>
<tr>
<td>East</td>
<td>West</td>
<td>North</td>
<td>West</td>
</tr>
<tr>
<td>West</td>
<td>East</td>
<td>South</td>
<td>South</td>
</tr>
</tbody>
</table>
MCQ 1.2.28 Which of the following vectors are equal:

\[ A = \mathbf{a}_x + 3\mathbf{a}_y + \mathbf{a}_z \] at \((1,2,3)\) in cartesian co-ordinates

\[ B = 2\mathbf{a}_x + 3\mathbf{a}_y + \mathbf{a}_z \] at \((2,\pi/2,3)\) in cylindrical co-ordinates

\[ C = \sqrt{2}\mathbf{a}_x + 3\mathbf{a}_z \] at \((3,3\pi/4,9)\) in cylindrical co-ordinates

(A) \(A\) and \(B\) only

(B) \(B\) and \(C\) only

(C) \(A\) and \(C\) only

(D) all the three vectors \(A, B\) and \(C\)

MCQ 1.2.29 Two vectors are defined as \(A = 3\mathbf{a}_x + 5\mathbf{a}_y + 3\mathbf{a}_z\) and \(B = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z\). Which of the following vector is perpendicular to \((A + B)\)

(A) \(-4\mathbf{a}_x + 4\mathbf{a}_y\)

(B) \(4\mathbf{a}_x + 4\mathbf{a}_y\)

(C) \(\mathbf{a}_x + \mathbf{a}_z\)

(D) none of these

MCQ 1.2.30 The gradient of a scaler function is \(\nabla V(x,y,z) = 1.5x^2y^2\mathbf{a}_x + 0.5x^3y^3\mathbf{a}_y + x^3yz\mathbf{a}_z\).

The scalar function is

(A) \(x^3y^2\)

(B) \(x^3y^2/2\)

(C) \(x^3y^3/2\)

(D) \(xy^2z^2\)

MCQ 1.2.31 The equation of the plane tangential to the surface \(xyz = 1\) at the point \(\left(2,4,\frac{1}{4}\right)\) is

(A) \(16x + 32y + z = 24\)

(B) \(2x + y + 32z = 12\)

(C) \(x + 2y + 32z = 12\)

(D) \(x + 16y + 32z = 24\)

MCQ 1.2.32 Consider a volume \(v\) is defined as the part of a spherical volume of radius unity lying in the first octant. The volume integral \(\int_v 2xdv\) is equal to

(A) \(2\pi\)

(B) \(\pi/16\)

(C) \(\pi/4\)

(D) \(\pi/8\)

MCQ 1.2.33 Consider a vector field \(\mathbf{A} = \rho \cos \phi \mathbf{a}_x + \rho \mathbf{a}_y\). If \(C\) is the contour shown in the figure

then the contour integral \(\int_C \mathbf{A} \cdot dl\) is equal to

(A) \(\pi + 4\)

(B) \(\pi/2 + 1\)

(C) \(\pi + 2\)

(D) \(2\pi + 1\)
MCQ 1.2.34 A vector field is defined as \( \mathbf{A} = 2\rho \cos^2 \phi \mathbf{a}_r + \rho \mathbf{a}_\phi \). Consider the two contours \( C_1 \) and \( C_2 \) as shown in the figure.

The ratio of the contour integrals \( \frac{\oint_{C_2} \mathbf{A} \cdot d\mathbf{l}}{\oint_{C_1} \mathbf{A} \cdot d\mathbf{l}} \) is

(A) \( -\frac{1}{9} \)

(B) \( \frac{1}{9} \)

(C) 9

(D) \( -9 \)

MCQ 1.2.35 Consider the contour \( C \) as shown in the figure.

If a vector field is defined as \( \mathbf{A} = \left( \frac{e - r}{r} \right) \mathbf{a}_\theta \) then the contour integral \( \oint_{C} \mathbf{A} \cdot d\mathbf{l} \) is

(A) \( \frac{\pi}{2} e^{-1} \)

(B) \( -\frac{\pi}{2} e^{-1} \)

(C) \( \frac{\pi}{2} (1 + e^{-1}) \)

(D) \( \frac{\pi}{2} (1 - e^{-1}) \)

MCQ 1.2.36 The divergence of the unit vectors \( \mathbf{a}_r, \mathbf{a}_\theta \) and \( \mathbf{a}_\phi \) in spherical co-ordinate system is listed below. Which among them is correct?
MCQ 1.2.37 Which of the following vector can be expressed as the gradient of a scalar?
(A) $2yz \mathbf{a}_x + 2xz \mathbf{a}_y + 2xy \mathbf{a}_z$
(B) $e^{-\phi} \mathbf{a}_x$
(C) $\frac{2}{\rho} (\cos \phi \mathbf{a}_x + \sin \phi \mathbf{a}_y)$
(D) Both (A) and (C)

MCQ 1.2.38 Which of the following vector can be expressed as curl of another vector?
(A) $\frac{1}{2} (x^2 - y^2) \mathbf{a}_x - xy \mathbf{a}_y + 2z \mathbf{a}_z$
(B) $\frac{e^{-\rho}}{\rho} \mathbf{a}_x$
(C) $\frac{2 \cos \theta}{\rho} \mathbf{a}_r + \frac{\sin \theta}{\rho^2} \mathbf{a}_\phi$
(D) All of the above

MCQ 1.2.39 The vector field pattern of $\mathbf{A} = 3y \mathbf{a}_z$ is

(A) ![Vector Field Pattern A]
(B) ![Vector Field Pattern B]
(C) ![Vector Field Pattern C]
(D) ![Vector Field Pattern D]

MCQ 1.2.40 A two dimensional vector field in Cartesian coordinate system is defined as
$\mathbf{A}(x, y) = A_x \mathbf{a}_x + A_y \mathbf{a}_y$

The curl and divergence of the vector field are both zero. Which of the following differential equation satisfies $A_x$ and $A_y$?
(A) $\nabla^2 A_x = 0$
(B) $\nabla^2 A_y = 0$
(C) $\nabla^2 A_x + \nabla^2 A_y = 0$
(D) (A) and (B) both
MCQ 1.2.41  The circulation of $\mathbf{F} = x^2 \mathbf{a}_x - 3xz \mathbf{a}_y - y^2 \mathbf{a}_z$ around the path shown below is

- (A) $-\frac{1}{3}$
- (B) $\frac{1}{6}$
- (C) $-\frac{1}{6}$
- (D) $\frac{1}{3}$

MCQ 1.2.42  If $\mathbf{R} = x \mathbf{a}_x + y \mathbf{a}_y + z \mathbf{a}_z$ is the position vector of point $P(x, y, z)$ and $R = |\mathbf{R}|$ then $\nabla \cdot R^n \mathbf{R}$ is equal to

- (A) $nr^n$ (B) $(n + 1)r^n$
- (C) $(n + 3)r^n$ (D) 0

MCQ 1.2.43  If $\mathbf{A} = \rho \sin \phi \mathbf{a}_\rho + \rho^2 \mathbf{a}_\phi$, and $L$ is the contour of figure given below, then circulation $\int_L \mathbf{A} \cdot d\mathbf{l}$ is

- (A) $7\pi + 2$
- (B) $7\pi - 2$
- (C) $7\pi$
- (D) 0

***********
MCQ 1.3.1  The direction of vector $\mathbf{A}$ is radially outward from the origin, with $|\mathbf{A}| = kr^n$, where $r^2 = x^2 + y^2 + z^2$ and $k$ is a constant. The value of $n$ for which $\nabla \cdot \mathbf{A} = 0$ is (A) $-2$ (B) $2$ (C) $1$ (D) $0$

MCQ 1.3.2  If $\mathbf{A} = xy \mathbf{a}_x + x^2 \mathbf{a}_y$, then $\int_C \mathbf{A} \cdot d\mathbf{l}$ over the path shown in the figure is (A) $0$ (B) $\frac{2}{\sqrt{3}}$ (C) $1$ (D) $2\sqrt{3}$

MCQ 1.3.3  If a vector field $\mathbf{V}$ is related to another vector field $\mathbf{A}$ through $\mathbf{V} = \nabla \times \mathbf{A}$, which of the following is true? (Note: $C$ and $S_C$ refer to any closed contour and any surface whose boundary is $C$.) (A) $\int_C \mathbf{V} \cdot d\mathbf{l} = \iint_{S_C} \mathbf{A} \cdot d\mathbf{S}$ (B) $\int_C \mathbf{A} \cdot d\mathbf{l} = \iint_{S_C} \mathbf{V} \cdot d\mathbf{S}$ (C) $\int_C (\nabla \times \mathbf{V}) \cdot d\mathbf{l} = \iint_{S_C} (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$ (D) $\int_C (\nabla \times \mathbf{V}) \cdot d\mathbf{l} = \iint_{S_C} \mathbf{V} \cdot d\mathbf{S}$

MCQ 1.3.4  Consider points A, B, C and D on a circle of radius 2 units as in the shown figure below. The items in List II are the values of $\mathbf{a}_a$ at different points on the circle. Match List I with List II and select the correct answer using the code given below the lists:
List I          List II

a.  A       1.  $a_x$

b.  B       2.  $a_y$

c.  C       3.  $-a_x$

d.  D       4.  $(a_x + a_y)/\sqrt{2}$

5.  $-(a_x + a_y)/\sqrt{2}$

6.  $(a_x - a_y)/\sqrt{2}$

Codes :

a  b  c  d

(A)  3  4  5  2
(B)  1  6  5  2
(C)  1  6  2  4
(D)  3  5  4  2

MCQ 1.3.5

If $V = 2\sinh x \cos y e^{z^2}$ is a solution of Laplace’s equation, what will be the value of $k$?

(A) $\frac{1}{\sqrt{1 + p^2}}$  
(B) $\sqrt{1 + p^2}$

(C) $\frac{1}{\sqrt{1 - p^2}}$  
(D) $\sqrt{1 - p^2}$

MCQ 1.3.6

The electric field intensity $E$ at a point $P$ is given by $10a_x + 10a_y + \|a_z$, where $a_x, a_y$ and $a_z$ are unit vectors in $x, y$ and $z$ directions respectively. If $\alpha, \beta, \gamma$ respectively the angles the $E$ vector makes with $x, y$ and $z$ axes respectively, they are given by which of the following ?

(A) $\alpha = \beta = \gamma = 30^\circ$  
(B) $\alpha = \beta = \gamma = 60^\circ$

(C) $\alpha = \beta = \gamma \cos^{-1} \frac{1}{\sqrt{3}}$  
(D) $\alpha = \beta = \gamma = \cos^{-1} \frac{1}{3}$

MCQ 1.3.7

Which one of the following potentials does NOT satisfy Laplace’s Equation ?

(A) $V = 10xy$  
(B) $V = r \cos \phi$

(C) $V = 10/r$  
(D) $V = \rho \cos \phi + 10$

MCQ 1.3.8

Laplacian of a scalar function $V$ is

(A) Gradient of $V$

(B) Divergence of $V$

(C) Gradient of the gradient of $V$

(D) Divergence of the gradient of $V$

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 1.3.9
A field $\mathbf{A} = 5x^2yz\mathbf{a}_z + 2x\mathbf{a}_y + (x^3y - 2z)\mathbf{a}_x$ can be termed as
(A) Harmonic
(B) Divergence less
(C) Solenoidal
(D) Rotational

MCQ 1.3.10
Laplace equation in cylindrical coordinates is given by
(A) $\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left( \frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = 0$
(B) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$
(C) $\nabla^2 V = -\frac{\rho e}{\varepsilon}$
(D) $\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho^2 \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2 \sin \theta} \frac{\partial}{\partial \phi} = 0$

MCQ 1.3.11
What is the value of the integral $\int dl$ along the curve $c$ ? ($c$ is the curve $ABCD$ in the direction of the arrow)

(A) $2R(a_x + a_y)/\sqrt{2}$
(B) $-2R(a_x + a_y)/\sqrt{2}$
(C) $2Ra_x$
(D) $-2Ra_y$

MCQ 1.3.12
Given a vector field $\mathbf{A} = \cos \phi \mathbf{a}_r$ in cylindrical coordinates. For the contour as shown below, $\oint \mathbf{A} \cdot dl$ is

(A) 1
(B) 1 - ($\pi/2$)
(C) 1 + ($\pi/2$)
(D) -1
MCQ 1.3.13
If a vector field $B$ is solenoidal, which of these is true?

(A) $\oint B \cdot dl = 0$
(B) $\oint B \cdot ds = 0$
(C) $\nabla \times B = 0$
(D) $\nabla \cdot B \neq 0$

MCQ 1.3.14
Which of the following equations is correct?

(A) $a_x \times a_x = \left| a_x \right|^2$
(B) $(a_x \times a_y) + (a_y \times a_z) = 0$
(C) $a_x \times (a_y \times a_z) = a_x \times (a_z \times a_y)$
(D) $a_x \cdot a_y + a_y \cdot a_z = 0$

MCQ 1.3.15
Match List I with List II and select the correct answer:

<table>
<thead>
<tr>
<th>List I (Term)</th>
<th>List II (Type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>curl ($F$) = 0</td>
</tr>
<tr>
<td>b</td>
<td>div ($F$) = 0</td>
</tr>
<tr>
<td>c</td>
<td>div Grad ($\phi$) = 0</td>
</tr>
<tr>
<td>d</td>
<td>div div ($\phi$) = 0</td>
</tr>
</tbody>
</table>

Codes :

(A) 2 3 1 4
(B) 4 1 3 2
(C) 2 1 3 4
(D) 4 3 1 2

MCQ 1.3.16
If $A = 2a_x + a_y + a_z$, the value of $\oint A \cdot dl$ around the closed circular quadrant shown in the given figure is

(A) $\pi$
(B) $\frac{\pi}{2} + 4$
(C) $\pi + 4$
(D) $\frac{\pi}{2} + 2$

**********
SOLUTIONS 1.1

SOL 1.1.1 Option (D) is correct.
Given the vector field has the only component in \( a_q \) direction and its magnitude is \( r \) so as \( r \) increase from origin to the infinity field lines will be larger and directed along \( a_q \) as shown in option (A).

SOL 1.1.2 Option (B) is correct.
\[
M - N = 5a_x - 2a_y + 4a_z - (-8a_x - 7a_y + 2a_z) \\
= 5a_x - 2a_y + 4a_z + 8a_x + 7a_y - 2a_z \\
= 13a_x + 5a_y + 2a_z
\]
So, the unit vector in the direction of \((M - N)\) is
\[
a = \frac{M - N}{|M - N|} = \frac{13a_x + 5a_y + 2a_z}{\sqrt{13^2 + 5^2 + 2^2}} \\
= 0.92a_x + 0.36a_y + 0.14a_z
\]

SOL 1.1.3 Option (C) is correct.
Vector \( \mathbf{G} \) at \((-2,1,3)\):
\[
G = 4(-2)(1)a_x + 2(2 + (-2)^2)a_y + 3(3)^2a_z \\
= -8a_x + 12a_y + 27a_z
\]
So, unit vector in the direction of \( \mathbf{G} \) at \( Q \) :
\[
a_G = \frac{G}{|G|} = \frac{-8a_x + 12a_y + 27a_z}{\sqrt{(-8)^2 + 12^2 + 27^2}} \\
= -0.26a_x + 0.49a_y + 0.88a_z
\]

SOL 1.1.4 Option (B) is correct.
Option (A), (B), (D) are the properties of vector product.
Now we check the relation defined in option (C). Since the triple cross product is not associative in general so, the given relation is incorrect. This inequality can be explained by considering vector \( \mathbf{A} = \mathbf{B} \) and \( \mathbf{C} \) perpendicular to \( \mathbf{A} \) as shown in the figure.
According to right hand rule we determine that \((\mathbf{B} \times \mathbf{C})\) points out of the page and so \( \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \) points down that has magnitude \( \mathbf{ABC} \).
But in L.H.S. of the relation, since \( \mathbf{A} = \mathbf{B} \)
So we have \((\mathbf{A} \times \mathbf{B}) = 0\)
and hence \((\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = 0\)
Therefore \((\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = 0 \neq \mathbf{A} \times (\mathbf{B} \times \mathbf{C})\)
SOL 1.1.5  Option (B) is correct.  
As the direction of cross vector is normal to the plane. So, direction of \( \mathbf{B} \times \mathbf{C} \) will be normal to the plane defined by the three vectors.  
Now the dot product of two mutually perpendicular vectors is always zero and since the direction of \( \mathbf{B} \times \mathbf{C} \) will be perpendicular to the plane of vector \( \mathbf{A} \). So \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \) will be zero.

SOL 1.1.6  Option (C) is correct.  
Consider the two vectors \( \mathbf{A} \) and \( \mathbf{B} \) are as shown below.

\[
\begin{align*}
\text{As the angle between the two vectors is } \alpha. \\
\text{So component of vector } \mathbf{A} \text{ along } \mathbf{B} \text{ is } A_1 = (\cos \alpha)A \\
\text{cosine of the angle between the two vectors is defined as } \\
\cos \alpha = \frac{\mathbf{A} \cdot \mathbf{B}}{AB} \\
\text{So, } A_1 = \left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right)A = \frac{\mathbf{A} \cdot \mathbf{B}}{B}
\end{align*}
\]

SOL 1.1.7  Option (C) is correct.  
Cross product of two parallel vector fields is always zero since the angle between them is \( \theta = 0^\circ \).  
i.e. \( \mathbf{A} \times \mathbf{B} = 0 \)
\[
\begin{vmatrix}
\alpha & \beta & \gamma \\
1 & 2 & 3 \\
\alpha & \beta & -6 \\
\end{vmatrix}
= 0
\]

\[(-12 - 3\beta)\mathbf{a}_\rho + (3\alpha + 6)\mathbf{a}_\phi + (\beta - 2\alpha)\mathbf{a}_z = 0\]

Solving it we have, \(\beta = -4\) and \(\alpha = -2\)

SOL 1.1.8  
Option (A) is correct.

Dot product of the two orthogonal vectors is always zero.  
i.e. \(\mathbf{A} \cdot \mathbf{B} = 0\)

\[(4)(1) + (2\hat{k})(4) + (\hat{k})(-4) = 0\]
\[4 + 8\hat{k} - 4\hat{k} = 0\]
\[4\hat{k} = -4\]
\[\hat{k} = -1\]

SOL 1.1.9  
Option (A) is correct.

From the given field vector we have the component 
\(E_x = 3zy^2\cos 2x\).

So for the given condition \(E_x = 0\)

We have, \(9zy^2\cos(2x) = 0\)

This condition met when, \(z = 0\)

or, \(y = 0\)

or, \(\cos 2x = 0 \Rightarrow 2x = \pi/2 \Rightarrow x = \pi/4\)

Therefore the planes on which field component \(E_x\) will be zero are 
\(z = 0\) and \(x = \pi/4\)

SOL 1.1.10  
Option (A) is correct.

From the given field vector we have the field components 
\(E_y = 2zysin 2x\)

and 
\(E_z = 2y^2\sin 2x\)

Now, in the plane \(y - 4z = 0 \Rightarrow y = 4z\)

So, 
\(E_x = 8z(4z)\sin 2x = 32z^2\sin 2x\)
\(E_y = 2(4z)^2\sin 2x = 32z^2\sin 2x\)

Thus 
\(E_y = E_z\)

SOL 1.1.11  
Option (C) is correct.

For the given condition \(\mathbf{E} = 0\), we must have 
\(E_x = E_y = E_z = 0\)

i.e. \(9zy^2\cos 2x = 8zysin 2x = 2y^2\sin 2x = 0\)

This condition met in the plane \(y = 0\).

SOL 1.1.12  
Option (B) is correct.

Since the two points are defined in different coordinate system so we represent the point \(Q\) in Cartesian system as 
\(x = \rho \cos \phi = 4 \cos(-50) = 2.57\)
For View Only
Shop Online at www.nodia.co.in

\[ y = \rho \sin \phi = 4 \sin(-50) = -3.064 \]
and
\[ z = 2 \]

So, the distance between the two points \( P(2,3,-1) \) and \( Q(2.57, -3.064, 2) \) is given as

\[
|PQ| = \sqrt{(2.57 - 2)^2 + (-3.064 - 3)^2 + (2 + 1)^2} = 6.78 \text{ units}
\]

SOL 1.1.13
Option (A) is correct.
Since, the options include spherical as well as cylindrical representation of \( \mathbf{A} \), so, we will transform the vector in both the forms to check the result.
The components of vector field \( \mathbf{A} \) are
\[ A_x = 1, \ A_y = 0 \text{ and } A_z = 0 \]

Now, we transform the vector components in cylindrical system as
\[
\begin{bmatrix}
A_r \\
A_\phi \\
A_z
\end{bmatrix} = \begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
\]
So, we get
\[ A_r = (\cos \phi)(1) = \cos \phi \]
\[ A_\phi = (-\sin \phi)(1) = -\sin \phi \]
\[ A_z = 0 \]

Thus, the vector field in cylindrical system is
\[ \mathbf{A}(\rho, \phi, z) = \cos \phi \mathbf{a}_r - \sin \phi \mathbf{a}_\phi \]

Hence, both the options (A) and (B) are incorrect.

Again, we transform the vector components in spherical system as
\[
\begin{bmatrix}
A_r \\
A_\theta \\
A_\phi
\end{bmatrix} = \begin{bmatrix}
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\
\cos \theta \cos \phi & \cos \theta \sin \phi & \sin \theta \\
\sin \phi & \cos \phi & 0
\end{bmatrix} \begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
\]
So, we get
\[ A_r = (\sin \theta \cos \phi)(1) = \sin \theta \cos \phi \]
\[ A_\theta = (\cos \theta \cos \phi)(1) = \cos \theta \cos \phi \]
\[ A_\phi = (-\sin \phi)(1) = -\sin \phi \]

Thus, the vector field in spherical system is
\[ \mathbf{A}(r, \theta, \phi) = \sin \theta \cos \phi \mathbf{a}_r + \cos \theta \cos \phi \mathbf{a}_\theta - \sin \phi \mathbf{a}_\phi \]

SOL 1.1.14
Option (C) is correct.

We transform the given vector field in spherical system.
Since the given vector field is \( \mathbf{F} = 10\mathbf{a}_z \)
The Cartesian components of the field are \( F_x = 10, \ F_y = 0, \ F_z = 0 \).

So, the spherical components of vector field can be determined as
\[
\begin{bmatrix}
F_r \\
F_\theta \\
F_\phi
\end{bmatrix} = \begin{bmatrix}
\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\
\cos \theta \cos \phi & \cos \theta \sin \phi & \sin \theta \\
-\sin \theta & \cos \phi & 0
\end{bmatrix} \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
\]
So, we get
\[ F_r = 10 \sin \theta \cos \phi \]
\[ F_\theta = 10 \cos \theta \cos \phi \]
and
\[ F_\phi = -10 \sin \theta \]
Now, for the given point \((x = 3, y = 2, z = -1)\) we have

\[
r = \sqrt{(3)^2 + (2)^2 + (-1)^2} = \sqrt{14}
\]

\[
\theta = \tan^{-1}\left(\frac{x}{y}\right) = \tan^{-1}\left(\frac{\sqrt{(3)^2 + (2)^2}}{-1}\right) = 105.5^\circ
\]

\[
\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{3}\right) = 33.7^\circ
\]

Putting all the values in the matrix transformation, we have

\[
F_r = 5\sin(105.5^\circ)\cos(33.7^\circ) = 8
\]

\[
F_\theta = 10\cos(105.5^\circ)\cos(33.7^\circ) = -2.2
\]

\[
F_\phi = -10\sin(33.7^\circ) = -5.5
\]

Therefore, the vector field in spherical coordinate is

\[
\mathbf{F} = F_r\mathbf{a}_r + F_\theta\mathbf{a}_\theta + F_\phi\mathbf{a}_\phi
\]

\[
= 8\mathbf{a}_r - 2.2\mathbf{a}_\theta - 5.5\mathbf{a}_\phi
\]

**SOL 1.1.15** Option (C) is correct.

Since \(z\)-axis is normal to the surface \(z = 5\), so first of all we will find the angle between \(z\)-axis and \(\mathbf{A}\) which can be easily obtained from the figure shown below:

\[
\cos \phi = \frac{A_z}{|\mathbf{A}|} = \frac{3}{\sqrt{(-5)^2 + (10)^2 + (3)^2}} = \frac{3}{\sqrt{134}}
\]

\[
\phi = \cos^{-1}\left(\frac{3}{\sqrt{134}}\right) \approx 75^\circ
\]

Therefore, the angle between surface \(z = 5\) and vector \(\mathbf{A}\) is \((90^\circ - \phi) = 15^\circ\).

**SOL 1.1.16** Option (B) is correct.

The given line \(x = 6, z = -2\) is parallel to \(y\)-axis. So, the component of \(\mathbf{A}\) parallel to the given line is

\[
A_y = (\mathbf{A} \cdot \mathbf{a}_y)\mathbf{a}_y
\]

\[
= \left[(-2a_x + 20a_\phi + 4a_z) \cdot a_y\right]a_y
\]

At point \(P, \phi = 90^\circ\), so, \(A_y = -2a_\phi\)

**SOL 1.1.17** Option (A) is correct.

The given point is shown below:
After 120° rotation looking down the axis the new coordinate axes \((x', y', z')\) will be as shown below:

So, the rotation carries \(z\) axis into \(y\); \(y\)-axis into \(x\) and \(x\) into \(z\).

Therefore the new coordinates of point \(P\) are:

\[
\begin{align*}
x' &= z = c \\
y' &= x = a \\
z' &= y = b
\end{align*}
\]

i.e. \((c, a, b)\) is the coordinates of point \(P\) in the transformed system.

**SOL 1.1.18**

Option (B) is correct.

The position vector can be defined as:

\[
R = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z
\]

\[
R = \sqrt{x^2 + y^2 + z^2}
\]

So,

\[
\text{grad}R = \frac{\partial R}{\partial x} \mathbf{a}_x + \frac{\partial R}{\partial y} \mathbf{a}_y + \frac{\partial R}{\partial z} \mathbf{a}_z
\]

\[
= \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2 + z^2}} \mathbf{a}_x + \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2 + z^2}} \mathbf{a}_y + \frac{1}{2} \frac{2z}{\sqrt{x^2 + y^2 + z^2}} \mathbf{a}_z
\]

\[
= \frac{x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z}{\sqrt{x^2 + y^2 + z^2}} = \frac{R}{R}
\]
SOL 1.1.19 Option (D) is correct.

\[ \nabla \cdot \mathbf{A} = \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(2xy + x^2 + z^2) + \frac{\partial}{\partial z}(4x + 2yz) = 0 + 2x + 2y = 2(x + y) \]

SOL 1.1.20 Option (A) is correct.

We have the vector field components as 
\[ F_{\rho} = \rho \sin \phi, \quad F_{\theta} = \rho^2 z \quad \text{and} \quad F_{z} = \cos \phi \]

Now, 
\[ \nabla \times \mathbf{F} = \frac{1}{\rho} \begin{vmatrix} a_{\rho} & \rho a_{\phi} & a_{z} \\ \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ F_{\rho} & \rho F_{\theta} & F_{z} \end{vmatrix} \]

\[ = \frac{1}{\rho} \left[ \frac{\partial}{\partial \theta} \cos \phi + \frac{\partial}{\partial z} \rho \cos \phi \right] a_{\rho} - \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} \rho \sin \phi - \frac{\partial}{\partial z} \rho \cos \phi \right] a_{\phi} + \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} \rho \cos \phi - \frac{\partial}{\partial \phi} \rho \sin \phi \right] a_{z} \]

\[ = \frac{1}{\rho} \left[ -z \sin \phi - \rho^2 \right] a_{\rho} - [0 - 0] a_{\phi} + \frac{1}{\rho} \left[ 3 \rho^2 \cos \phi - \rho \cos \phi \right] a_{z} \]

At point \( P(1, \pi/2, 2) \)
\[ \nabla \times \mathbf{F} = -1 \left( 2 \times 1 + 1^3 \right) a_{\rho} + (3 \times 1 \times 2 - 0) a_{\phi} = -3a_{\rho} + 6a_{\phi} \]

SOL 1.1.21 Option (A) is correct.

\[ \mathbf{D} = yz \mathbf{a}_x + xz \mathbf{a}_y + xy \mathbf{a}_z \]

\[ \nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy) = 0 \]

\[ \nabla \times \mathbf{D} = \begin{vmatrix} a_{x} & a_{y} & a_{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = \left( x - z \right) a_{x} - \left( y - z \right) a_{y} + \left( x - z \right) a_{z} = 0 \]

SOL 1.1.22 Option (D) is correct.

Curl of the unit vector \( \mathbf{a}_\rho \) is

\[ \nabla \times \mathbf{a}_{\rho} = \frac{1}{\rho} \begin{vmatrix} a_{\rho} & \rho a_{\phi} & a_{z} \\ \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 1 & 0 & 0 \end{vmatrix} = 0 \]

The curl of unit vector \( \mathbf{a}_\phi \) is

\[ \nabla \times \mathbf{a}_{\phi} = \frac{1}{\rho} \begin{vmatrix} a_{\rho} & \rho a_{\phi} & a_{z} \\ \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & 0 & \rho \end{vmatrix} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \mathbf{a}_z = \frac{a_z}{\rho} \]

and curl of unit vector \( \mathbf{a}_z \) is

\[ \nabla \times \mathbf{a}_{z} = \frac{1}{\rho} \begin{vmatrix} a_{\rho} & \rho a_{\phi} & a_{z} \\ \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ 0 & 0 & 1 \end{vmatrix} = 0 \]
SOL 1.1.23 Option (A) is correct.
Options (A), (B) and (C) are properties of \( \nabla \) operator where as:
\[
\nabla \times \left( \frac{\mathbf{A}}{g} \right) = g \left( \nabla \times \mathbf{A} \right) + \mathbf{A} \times \left( \nabla g \right)
\]

SOL 1.1.24 Option (A) is correct.
In a cylindrical coordinate system Laplacian of a scalar field is defined as
\[
\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}
\]
\[
= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( 2 \rho \sin \phi + 8 \rho \right) + \frac{1}{\rho^2} \left( -2 \rho \sin \phi - 6 \rho^2 \frac{\partial}{\partial \phi} \sin \phi \cos \phi \right)
\]
\[
+ \frac{\partial}{\partial z} \left( 2 \rho \sin \phi + 6 \rho \cos^2 \phi \right)
\]
\[
= \frac{1}{\rho} \left( 2 \rho \sin \phi + 16 \rho \right) - \frac{1}{\rho^2} \left( 2 \rho \sin \phi + 6 \rho \cos 2 \phi \right) + 6 \cos \phi
\]
\[
= 16 + 6 \cos^2 \phi - \frac{6 \rho^2}{\rho^2} \cos 2 \phi
\]
At point \( P(3, \pi/2, 6) \)
\[
\nabla^2 f = 16 + 0 - \frac{6 \times 36}{9} \times (-1) = 40
\]

SOL 1.1.25 Option (A) is correct.
A vector field is called conservative (irrotational) if its curl is zero.
i.e.
\[
\nabla \times \mathbf{M} = 0
\]
\[
\begin{vmatrix}
\frac{\partial a_z}{\partial x} & \frac{\partial a_y}{\partial y} & \frac{\partial a_z}{\partial z} \\
\cos xz + y & 2kz & \cos xz \\
\end{vmatrix} = 0
\]
\[
a_z(0 - 0) - a_y(\cos xz - xz \sin xz - \cos xz + xz \sin xz) + (2k - 1) a_z = 0
\]
\[
2k - 1 = 0
\]
\[
k = \frac{1}{2}
\]

SOL 1.1.26 Option (B) is correct.
For a scalar field to be harmonic,
\[
\nabla^2 g = 0
\]
\[
\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} + \frac{\partial^2 g}{\partial z^2} = 0
\]
\[
2(1 + 2k)y = 0 \text{ which results in } k = -\frac{1}{2}
\]

SOL 1.1.27 Option (D) is correct.
Consider \( V \) is a scalar field. So the gradient of the field is
\[
\nabla V = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}
\]
and the curl of the gradient of the field is
\[ \nabla \times ( \nabla V ) = \begin{bmatrix} \frac{\partial}{\partial x} V \\ \frac{\partial}{\partial y} V \\ \frac{\partial}{\partial z} V \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \]

\[ = \left( \frac{\partial^2 V}{\partial z \partial y} - \frac{\partial^2 V}{\partial y \partial z} \right) a_x + \left( \frac{\partial^2 V}{\partial x \partial z} - \frac{\partial^2 V}{\partial z \partial x} \right) a_y + \left( \frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 V}{\partial y \partial x} \right) a_z = 0 \]

So the curl of the gradient of any scalar field is zero everywhere.

**SOL 1.1.28** Option (D) is correct.

For a vector \( \mathbf{A} \) to be solenoidal its divergence must be zero.

\[ \nabla \cdot \mathbf{A} = 0 \]

\[ \frac{\partial}{\partial x}(x + 4z) + \frac{\partial}{\partial y}(2x - 3z) + \frac{\partial}{\partial z}(4x + 3y - c^2) = 0 \]

\[ 1 + 0 - c = 0 \]

\[ c = 1 \]

**SOL 1.1.29** Option (B) is correct.

For a vector function to be irrotational its curl must be zero.

\[ \nabla \times \mathbf{A} = 0 \]

\[ \begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = 0 \]

\[ \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (4x + k_1 z) & (k_2 x - 5 z) & (4x - k_3 y + 2 z) \end{vmatrix} = 0 \]

\[ (- k_3 - (- 5)) a_z - (4 - k_1) a_y + (k_2) a_x = 0 \]

\[ - k_3 - (- 5) = 0 \Rightarrow k_3 = 5 \]

\[ 4 - k_1 = 0 \Rightarrow k_1 = 4 \]

and \( k_2 - 0 = 0 \Rightarrow k_2 = 0 \)

So \( k_1, k_2 \) and \( k_3 \) are 4, 0 and 5 respectively.

**SOL 1.1.30** Option (C) is correct.

The unit vector normal to a given plane \( f = 0 \) is

\[ \mathbf{a}_n = \frac{\nabla f}{|\nabla f|} \]

The given equation is

\[ 2x + 4y + 6z = 7 \]

\[ 2x + 4y + 6z - 7 = 0 \]

So,

\[ f = 2x + 4y + 6z - 7 \]

and gradient of \( f \) is

\[ \nabla f = -2a_x + 3a_y + 6a_z \]

\[ |\nabla f| = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{56} \]

So,

\[ a_n = \frac{1}{\sqrt{56}}(2a_x + 4a_y + 6a_z) = \frac{1}{\sqrt{14}}(a_x + 2a_y + 3a_z) \]
SOL 1.1.31 Option (D) is correct.
Consider the differential displacement,
\[ dl = dx \mathbf{a}_x + dy \mathbf{a}_y + dz \mathbf{a}_z \]
So
\[ \oint_C dl = \left( \oint_C dx \right) \mathbf{a}_x + \left( \oint_C dy \right) \mathbf{a}_y + \left( \oint_C dz \right) \mathbf{a}_z \]
For a contour the initial and final points are same. So, all the individual integrals described above will be zero. Therefore,
\[ \oint_C dl = 0 \]

SOL 1.1.32 Option (B) is correct.
According to stoke's theorem.
\[ \oint_C \mathbf{A} \cdot dl = \int (\nabla \times \mathbf{A}) \cdot dS \]
So
\[ \oint_C (\nabla \mathbf{U}) \cdot dl = \int [\nabla \times (\nabla \mathbf{U})] \cdot dS \]
Since \( \nabla \times (\nabla \mathbf{U}) = 0 \) (curl of the gradient of a scalar field is always zero)
So the contour integral is zero.

SOL 1.1.33 Option (B) is correct.
According to the divergence theorem surface integral of vector over a closed surface is equal to the volume integral of its divergence inside the region defined by closed surface.
i.e.
\[ \int_S \mathbf{A} \cdot dS = \int_V (\nabla \cdot \mathbf{A}) dv \]
Now,
\[ \nabla \cdot \mathbf{A} = \frac{\partial}{\partial x} (2yz) + \frac{\partial}{\partial y} (x^2) + \frac{\partial}{\partial z} (2xy) \]
\[ = 0 + 0 + 0 \]
So
\[ \oint \mathbf{A} \cdot dS = 0 \]
SOL 1.2.1 Option (D) is correct.
Consider that the cube has its edges on the $x$, $y$ and $z$-axes respectively as shown in the figure. As the angle between any of the two body diagonals of the cube will be same so we determine the angle $\theta$ between the diagonals $\mathbf{OB}$ and $\mathbf{AC}$ of the cube.

From the figure we get the co-ordinates of points $A$, $B$ and $C$ as:
$A \rightarrow (0,0,3)$  $B \rightarrow (3,3,3)$ and $C \rightarrow (3,3,0)$

So, the vector length, $\mathbf{OB} = 3\mathbf{a}_x + 3\mathbf{a}_y + 3\mathbf{a}_z$
and
$\mathbf{AC} = 3\mathbf{a}_x + 3\mathbf{a}_y - 3\mathbf{a}_z$

For determining the angle $\theta$ between them, we take their dot product as

$$\mathbf{OB} \cdot \mathbf{AC} = |\mathbf{OB}| \cdot |\mathbf{AC}| \cos \theta$$

$$9 + 9 - 9 = (3\sqrt{3})(3\sqrt{3}) \cos \theta$$

So, the angle formed between the diagonals is

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.53^\circ$$

SOL 1.2.2 Option (C) is correct.
For the given points $A$, $B$, $C$, the vector length,
and
\[ \mathbf{AC} = -10\mathbf{a}_x + 8\mathbf{a}_y + 15\mathbf{a}_z \]

Since the cross product of two vectors is always perpendicular to the plane of vectors. So the unit vector perpendicular to the plane of triangle is given by
\[ \mathbf{a}_n = \frac{\mathbf{AB} \times \mathbf{AC}}{\left| \mathbf{AB} \times \mathbf{AC} \right|} \]
now,
\[ \mathbf{AB} \times \mathbf{AC} = \begin{vmatrix} a_x & a_y & a_z \\ 20 & 18 & -10 \\ -10 & 8 & 15 \end{vmatrix} \]
\[ = \left[ 18 \times 15 - (-10 \times 8) \right] \mathbf{a}_x - \left[ 20 \times 15 - (-10 \times -10) \right] \mathbf{a}_y + \left[ 20 \times 8 - (-10 \times 15) \right] \mathbf{a}_z \]
\[ = 350\mathbf{a}_x - 200\mathbf{a}_y + 340\mathbf{a}_z \]
So,
\[ \mathbf{a}_n = \frac{350\mathbf{a}_x - 200\mathbf{a}_y + 340\mathbf{a}_z}{\sqrt{(350)^2 + (-200)^2 + (340)^2}} \]
\[ = \frac{0.664\mathbf{a}_x - 0.379\mathbf{a}_y + 0.645\mathbf{a}_z}{0.550\mathbf{a}_x - 0.832\mathbf{a}_y + 0.207\mathbf{a}_z} \]

\[ \text{SOL 1.2.3} \]
Option (D) is correct.
The unit vector in the direction of vector \( \mathbf{AC} \) is given by
\[ \mathbf{a}_{AC} = \frac{\mathbf{AC}}{\left| \mathbf{AC} \right|} = \frac{-10\mathbf{a}_x + 8\mathbf{a}_y + 15\mathbf{a}_z}{\sqrt{(-10)^2 + (8)^2 + (15)^2}} \]
\[ = -0.507\mathbf{a}_x + 0.406\mathbf{a}_y + 0.761\mathbf{a}_z \]
\[ = -0.61\mathbf{a}_x + 0.41\mathbf{a}_y + 0.76\mathbf{a}_z \]
Since the cross product of two vectors is always perpendicular to the plane of vectors. So, the unit vector in the plane of the triangle which is perpendicular to \( \mathbf{AC} \) is given by cross product of the unit vector perpendicular to the plane of the triangle and the unit vector \( \mathbf{a}_{AC} \), i.e.
\[ \mathbf{a}_p = \mathbf{a}_n \times \mathbf{a}_{AC} = 0.550\mathbf{a}_x - 0.832\mathbf{a}_y + 0.077\mathbf{a}_z \]

\[ \text{SOL 1.2.4} \]
Option (A) is correct.
Unit vector in the direction of \( \mathbf{AB} \) is given by
\[ \mathbf{a}_{AB} = \frac{-10\mathbf{a}_x + 18\mathbf{a}_y - 10\mathbf{a}_z}{\sqrt{(-10)^2 + (18)^2 + (-10)^2}} \]
\[ = 0.697\mathbf{a}_x + 0.627\mathbf{a}_y - 0.348\mathbf{a}_z \]
A non unit vector in the direction of bisector of interior angle at \( \mathbf{A} \) is defined as
\[ \frac{1}{2}(\mathbf{a}_{AB} + \mathbf{a}_{AC}) = \frac{1}{2}[0.697\mathbf{a}_x + 0.627\mathbf{a}_y - 0.348\mathbf{a}_z - 0.507\mathbf{a}_x + 0.406\mathbf{a}_y + 0.761\mathbf{a}_z] \]
\[ = 0.095\mathbf{a}_x + 0.516\mathbf{a}_y + 0.207\mathbf{a}_z \]
So the unit vector in the direction of bisector of interior angle at \( \mathbf{A} \) is given by
\[ \mathbf{a}_{bis} = \frac{0.095\mathbf{a}_x + 0.516\mathbf{a}_y + 0.207\mathbf{a}_z}{\sqrt{(0.095)^2 + (0.516)^2 + (0.207)^2}} \]
\[ = 0.168\mathbf{a}_x + 0.915\mathbf{a}_y + 0.367\mathbf{a}_z \]
SOL 1.2.5
Option (B) is correct.

The vector field \( \mathbf{F} \) can be written in cartesian system as

\[
\mathbf{F}(x, y, z) = \frac{x}{x^2 + y^2} \mathbf{a}_x + \frac{y}{x^2 + y^2} \mathbf{a}_y
\]

\[
\mathbf{F}(x, y, z) = \frac{\rho \cos \phi}{\rho^2} \mathbf{a}_x + \frac{\rho \sin \phi}{\rho^2} \mathbf{a}_y
\]

\[
(x = \rho \cos \phi, y = \rho \sin \phi)
\]

The components of vector field \( \mathbf{F} \) are

\[
F_x = \frac{1}{\rho} \cos \phi, \quad F_y = \frac{1}{\rho} \sin \phi \quad \text{and} \quad F_z = 0
\]

So the components of vector field \( \mathbf{F} \) in cylindrical system can be expressed as

\[
\begin{bmatrix}
F_r \\
F_\theta \\
F_z
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
\]

\[
F_r = \frac{1}{\rho} \left[ \cos^2 \phi + \sin^2 \phi \right] = \frac{1}{\rho}
\]

\[
F_\theta = \frac{1}{\rho} \left[ \cos \phi (-\sin \phi) + \sin \phi \cos \phi \right] = 0
\]

\[
F_z = 0
\]

So the vector field \( \mathbf{F}(\rho, \phi, z) = \frac{1}{\rho} \mathbf{a}_\rho \)

At the point \( P(\rho = 2, \phi = \pi/4, z = 0.1) \)

\[
\mathbf{F} = \frac{1}{2} \mathbf{a}_\rho = 0.5 \mathbf{a}_\rho
\]

SOL 1.2.6
Option (D) is correct.

Any vector field can be represented as the sum of its normal and tangential component to any surface as

\[
\mathbf{A} = \mathbf{A}_t + \mathbf{A}_n
\]

where \( \mathbf{A}_t \) is tangential component and \( \mathbf{A}_n \) is normal component to the surface \( r = 20 \) at point \( P(20, 150^\circ, 330^\circ) \).

So,

\[
\mathbf{A}_n = r \mathbf{a}_r = 20 \mathbf{a}_r
\]

and therefore,

\[
\mathbf{A}_t = \mathbf{A} - \mathbf{A}_n = -\frac{2}{r} \sin \theta \cos \phi \mathbf{a}_\theta + \frac{r^3}{4} \mathbf{a}_\phi
\]

\[
= 0.043 \mathbf{a}_\theta + 100 \mathbf{a}_\phi
\]

SOL 1.2.7
Option (A) is correct.

Any vector field can be represented as the sum of its normal and tangential components to any surface as

\[
\mathbf{A} = \mathbf{A}_t + \mathbf{A}_n
\]

Here \( \mathbf{A}_t \) and \( \mathbf{A}_n \) are the tangential and normal components to the conical surface.
Since the unit vector normal to the conical surface is \( \mathbf{a}_n \).
So, \( \mathbf{A}_n = -8 \mathbf{a}_n \)
and therefore the tangential component to the cone is
\[
\mathbf{A}_t = \mathbf{A} - \mathbf{A}_n = -12 \mathbf{a}_t + 9 \mathbf{a}_n
\]

SOL 1.2.8 Option (B) is correct.
Consider the unit vector perpendicular to \( \mathbf{A} \) and tangent to the cone \( \theta = 150^\circ \) is
\( \mathbf{b} = b_r \mathbf{a}_r + b_\theta \mathbf{a}_\theta \) (Tangential component to the cone will have \( b_\phi = 0 \))
Now the magnitude of unit vector is 1
So, \( b_r^2 + b_\theta^2 = 1 \)  
and the dot product of mutually perpendicular vectors is zero.
So, \( \mathbf{A} \cdot \mathbf{b} = 0 \)
\[
-6b_r + 9b_\theta = 0
\]
\( b_\theta = \frac{3}{4} b_r \)  
....(i)
So, from equation (i) and (ii) we have
\[
b_r^2 \left( 1 + \frac{16}{9} \right) = 1
\]
\[
b_r = \frac{3}{5}, \quad b_\theta = \frac{4}{5}
\]
Therefore, \( \mathbf{b} = \frac{1}{9} (3 \mathbf{a}_r + 4 \mathbf{a}_\theta) \)

SOL 1.2.9 Option (D) is correct.
The separation vector \( \mathbf{R} \) can be defined as :
\[
\mathbf{R} = (x-a) \mathbf{a}_x + (y-b) \mathbf{a}_y + (z-c) \mathbf{a}_z
\]
and \( \mathbf{R} = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} \)
So,
\[
\nabla \left( \frac{1}{\mathbf{R}} \right) = \frac{\partial}{\partial x} [(x-a)^2 + (y-b)^2 + (z-c)^2]^{-1/2} \mathbf{a}_x
\]
\[
+ \frac{\partial}{\partial y} [(x-a)^2 + (y-b)^2 + (z-c)^2]^{-1/2} \mathbf{a}_y
\]
\[
+ \frac{\partial}{\partial z} [(x-a)^2 + (y-b)^2 + (z-c)^2]^{-1/2} \mathbf{a}_z
\]
\[
= -\frac{1}{2} \mathbf{R}^{-3/2} (x-a) \mathbf{a}_x - \frac{1}{2} \mathbf{R}^{-3/2} (y-b) \mathbf{a}_y - \frac{1}{2} \mathbf{R}^{-3/2} (z-c) \mathbf{a}_z
\]
\[
= \frac{x-a}{R^{3/2}} \mathbf{a}_x + \frac{y-b}{R^{3/2}} \mathbf{a}_y + \frac{z-c}{R^{3/2}} \mathbf{a}_z = -\frac{\mathbf{R}}{R^2}
\]

SOL 1.2.10 Option (C) is correct.
The gradient of a scalar field at its maxima is zero. So at the top of hill
\( \nabla h = 0 \)
or, \( (12x - 4y + 36) \mathbf{a}_x + (16y - 4x - 56) \mathbf{a}_y = 0 \)
Therefore both the components will be equal to zero
i.e., \( 12x - 4y + 36 = 0 \)
and, \( 16y - 4x - 56 = 0 \)
Solving the two equations, we get,
\[ x = -2, \ y = 3 \]
Thus the top of the hill is located at 2 miles south (−2 miles north) and 3 miles
east of the railway station.

**SOL 1.2.11** Option (C) is correct.
Consider the position vector of point \( P \) is
\[ \mathbf{R} = x^4 \mathbf{a}_x + 5y \mathbf{a}_y + z \mathbf{a}_z \]
So, the magnitude of \( \mathbf{R} \) is
\[ R = \sqrt{x^4 + y^2 + z^2} \]
and unit vector in the direction of \( \mathbf{R} \) is
\[ \mathbf{a}_R = \frac{x \mathbf{a}_x + y \mathbf{a}_y + z \mathbf{a}_z}{\sqrt{x^4 + y^2 + z^2}} \]
Therefore, the vector field \( \mathbf{F} \) at point \( P \) is
\[ \mathbf{F} = \frac{10}{R^2} \mathbf{a}_R = \frac{10}{R^2} \left( \frac{x \mathbf{a}_x + y \mathbf{a}_y + z \mathbf{a}_z}{\sqrt{x^4 + y^2 + z^2}} \right) \]
The divergence of the field \( \mathbf{F} \) is given as
\[ \nabla \cdot \mathbf{F} = 10 \left| \frac{\partial}{\partial x} \left( \frac{x}{(x^4 + y^2 + z^2)^{3/2}} \right) + \frac{\partial}{\partial y} \left( \frac{y}{(x^4 + y^2 + z^2)^{3/2}} \right) + \frac{\partial}{\partial z} \left( \frac{z}{(x^4 + y^2 + z^2)^{3/2}} \right) \right| 
= 10 \left[ \frac{1}{(x^4 + y^2 + z^2)^{3/2}} - 3 \frac{x^2}{2(x^4 + y^2 + z^2)^{5/2}} + \frac{1}{(x^4 + y^2 + z^2)^{3/2}} \frac{y^2}{2(x^4 + y^2 + z^2)^{3/2}} \right] 
= 10 \left[ \frac{3}{R^3} - 3 \frac{(x^4 + y^2 + z^2)}{(x^4 + y^2 + z^2)^{5/2}} \right] 
= 10 \left[ \frac{3}{R^3} - 3 \frac{1}{R} \right] = 0 \]
But at origin \( (x = 0, \ y = 0, \ z = 0) \) the position vector \( \mathbf{R} = 0 \) and so the expression for field \( \mathbf{F} \) blows up. Therefore, \( \nabla \cdot \mathbf{F} \) is infinite at origin and zero else where.

**SOL 1.2.12** Option (D) is correct.
The circulation of \( \mathbf{A} \) around the route is given by
\[ \oint \mathbf{A} \cdot d\ell = \left( \int_{\gamma_1} + \int_{\gamma_2} + \int_{\gamma_3} \right) \mathbf{A} \cdot d\ell \]
where the route is broken into segments numbered 1 to 3 as described below
1st segment : \((0,0,0) \rightarrow (2,0,0)\)
x changes from 0 to 2, \( y = 0, \ z = 0 \)
So,
\[ \int_{\gamma_1} \mathbf{A} \cdot d\ell = \int_0^2 3x \, dx = 3 \left[ \frac{x^3}{3} \right]_0^2 = 8 \quad (d\ell = dx \mathbf{a}_x) \]
2nd segment : \((2,0,0) \rightarrow (2,2,0)\)
x = 2, \( y \) changes from 0 to 2, \( z = 0 \)
\[ \int_{\gamma_2} \mathbf{A} \cdot d\ell = \int_0^2 6yz \, dy = 0 \quad (d\ell = dy \mathbf{a}_y) \]
3rd segment : \((2,2,0) \rightarrow (2,2,2)\)
\[ x = 2, \ y = 2, \ z \text{ changes from 0 to 2}. \]
\[
\int A \cdot dl = \int_0^2 3y^2 \, dz = (3 \times 4)\, z^3 \Bigg|_0^2 = 24
\]
\((dl = dz\, a_z)\)

So total line integral will be:
\[
\int A \cdot dl = 8 + 0 + 24 = 32 \text{ units}
\]

**SOL 1.2.13** Option (A) is correct.

For the straight line from origin to the point \((2, 2, 2)\) we have the relation between the coordinates as
\[ x = y = 2z \]

or,
\[ dx = dy = dz \]

and the line integral along straight line is given as
\[ dl = dx\, a_x + dy\, a_y + dz\, a_z \]

Therefore, the line integral of the vector field along the straight line is given as
\[
\int A \cdot dl = \int 3x^2 \, dx + \int 6y \, dy + \int 3z^2 \, dz
\]
\[
= \int 3x^2 \, dx + \int 6x^2 \, dx + \int 3x^2 \, dx
\]
\[
= 12 \int x^2 \, dx = \left(12 \frac{x^3}{3}\right) \Bigg|_0^2 = \left(4x^3\right) \Bigg|_0^2
\]
\[
= 3 \times 8 = 24 \text{ units}
\]

**SOL 1.2.14** Option (C) is correct.

For the closed path defined,
the line integral in forward path = 32 units
the line integral in return path = −32 units.
So, total integral in the closed path is:
\[
\int A \cdot dl = 32 - 32 = 0 \text{ units}
\]

**SOL 1.2.15** Option (A) is correct.

The circulation of \( A \) around the path \( L \) can be given as
\[
\oint A \cdot dl = \left(\int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3}\right) A \cdot dl
\]
where the route is broken into segments numbered 1 to 3 as shown in figure below:

1st segment: \((\phi = 30^\circ, \ z = 2, \ 0 \leq \rho \leq 5)\) and \(dl = d\rho a_\rho\)
So, 
\[ \int A \cdot dl = \int_{0}^{\phi} \rho \sin \phi d\rho = \int_{0}^{\phi} \frac{\rho}{2} d\rho = \frac{25}{4} \quad (\phi = 30^\circ) \]

2nd segment: \((\rho = 5, z = 2, 30^\circ \leq \phi \leq 180^\circ)\) and \(dl = d\phi a_\phi\)

So, 
\[ \int A \cdot dl = \int_{0}^{\phi} 0 d\phi = 0 \]

3rd segment: \((\phi = 180^\circ, z = 2, 5 \leq \rho \leq 0)\) and \(dl = d\rho a_\rho\)

\[ \int A \cdot dl = \int_{0}^{\phi} \rho \sin \phi d\rho = 0 \quad (\phi = 180^\circ) \]

Therefore, the circulation of vector field along the edge \(L\) is

\[ \int A \cdot dl = \frac{25}{4} + 0 + 0 = \frac{25}{4} \text{ units} \]

SOL 1.2.16 Option (D) is correct.

Volume integral of the function is given by 
\[ V = \iiint f \, dx dy dz = \iiint 30z^2 \, dx dy dz \]

The surface of the tetrahedron will have a slope 
\[ x + y + z = -1 \]

So, for a given value of \(y\) and \(z\), \(x\) varies from 0 to \((-1 - y - z)\) and \(x\) integral will be

\[ \int_{0}^{(-1 - y - z)} dx = -1 - y - z \]

again for a given value of \(z\), \(y\) ranges from 0 to \((-1 - z)\). So \(y\)-integral will be:

\[ \int_{0}^{(-1 - z)} (-1 - y - z) \, dy = \left[ (-1 - z) y - \frac{y^2}{2} \right]_{0}^{(-1 - z)} \]

\[ = (-1 - z)^2 - \frac{(-1 - z)^2}{2} = \frac{(-1 - z)^2}{2} \]

\[ = \frac{1}{2} + \frac{z^2}{2} \]

Now there is only one remaining variable \(z\) that ranges from -1 to 0. So we have the volume integral of the function as

\[ V = \int_{-1}^{0} 30z^2 \left( \frac{1}{2} + z + \frac{z^2}{2} \right) \, dz \]

\[ = 30 \left[ \frac{z^3}{6} + \frac{z^4}{4} + \frac{z^5}{10} \right]_{-1}^{0} \]

\[ = 30 \left[ 0 + \frac{1}{6} - \frac{1}{4} + \frac{1}{10} \right] \]

\[ = 30 \times \frac{1}{20} = \frac{3}{2} \]

SOL 1.2.17 Option (D) is correct.

The net outward flux through the closed cylindrical surface will be summation of the fluxes through the top(in \(a_z\) direction), bottom(in \(-a_z\) direction) and the curved surfaces(in \(a_\rho\) direction) as shown in the figure.
Since, the vector field has no \( z \)-component so, the outward flux through the top and bottom surfaces will be zero. Therefore, the total outward flux through the closed cylindrical surface will be only due to the field component in \( a_r \) direction (flux through the curved surfaces) which is given as

\[
\int A \cdot dS = \int_{\rho=0}^{2} \int_{\phi=0}^{2\pi} (A_r) (\rho d\phi dz) = \int_{\rho=0}^{2} \int_{\phi=0}^{2\pi} \left( \rho^2 \cos^2 \phi \right) (\rho d\phi dz)
\]

At \( \rho = 2 \),

\[
\int A \cdot dS = \left( \frac{2}{4} \right) \left[ \int_0^{\pi} \cos^2 \phi d\phi \right] \left[ \int_0^{2\pi} d\phi \right]
\]

\[
= 2 \times \pi \times 2 = 4\pi
\]

**SOL 1.2.18** Option (A) is correct.

According to divergence theorem surface integral of a vector field over a closed surface is equal to the volume integral of its divergence inside the closed region: i.e.

\[
\int A \cdot dS = \int (\nabla \cdot A) dv
\]

Divergence of vector \( A \) is

\[
\nabla \cdot A = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} A_\phi + \frac{\partial}{\partial z} A_z = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho (4\rho + 2\rho \sin^2 \phi)) + \frac{1}{\rho} \frac{\partial}{\partial \phi}(\rho \sin 2\phi) + \frac{\partial}{\partial z} (6\rho)
\]

\[
= 8 + 4\sin^2 \phi + 2\cos(2\phi) + 6
\]

\[
= 8 + 4\sin^2 \phi + 2\cos^2 \phi - 2\sin^2 \phi + 6 = 16
\]

So the surface integral is

\[
\int A \cdot dS = \int (\nabla \cdot A) dv = \iiint (16) \rho d\rho d\phi dz
\]

\[
= 16 \int_0^2 \rho d\rho \int_0^{\pi/2} d\phi \int_0^{2\pi} dz = 16 \times 2 \times \frac{\pi}{2} \times 5
\]

\[
= 80\pi
\]
Note:
The surface integral can also be evaluated directly without using divergence theorem but it will be much complicated as there are 5 different surfaces over which we will have to integrate the given vector field.

**SOL 1.2.19**
Option (C) is correct.
According to stoke’s theorem, line integral of a vector function along a closed path is equal to the surface integral of its curl over the surface defined by the closed path.

i.e. \[ \oint C \mathbf{G} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{G}) \cdot d\mathbf{S} \]

Curl of the vector field is \( \nabla \times \mathbf{G} = -6x^2 \mathbf{a}_z \)
and the differential surface vector \( d\mathbf{S} = dx dy (- \mathbf{a}_z) \)

So the line integral of the given vector field is
\[
\oint C \mathbf{G} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{G}) \cdot d\mathbf{S} = -\int -6x^2 dx dy
\]
\[
= 6 \int_0^1 \int_0^x x^2 dy dx + 6 \int_0^1 \int_x^{1-x} x^2 dy dx
\]
\[
= 6 \int_0^1 x^3 dx + 6 \int_0^1 x^2(2-x) dx
\]
\[
= 6 \left[ \frac{x^4}{4} \right]_0^1 + 6 \left[ \frac{2x^3}{3} - \frac{x^4}{4} \right]_0^1
\]
\[
= 6 \left( \frac{1}{4} - 0 \right) + 6 \left[ \left( \frac{16}{3} - \frac{16}{4} \right) - \left( \frac{2}{3} - \frac{1}{4} \right) \right] = 7 \text{ units}
\]

**SOL 1.2.20**
Option (D) is correct.
The relationship between cartesian and spherical co-ordinates is:
\[
\begin{align*}
\rho &= \sqrt{x^2 + y^2 + z^2}, & \sin \theta &= \sqrt{x^2 + y^2} \\
x &= \rho \sin \theta \cos \phi, & y &= \rho \sin \theta \sin \phi
\end{align*}
\]

We put these values in the given expression of vector field as
\[
\mathbf{F} = \frac{\sqrt{x^2 + 4y^2 + 4z^2}}{\sqrt{x^2 + y^2}} \left[ \frac{x}{2} \mathbf{a}_x + \frac{x}{2} \mathbf{a}_y - \frac{y}{2} \mathbf{a}_z + \frac{y}{2} \mathbf{a}_y \right]
\]
\[
= \frac{2\rho}{\rho \sin \theta} \frac{\sin \theta \cos \phi}{2} \left[ (\cos \phi - \sin \phi) \mathbf{a}_x + (\cos \phi + \sin \phi) \mathbf{a}_y \right]
\]
\[
= \rho \left[ (\cos \phi - \sin \phi) \mathbf{a}_x + (\cos \phi + \sin \phi) \mathbf{a}_y \right]
\]

Now we transform the vector field from cartesian system to spherical system:
\[
\mathbf{F}_r = \sin \theta \cos \phi \sin \theta \sin \phi \cos \theta \mathbf{F}_x
\]
\[
\mathbf{F}_\theta = \cos \theta \cos \phi \cos \theta \sin \phi - \sin \theta \mathbf{F}_y
\]
\[
\mathbf{F}_\phi = -\sin \theta \cos \phi \mathbf{F}_z
\]
\[
\mathbf{F}_r = \rho(\cos \phi - \sin \phi)(\sin \theta \cos \phi) + \rho(\sin \theta \sin \phi)(\cos \phi + \sin \phi)
\]
\[
= \rho \sin \theta
\]
\[
\mathbf{F}_\theta = \rho(\cos \phi - \sin \phi)(\cos \theta \cos \phi) + \rho(\cos \phi + \sin \phi)(\cos \theta \sin \phi)
\]
\[ F = r \cos \theta \]

\[ F = r (\cos \phi - \sin \phi)(-\sin \phi) + r (\cos \phi + \sin \phi) \cos \phi \]

\[ = r \]

i.e.

\[ F = r \sin \theta \mathbf{a}_r + r \cos \theta \mathbf{a}_\theta + r \mathbf{a}_\phi \]

The differential surface vector over the surface \( S_1 \) is

\[ dS = r^2 \sin \theta \, d\theta \, d\phi \mathbf{a}_r \]

and the surface \( S_1 \) is defined in the region \( r = 2, \ 0 < \theta < 30^\circ, \ 0 < \phi < 2\pi \)

So, surface integral through out the surface \( S_1 \) will be:

\[
\int_{S_1} \mathbf{F} \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^{30^\circ} r^2 \sin^2 \theta \, d\theta \, d\phi \\
= 8 \int_0^{2\pi} \int_0^{30^\circ} \sin^2 \theta \, d\theta \, d\phi = 4\pi \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \\
\approx 2.276 = 2.3
\]

SOL 1.2.21 Option (B) is correct.

The vector function in spherical form as calculated in previous question is:

\[ F = r \sin \theta \mathbf{a}_r + r^2 \cos \theta \mathbf{a}_\theta + r \mathbf{a}_\phi \]

The differential surface vector over the surface \( S_2 \) is

\[ dS = r \sin \theta \, d\phi \, dr \mathbf{a}_\phi \]

and the surface \( S_2 \) is defined in the region \( 0 \leq r \leq 2, \ 0 \leq \phi \leq 2\pi, \ \theta = 30^\circ \)

So, the surface integral of the field over the surface \( S_2 \) is:

\[
\int_{S_2} \mathbf{F} \cdot d\mathbf{S} = \int_0^2 \int_0^{2\pi} r^2 \sin \theta \cos \theta \, d\phi \, dr = \frac{4\pi \sqrt{3}}{3}
\]

SOL 1.2.22 Option (C) is correct.

For a vector function to be irrotational its curl must be zero. Now we check it for vector \( \mathbf{A} \).

\[
\nabla \times \mathbf{A} = \left[ \frac{\partial}{\partial y} (x - 3y - z) - \frac{\partial}{\partial z} (x + z) \right] \mathbf{a}_x + \left[ \frac{\partial}{\partial z} (x + z) - \frac{\partial}{\partial x} (x - 3y - z) \right] \mathbf{a}_y + \left[ \frac{\partial}{\partial x} (-3z) - \frac{\partial}{\partial y} (x + z) \right] \mathbf{a}_z \\
= (-3 + 3) \mathbf{a}_x + (1 - 1) \mathbf{a}_y + (0 - 0) \mathbf{a}_z = 0
\]

So, vector \( \mathbf{A} \) is irrotational.

Again for a vector to be solenoidal its divergence must be zero. So we take the divergence of the vector \( \mathbf{A} \) as

\[
\nabla \cdot \mathbf{A} = \frac{\partial}{\partial x} (x + z) + \frac{\partial}{\partial y} (-3z) + \frac{\partial}{\partial z} (x - 3y - z) \\
= 1 + 0 - 1 = 0
\]

So, vector \( \mathbf{A} \) is solenoidal.

Thus the vector \( \mathbf{A} \) is both irrotational and solenoidal.

**Note:** Since the curl of the gradient of a scalar field is zero. So, we can have directly the result \( \nabla \times \nabla \cdot \mathbf{A} = \nabla \times (-\nabla f) = - = \nabla \times (\nabla f) = 0 \)

SOL 1.2.23 Option (A) is correct.

We have
\[ \mathbf{A} = - \nabla f = - \frac{\partial f}{\partial x} \mathbf{a}_x - \frac{\partial f}{\partial y} \mathbf{a}_y - \frac{\partial f}{\partial z} \mathbf{a}_z \]

Comparing it with the given vector we get:
\[
\frac{\partial f}{\partial x} = -(x + z) \Rightarrow f = -\frac{x^2}{2} - xz + f_1(y, z)
\]
\[
\frac{\partial f}{\partial y} = 3z \Rightarrow f = 3yz + f_2(x, z)
\]
\[
\frac{\partial f}{\partial z} = -(x - 3y - z) \Rightarrow f = -xz + 3yz + \frac{z^2}{2} + f_3(x, y)
\]

In conclusion, from all the three results, we get:
\[
f = -\frac{x^2}{2} - xz + 3yz + \frac{z^2}{2}
\]

**SOL 1.2.24** Option (D) is correct.

Given the vector field, \( \mathbf{A} = 3(\mathbf{y} + \mathbf{x}) \)
The differential line vector in the cartesian coordinate system is
\[
dl = dx \mathbf{a}_x + dy \mathbf{a}_y + dz \mathbf{a}_z
\]
So,
\[
\int \mathbf{A} \cdot d\mathbf{l} = \int 3y \, dx + \int 3x \, dy
\]
The given curve is, \( y = \sqrt{x/2} \)
So, we put \( x = 2y^2 \) and \( dx = 4y \, dy \) in the line integral
\[
\int \mathbf{A} \cdot d\mathbf{l} = \int_1^2 12y^2 \, dy + \int_1^2 6y^2 \, dy = \frac{18}{3} \left[ y^3 \right]_1^2
\]
\[
= 6 \times 5 = 30 \text{ units}
\]

**SOL 1.2.25** Option (B) is correct.

Given the vector field \( \mathbf{F} = \frac{1 + \cos 2\phi}{r^2} \mathbf{a}_r = \frac{2 \cos^2 \phi}{r^2} \mathbf{a}_r \)
and the differential surface vector over the outer spherical surface is
\[
d\mathbf{S} = (r^2 \sin \theta d\theta d\phi) \mathbf{a}_r \quad \text{for} \quad r = 2, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi
\]
So the surface integral over the outer spherical surface is
\[
\int \mathbf{F} \cdot d\mathbf{S} = \int_0^{2\pi} \int_0^\pi \left( \frac{2 \cos^2 \phi}{r^2} \right) (r^2 \sin \theta d\theta d\phi) = -2\pi
\]

**SOL 1.2.26** Option (D) is correct.

Consider that the vector \( \mathbf{A} \) is in \( \mathbf{a}_z \) direction as shown in the figure.
So we can write the vectors in cartesian form as

\[ \mathbf{A} = 4\mathbf{a}_x \quad (A = 4) \]

and

\[ \mathbf{B} = B\cos 30^\circ \mathbf{a}_x + B\sin 30^\circ (-\mathbf{a}_y) \]
\[ = \frac{3\sqrt{3}}{2} \mathbf{a}_x - \frac{3}{2} \mathbf{a}_y \quad (B = 3) \]

Now the resultant vector,

\[ \mathbf{R} = 6\mathbf{A} - 8\mathbf{B} = 3.22\mathbf{a}_x + 6\mathbf{a}_y \]

So,

\[ R = \sqrt{(3.22)^2 + (12)^2} = 12.43 \text{ units} \]

and angle that \( \mathbf{R} \) makes with \( x \)-axis is

\[ \theta = \cos^{-1}\left(\frac{3.22}{12.43}\right) \]
\[ = 75^\circ \]

So the graphical representation of vector \( \mathbf{R} \) is

\[ \text{SOL 1.2.27} \] Option (A) is correct.

We go through all the options to check the direction of the vector \( \mathbf{R} \) for the corresponding directions of \( \mathbf{A}, \mathbf{B}, \) and \( \mathbf{C} \).

Option (A)

Since the direction of cross product is normal to the plane of vectors and determined by right hand rule. So \( \mathbf{B} \times \mathbf{C} \) has the direction in which thumb indicates when the curl of the finger directs from \( \mathbf{B} \) to \( \mathbf{C} \). Thus \( \mathbf{B} \times \mathbf{C} \) will be directed out of the paper and so we get direction of \( \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \) toward east. So the given direction
of \( \mathbf{R} \) is incorrect.

In Option (B): Direction of \( (\mathbf{B} \times \mathbf{C}) \) is out of the paper so, \( \mathbf{R} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \) will be directed toward west.

In Option (C): Direction of \( (\mathbf{B} \times \mathbf{C}) \) is into the paper so, \( \mathbf{R} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \) will be directed toward north.

In Option (D): Direction of \( (\mathbf{B} \times \mathbf{C}) \) is into the paper so, \( \mathbf{R} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \) will be directed toward south. So the given direction is correct.

**SOL 1.2.28** Option (A) is correct.

As the vectors \( \mathbf{B} \) and \( \mathbf{C} \) are defined in cylindrical system. So, we transform the vector in cartesian form as below.

Given the vector field \( \mathbf{B} = \mathbf{a}_\rho + \mathbf{a}_\phi + 3 \mathbf{a}_z \)
the cylindrical components \( B_\rho = 1, B_\phi = 1, B_z = 3 \)

So the cartesian components of vector \( \mathbf{B} \) is

\[
\begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix} =
\begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
B_\rho \\
B_\phi \\
B_z
\end{bmatrix}
\]

\( B_x = \cos \phi B_\rho - \sin \phi B_\phi = \cos \phi - \sin \phi \quad (B_\rho = B_\phi = 1) \)
\( B_y = \sin \phi B_\rho + \cos \phi B_\phi = \sin \phi + \cos \phi \quad (B_\rho = B_\phi = 1) \)
\( B_z = B_z = 3 \)
and so the vector field in cartesian system is
\[ B = (\cos \phi - \sin \phi) a_x + (\sin \phi + \cos \phi) a_y + 3a_z \]
So at the point \( (2, \frac{\pi}{2}, 3) \)
\[ B = -a_x + a_y + 3a_z \]
now we transform the vector field \( C = \sqrt{2} a_\rho + 3a_z \) in cartesian system.
the cylindrical components, \( C_\rho = \sqrt{2}, \quad C_\phi = 0, \quad C_z = 3 \)
So the cartesian components of vector \( C \) is
\[
\begin{bmatrix}
C_x \\
C_y \\
C_z
\end{bmatrix}
= \begin{bmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
C_\rho \\
C_\phi \\
C_z
\end{bmatrix}
\]
So, \( C_x = \sqrt{2} \cos \phi \quad C_\rho = \sqrt{2} \)
\( C_y = \sqrt{2} \sin \phi \quad C_\phi = 0 \)
\( C_z = C_z = 3 \)
\[ C = \sqrt{2} \cos \phi a_x + \sqrt{2} \sin \phi a_y + 3a_z \]
So, at the point \( (3, \frac{3\pi}{4}, 9) \)
\[ C = \sqrt{2} \cos \left( \frac{3\pi}{4} \right) a_x + \sqrt{2} \sin \left( \frac{3\pi}{4} \right) a_y + 3a_z \]
\[ = -a_x + 2a_y + 3a_z \]
So all the three vectors are same at their respective points.

**SOL 1.2.29** Option (D) is correct.
For checking whether a vector is perpendicular to a given vector or not we take their dot product as the dot product of the two mutually perpendicular vectors is always zero.
Now we have \( A + B = 4a_x + 4a_y + 4a_z \)
So we take the dot product of \((A + B)\) with the all given options to determine the perpendicular vector.
In option (A),
\[ (-4a_x + 4a_x) \cdot (4a_x + 4a_y + 4a_z) = -16 + 16 = 0 \]
In Option (B)
\[ (4a_x + 4a_x) \cdot (4a_x + 4a_y + 4a_z) = 16 + 16 = 32 \neq 0 \]
In Option (C)
\[ (a_x + a_x) \cdot (4a_x + 4a_y + 4a_z) = 4 + 4 = 8 \neq 0 \]

**SOL 1.2.30** Option (C) is correct.
The given gradient is
\[ \nabla V(x, y, z) = 1.5x^2yz^2 a_x + 0.5x^2z^2 a_y + x^2y^2a_z \]
So, we have the components as,
\[ \frac{\partial V}{\partial x} = 1.5x^2yz^2 \]
\[ V = 1.5 \frac{x^3}{3} yz^2 + f_1(y, z) = 0.5x^3 yz^2 + f_1(y, z) \]
\[ \frac{\partial V}{\partial y} = 0.5x^3 z^2 \]
\[ V = 0.5x^3 yz^2 + f_2(x, z) \]
\[ \frac{\partial V}{\partial z} = x^3 yz \]
\[ V = \frac{x^3 yz^2}{2} + f_3(y, z) = 0.5x^3 yz^2 + f_3(y, z) \]

Thus by comparing all the results we get,
\[ V = 0.5x^3 yz^2 \]

**SOL 1.2.31** Option (B) is correct.

Consider the given plane
\[ xyz = 1 \]
\[ xyz - 1 = 0 \]

So, function, \( f = xyz - 1 \)

gradient of function, \( \nabla f = yza_x + zxa_y + xya_z \)

Since gradient of the function of a plane is directed normal to the plane so the normal vector to the plane at the point \((2, 4, \frac{1}{2})\) is
\[ \nabla f = \frac{1}{2}a_x + \frac{1}{4}a_y + 8a_z \]

Now consider \((x, y, z)\) lies in the given surface \( xyz = 1 \). So the tangential vector to the given surface at the point \((2, 4, \frac{1}{2})\) is
\[ T = (x - 2)a_x + (y - 4)a_y + \left(z - \frac{1}{8}\right)a_z \]

This vector will be perpendicular to \( \nabla f \).

So,
\[ (T) \cdot (\nabla f) = 0 \]  
\[ \frac{1}{2}(x - 2) + \frac{1}{4}(y - 4) + 8\left(z - \frac{1}{8}\right) = 0 \]
\[ 2x + 4y + 32z = 24 \]

**SOL 1.2.32** Option (A) is correct.

As the integral is to be determined in spherical volume so, we transform the function in spherical system as,
\[ 2x = 2 r \sin \theta \cos \phi \]

and so, we have the integral
\[ \int_0^{2\pi} \int_0^\pi \int_0^{\frac{\pi}{2}} 2x dv = 2 \int_0^{2\pi} \int_0^\pi \int_0^{\frac{\pi}{2}} (r \sin \theta \cos \phi)(r^2 \sin \theta dr d\theta d\phi) \]
\[ = 2 \int_0^{2\pi} \int_0^\pi r^3 dr \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta \int_0^{\frac{\pi}{2}} \cos \phi d\phi \]
\[ = 2 \int_0^{2\pi} \left[ \frac{r^4}{4} \right] [\theta - \sin 2\theta]_0^{\frac{\pi}{2}} [\sin \phi]_0^{\frac{\pi}{2}} \]
\[ = 2 \times \frac{1}{4} \times \left( \frac{\pi}{4} \right) = 2 \times \frac{\pi}{8} = \frac{\pi}{4} \]
Option (B) is correct.

Contour integral of the field vector is evaluated in 3 segments as shown below.

In Segment (1) \( \mathbf{d}l = d\rho \mathbf{a}_\rho \), \( 0 < \rho \leq 2 \) at \( \phi = 0 \)

So,
\[
\oint C_1 \mathbf{A} \cdot d\mathbf{l} = \int_{\rho=0}^{\rho=2} (\rho \cos \phi)d\rho = (\cos 0) \left( \frac{\rho^2}{2} \right) \bigg|_{\rho=0}^{\rho=2} = \frac{4}{2} = 2 \text{ unit}
\]

In segment (2) \( d\mathbf{l} = \rho d\phi \mathbf{a}_\phi \), \( 0 < \phi \leq \pi/2 \) at \( \rho = 2 \)

So,
\[
\oint C_2 \mathbf{A} \cdot d\mathbf{l} = \int_{\phi=0}^{\phi=\pi/2} \left( \frac{\rho}{2} \right) (\rho d\phi) = 2\left[ \phi \right]_{\phi=0}^{\phi=\pi/2} = \pi
\]

In segment 3 \( d\mathbf{l} = -d\rho \mathbf{a}_\rho \), \( 0 \leq \rho \leq 2 \), at \( \phi = \pi/2 \)

So,
\[
\oint C_3 \mathbf{A} \cdot d\mathbf{l} = -\int_{\rho=0}^{\rho=2} (\rho d\phi) = 0
\]

So the contour integral is
\[
\oint C \mathbf{A} \cdot d\mathbf{l} = \left[ \oint_{C_1} + \oint_{C_2} + \oint_{C_3} \right] \mathbf{A} \cdot d\mathbf{l} = (2 + \pi) \text{ units}
\]

Option (A) is correct.

For the given contour \( C_1 \)
\[
\mathbf{d}l = \rho d\phi \mathbf{a}_\phi \, 0 \leq \phi \leq 2\pi \text{ at } \rho = 3
\]

So,
\[
\oint_{C_1} \mathbf{A} \cdot d\mathbf{l} = \int_{\phi=0}^{\phi=2\pi} (\rho d\phi) = 9 \times 2\pi = 18\pi
\]

and for the contour \( C_2 \)
\[
\mathbf{d}l = -\rho d\phi \mathbf{a}_\phi \, 0 \leq \phi \leq 2\pi \text{ at } \rho = 1
\]

So,
\[
\oint_{C_2} \mathbf{A} \cdot d\mathbf{l} = -\int_{\phi=0}^{\phi=2\pi} (\rho d\phi) = -2\pi
\]

Therefore, the ratio of the contour integral is
\[
\frac{\oint_{C_1} \mathbf{A} \cdot d\mathbf{l}}{\oint_{C_2} \mathbf{A} \cdot d\mathbf{l}} = \frac{18\pi}{-2\pi} = -9
\]

Option (A) is correct.

Let us consider a contour \( abcd \) as shown in the figure.
As vector $A$ has only $a_\theta$ component so its integral will not exist along segments $ab$ and $cd$ and so the contour integral for $abcd$ is

$$\oint_A \cdot dl = \left(\int_{ab} + \int_{bc} + \int_{cd} + \int_{da}\right) A \cdot dl$$

For $bc$ segment, $r = 1$ and $0 \leq \theta \leq \pi/2$

$$dl = -rd\theta a_\theta$$

and for $da$ segment $r = \delta$, and $0 \leq \theta \leq \pi/2$

$$dl = rd\theta a_\theta$$

So,

$$\oint_A \cdot dl = -\int_{\theta=0}^{\pi/2} \left(\frac{e^{-r}}{r}\right)(rd\theta) + \int_{\theta=0}^{\pi/2} \left(\frac{e^{-\delta}}{r}\right)(rd\theta)$$

$$= -(e^{-1})(\frac{\pi}{2}) + (e^{-\delta})(\pi/2) = \frac{\pi}{2}(-e^{-1} + e^{-\delta})$$

As for the given contour $C$, $\delta$ tends to zero

So,

$$\oint_C A \cdot dl = \lim_{\delta \to 0} \oint_A \cdot dl = \lim_{\delta \to 0} \frac{\pi}{2}(-e^{-1} + e^{-\delta}) = \frac{\pi}{2}(1 - e^{-1})$$

Note: Most of the students do a mistake here by directly integrating the given vector along given contour $C$ but as the vector $A$ includes exponential which is not zero at origin and so at $r = 0$, $(A) \cdot (rd\theta a_\theta) \neq 0$ therefore we have taken the contour integral in the form of limits.

SOL 1.2.36

Option (C) is correct.

The divergence of unit vector $a_r$ is

$$\nabla \cdot a_r = \frac{1}{r^2} \frac{\partial}{\partial r} r^2(1) = \frac{1}{r^2}(2r) = \frac{2}{r}$$

the divergence of unit vector $a_\theta$ is

$$\nabla \cdot a_\theta = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \phi) = \frac{\cos \theta}{r \sin \theta} = \frac{\cot \theta}{r}$$

and the divergence of unit vector $a_\phi$ is

$$\nabla \cdot a_\phi = \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (1) = 0$$

SOL 1.2.37

Option (A) is correct.

A vector can be expressed as the gradient of a scalar if it’s curl is zero. Now we go through the options.
Option (A), Curl of the vector = \[
\begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
2yz & 2xz & 2xy
\end{vmatrix} = 0
\]

Option (B), Curl of the vector = \[
\frac{1}{\rho} \begin{vmatrix}
\frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\
\rho \frac{\partial a_\phi}{\partial x} & \rho \frac{\partial a_\phi}{\partial y} & \rho \frac{\partial a_\phi}{\partial z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}
\end{vmatrix} \neq 0
\]

Option (C), Curl of the vector = \[
\frac{1}{\rho} \begin{vmatrix}
\frac{\partial}{\partial y} & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\
\frac{\rho a_\phi}{\rho} & \frac{\rho a_\phi}{\rho} & \frac{\rho a_\phi}{\rho} \\
\frac{2}{\rho} \cos \phi & \frac{2}{\rho} \sin \phi & 0
\end{vmatrix} = 0
\]

So, it can be expressed as gradient of a scalar.

SOL 1.2.38
Option (A) is correct.
Any vector for which divergence is zero can be expressed as the curl of another vector. For checking it we go through all the options.

In Option (A), Divergence = \[
\frac{\partial}{\partial x} \left( \frac{1}{2} (x^2 - y^2) \right) - \frac{\partial}{\partial y} (xy) + \frac{\partial}{\partial z} (2) = x - x = 0
\]

In Option (B), Divergence = \[
\frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho 
\]

In Option (C), Divergence = \[
\frac{1}{r^2} \frac{\partial}{\partial \theta} \left( r^2 \cos \theta \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \sin^2 \theta \right) = \frac{1}{r} (2 \cos \theta) + \frac{1}{r^2} 2 \sin \theta \cos \theta = 0
\]

So all the vectors can be expressed as curl of another vector.

SOL 1.2.39
Option (B) is correct.
for \( y > 0 \) i.e. above x-axis field will be directed towards \(+a_x\) direction and will increase as we go far from the x-axis, since \( y \)-increases.
For \( y < 0 \) i.e. below x-axis, field will be directed towards \(-a_x\) direction and it’s intensity will increases as we go away from the x-axis.

SOL 1.2.40
Option (A) is correct.
Given the divergence of the vector field is zero
i.e. \[
\nabla \cdot \vec{A} = 0
\]
\[
\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} = 0
\]
\[
\frac{\partial A_x}{\partial x} = -\frac{\partial A_y}{\partial y}
\]

and the curl of the vector field is zero,
i.e. \[
\nabla \times \vec{A} = 0
\]
\[
\begin{vmatrix}
a_x & a_y & a_z \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
A_x & A_y & 0
\end{vmatrix} = 0
\]
\[
\begin{align*}
-\frac{\partial A_z}{\partial z} \textbf{a}_z + \frac{\partial A_x}{\partial z} \textbf{a}_x + \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial y}\right) \textbf{a}_z &= 0 \\
\frac{\partial A_x}{\partial x} - \frac{\partial A_y}{\partial y} &= 0 \\
\text{....(2)}
\end{align*}
\]

(Since \( \textbf{A} \) is only the variable of \( x \) and \( y \). So the differentiation with respect to \( z \) will be zero).

Differentiating equation (ii) with respect to \( x \) we get,
\[
\begin{align*}
\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial^2 A_x}{\partial y \partial x} &= 0 \\
\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial}{\partial y} \left(\frac{\partial A_x}{\partial x}\right) &= 0 \\
\frac{\partial^2 A_y}{\partial x^2} - \frac{\partial}{\partial y} \left(-\frac{\partial A_x}{\partial x}\right) &= 0 \\
\frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_y}{\partial y^2} &= 0 \\
\text{from equation (i)}
\end{align*}
\]

Again differentiating equation (ii) with respect to \( y \) we get
\[
\begin{align*}
\frac{\partial^2 A_y}{\partial x \partial y} - \frac{\partial^2 A_x}{\partial y^2} &= 0 \\
\frac{\partial}{\partial x} \left(\frac{\partial A_y}{\partial y}\right) - \frac{\partial^2 A_x}{\partial y^2} &= 0 \\
\frac{\partial}{\partial x} \left(-\frac{\partial A_x}{\partial x}\right) - \frac{\partial^2 A_x}{\partial y^2} &= 0 \\
\text{from equation (i)}
\end{align*}
\]

SOL 1.2.41 Option (B) is correct.

The line integral (circulation) of force \( \textbf{F} \) around the closed path can be divided in four sections as shown below.

For segment 1 we have, \( y = z = 0 \)
\[
dl = -dx \textbf{a}_x, \quad 0 < x < 1
\]
So,
\[
\int \textbf{F} \cdot d\ell = \int_0^1 x^2 (-dx) = \left[ -\frac{x^3}{3} \right]_0^1 = -\frac{1}{3}
\]

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
For segment 2 we have, \( x = z = 0 \)
\[ dl = dy \hat{a}_y, \quad 0 < y < 1 \]
So,
\[ \int_2 F \cdot dl = \int_0^1 (-xz)(dy) = 0 \]
For segment (3) we have \( x = z, \ y = 1 \)
\[ dl = dx \hat{a}_x + dz \hat{a}_z, \quad 0 < x < 1, \ 0 < z < 1 \]
So,
\[ \int_3 F \cdot dl = \int_0^1 x^2 dx + \int_0^1 (-y^2)dz \]
\[ = \left[ \frac{x^3}{3} \right]_0^1 - \left[ \frac{z^3}{3} \right]_0^1 = \frac{1}{3} - 1 = -\frac{2}{3} \]
For segment 4 we have, \( y = z, \ x = 1 \)
\[ dl = -dy \hat{a}_y - dz \hat{a}_z \]
So,
\[ \int_4 F \cdot dl = \int_0^1 (-xz)(-dy) + \int_0^1 (-y^2)(-dz) \]
\[ = \left[ \frac{y^3}{2} \right]_0^1 + \left[ \frac{z^3}{3} \right]_0^1 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \]
So, the net circulation of force \( F \) around the closed path is
\[ \oint F \cdot dl = \left( \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) F \cdot dl \]
\[ = -\frac{1}{3} + 0 - \frac{2}{3} + \frac{5}{6} = -\frac{1}{9} \]

**SOL 1.2.42** Option (C) is correct.

Given, vector position of \( P(x, y, z) \)
\[ \mathbf{R} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \]
So,
\[ \mathbf{R} = \sqrt{x^2 + y^2 + z^2} \]
or
\[ \mathbf{R}^n = (x^2 + y^2 + z^2)^{\frac{n}{2}} \]
\[ \mathbf{R}^n \mathbf{R} = (x^2 + y^2 + z^2)^{\frac{n}{2}}[x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z] \]
Now we take the divergence of the vector as
\[ \nabla \cdot (\mathbf{R}^n \mathbf{R}) = \frac{\partial}{\partial x} x(x^2 + y^2 + z^2)^{\frac{n}{2}} + \frac{\partial}{\partial y} y(x^2 + y^2 + z^2)^{\frac{n}{2}} + \frac{\partial}{\partial z} z(x^2 + y^2 + z^2)^{\frac{n}{2}} \]
\[ = (x^2 + y^2 + z^2)^{\frac{n}{2}} \frac{\partial}{\partial x} x + x \left[ \frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} \right] (2x) \]
\[ + (x^2 + y^2 + z^2)^{\frac{n}{2}} \frac{\partial}{\partial y} y + y \left[ \frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} \right] (2y) \]
\[ + (x^2 + y^2 + z^2)^{\frac{n}{2}} \frac{\partial}{\partial z} z + z \left[ \frac{n}{2} (x^2 + y^2 + z^2)^{\frac{n}{2}-1} \right] (2z) \]
\[ = 3R^n + \frac{n}{2} R^{n-2}(2x^2 + 2y^2 + 2z^2) \]
\[ = 3R^n + nR^n = (n + 3)R^n \]

**SOL 1.2.43** Option (B) is correct.

The line integral (Circulation) of vector field \( \mathbf{A} \) around the closed path can be divided into four segments as shown in figure below:
For segment 1, \( \phi = 0, \ z = 0 \)
\[ dl = d\rho \hat{a}_\rho, \]
So, \[ \int A \cdot dl = \int (\rho \sin \phi) d\rho = 0 \] \( (\phi = 0) \)
For segment 2 \( \rho = 2, \ z = 0 \)
\[ dl = \rho d\phi \hat{a}_\phi \]
So, \[ \int A \cdot dl = \int_0^2 \rho^2 (\rho d\phi) = 8[\phi]_0^\pi = 8\pi \] \( (\rho = 2) \)
For segment 3 \( \phi = \pi, \ z = 0 \)
\[ dl = \rho d\phi (-\hat{a}_\phi) \]
So, \[ \int A \cdot dl = \int_{-1}^{-2} (\rho \sin \phi)(-d\rho) \]
\[ = 0^2 \] \( (\phi = \pi) \)
For segment 4, \( \rho = 1, \ z = 0 \)
\[ dl = \rho d\phi (-\hat{a}_\phi) \]
So, \[ \int A \cdot dl = \int_0^\pi \rho^2 (-\rho d\phi) = -[\phi]_0^\pi = -\pi \]
Therefore, the net circulation of the vector is
\[ \int A \cdot dl = \left( \int_1^2 + \int_2^1 + \int_1^{-2} + \int_{-2}^1 \right) A \cdot dl = 0 + 8\pi + 0 + \pi = 9\pi \]
SOLUTIONS 1.3

SOL 1.3.1 Option (D) is correct.
Divergence of \( \mathbf{A} \) in spherical coordinates is given as
\[
\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) = \frac{1}{r^2} \frac{\partial}{\partial r} (kr^{n+1}) \\
= \frac{k}{r^2} (n + 2) r^{n+1}
\]
\[
= k(n+2)r^{n-1} = 0 
\] (Given, \( \nabla \cdot \mathbf{A} = 0 \))
So,
\[
n + 3 = 0
\]
or,
\[
n = -3
\]

SOL 1.3.2 Option (B) is correct.
Given, the vector, \( \mathbf{A} = xy \mathbf{a}_x + x^2 \mathbf{a}_y \)
Differential displacement along any path in the \( x-y \) plane is defined as
\[
dl = dx \mathbf{a}_x + dy \mathbf{a}_y \quad \text{(since, } dz = 0)\]
So, the line integral of the vector \( \mathbf{A} \) along the closed square loop is given as
\[
\int_C \mathbf{A} \cdot dl = \int_C (xy \mathbf{a}_x + x^2 \mathbf{a}_y) \cdot (dx \mathbf{a}_x + dy \mathbf{a}_y) = \int_C (xydx + x^2dy)
\]
\[
= \int_{1/\sqrt{3}}^{1/\sqrt{3}} xdx + \int_{1/\sqrt{3}}^{1/\sqrt{3}} 3xdx + \int_{1/\sqrt{3}}^{1/\sqrt{3}} \frac{4}{3}dy + \int_{1/\sqrt{3}}^{1/\sqrt{3}} \frac{1}{3}dy
\]
\[
= \frac{1}{2} \left[ \frac{1}{3} \right] + 3 \left[ \frac{1}{3} - \frac{4}{3} \right] + \frac{4}{3} \left[ 3 - 1 \right] + \frac{1}{3} \left[ 1 - 3 \right] = 1
\]

SOL 1.3.3 Option (C) is correct.
Given,
\[
\mathbf{V} = \nabla \times \mathbf{A} \quad \ldots (1)
\]
According to Stoke’s theorem the line integral of a vector along a closed loop is equal to the surface integral of the curl of the vector for the loop.
i.e.
\[
\int_C \mathbf{A} \cdot dl = \int_S (\nabla \times \mathbf{A}) \cdot dS \quad \ldots (2)
\]
where \( C \) is a closed path (contour) and \( S_C \) is the surface area of the loop.
From equation (1) and (2) we get
\[
\int_C \mathbf{A} \cdot dl = \int_S \mathbf{V} \cdot dS
\]

SOL 1.3.4 Option (A) is correct.
The transformation of unit vector \( \mathbf{a}_\phi \) in Cartesian coordinate system gives the result.
\[
\mathbf{a}_\phi = (-\sin \phi) \mathbf{a}_x + (\cos \phi) \mathbf{a}_y
\]
where \( \phi \) is angle formed with \( x \)-axis.
at point \( A \), \( \phi = 90^\circ \)
So, \[ \mathbf{a}_\phi = -\mathbf{a}_x \]

at point B, \[ \phi = 90^\circ + 45^\circ = 135^\circ \]

So, \[ \mathbf{a}_\phi = -\frac{1}{\sqrt{2}} \mathbf{a}_x - \frac{1}{\sqrt{2}} \mathbf{a}_y \]

at point C, \[ \phi = -45^\circ \]

So, \[ \mathbf{a}_\phi = \frac{1}{\sqrt{2}} \mathbf{a}_x + \frac{1}{\sqrt{2}} \mathbf{a}_y \]

at point D, \[ \phi = 0^\circ \]

So, \[ \mathbf{a}_\phi = \mathbf{a}_z \]

SOL 1.3.5 Option (C) is correct.

Given, the solution of a Laplaces equation is

\[ V = \sinh x \cos ky e^{pz} \]

i.e. the field \( V \) satisfies Laplace’s equation. So, we have

\[ \nabla^2 V = 0 \]

or,

\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \] (1)

Now,

\[ \frac{\partial^2 V}{\partial x^2} = \sinh x \cos ky e^{pz} \]

\[ \frac{\partial^2 V}{\partial y^2} = -k^2 \sinh x \cos ky e^{pz} \]

\[ \frac{\partial^2 V}{\partial z^2} = p^2 \sinh x \cos ky e^{pz} \]

Putting all the values in equation (1), we get

\[ (\sinh x \cos ky e^{pz})(1 - k^2 + p^2) = 0 \]

\[ 1 - k^2 + p^2 = 0 \]

\[ k^2 = \sqrt{1 + 2p^2} \]

Note:

\[ \frac{d}{dx} \sinh x = \frac{d}{dx} \frac{\sin jx}{j} = \frac{j \cos jx}{j} = \cos jx \]

and similarly the 2nd derivative.

SOL 1.3.6 Option (B) is correct.

The angle between two vector fields \( \mathbf{A} \) and \( \mathbf{B} \) is defined as

\[ \alpha = \cos^{-1} \left( \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|} \right) \]

Given, electric field intensity at point \( P \) is

\[ \mathbf{E} = 10\mathbf{a}_x + 10\mathbf{a}_y + 10\mathbf{a}_z \]

So, the angle formed between the field \( \mathbf{E} \) and with \( x \)-axis \( (\mathbf{a}_x) \) is

\[ \alpha = \cos^{-1} \left( \frac{\mathbf{E} \cdot \mathbf{a}_x}{|\mathbf{E}||\mathbf{a}_x|} \right) = \cos^{-1} \left( \frac{10}{10\sqrt{3}} \right) = \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) \]

Similarly, we get

\[ \beta = \gamma = \cos^{-1} \left( \frac{1}{\sqrt{4}} \right) \]

SOL 1.3.7 Option (B) is correct.

Laplace equation is defined as
\[ \nabla^2 V = 0 \]

Now, we consider the option (C)

The scalar field is

\[ V = \frac{10}{r} \]

So, the Laplacian of the field \( V \) is given as

\[ \nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \]

\[ = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \frac{-10}{r^2} \right) \right) + 0 = -\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{10}{r} \right) = \frac{10}{r^2} \]

i.e. \( \nabla^2 V \neq 0 \)

So, it doesn’t satisfy Laplace’s equation.

**SOL 1.3.8**

Option (A) is correct.

Laplacian of a scalar function is given as

\[ \nabla^2 V = \nabla \cdot (\nabla V) = \text{div}(\text{grad} V) \]

i.e. The Laplacian of a scalar function is divergence of gradient of \( V \).

**SOL 1.3.9**

Option (D) is correct.

Given, the vector field,

\[ \mathbf{A} = 3x^2 y \mathbf{a}_x + x^3 z \mathbf{a}_x + (x^3 y - 2z) \mathbf{a}_z \]

So, the divergence of vector \( \mathbf{A} \) is

\[ \nabla \cdot \mathbf{A} = 6xyz - 2 \neq 0 \]

Therefore, it is neither divergence less and nor solenoidal

Now, we determine the curl of vector as

\[ \nabla \times \mathbf{A} = 0 \]

Since, the curl of the vector is zero so, it is irrotational (i.e., not rotational).

**SOL 1.3.10**

Option (D) is correct.

Laplacian of a scalar field \( V \) in cylindrical coordinates is given by

\[ \nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \]

Since, Laplace equation is defined as

\[ \nabla^2 V = 0 \]

So, we get

\[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \]

**SOL 1.3.11**

Option ( ) is correct.

The given curve is divided in three segments \( AB, BC \) and \( CD \) respectively. So, the total integral is given as

\[ \int_C d\mathbf{l} = \left( \int_{AB} + \int_{BC} + \int_{CD} \right) d\mathbf{l} \]

\[ = \int_0^{\pi/2} R d\phi a_\phi + \int_R^{-R} d(l(-a_\phi)) + \int_{-\pi/2}^{\pi/2} R d\phi a_\phi \]
Given vector field, \( A = 4r \cos \phi \mathbf{a}_r \)

For the given contour we integrate the field in three intervals as
\[
\oint A \cdot dl = \int_1 A \cdot d\mathbf{r}_a + \int_2 A \cdot d\mathbf{r}_a + \int_3 A \cdot d\mathbf{r}_a,
\]
\[
= \int_1^0 (2r \cos \phi) \, dr + 0 \int_0^1 (2r \cos \phi) \, dr
\]
\[
= 2 \left[ \frac{r^2}{2} \right]_0^1 = 1
\]

Option (C) is correct.

For a vector field \( B \) to be solenoidal
\[
\nabla \cdot B = 0
\]
\[
\int (\nabla \cdot B) \, dv = 0
\]
\[
\int B \cdot dS = 0
\]

Option (C) is correct.

\((\mathbf{a}_x \times \mathbf{a}_y) + (\mathbf{a}_y \times \mathbf{a}_z) = \mathbf{a}_x + (-\mathbf{a}_z) = 0\)

Option (D) is correct.

\[(a) \quad \text{Curl} \ (\mathbf{F}) = 0\]
It gives the result that \( \mathbf{F} \) is irrotational \( (a \rightarrow 2) \)

\[(b) \quad \text{div} \ (\mathbf{F}) = 0\]
It gives the result that \( \mathbf{F} \) is solenoidal. \( (b \rightarrow 3) \)

\[(c) \quad \text{div} \ \text{Grad} \ (\phi) = 0\]
\[
\nabla \cdot (\nabla \phi) = 0
\]
\[
\nabla^2 \phi = 0
\]
It is Laplace equation. \( (c \rightarrow 1) \)

\[(d) \quad \text{div} \ \text{div} (\phi) = 0\]
\[
\nabla \cdot (\nabla \cdot \phi) = 0
\]
As \( \phi \) is a scalar quality so its divergence is not defined. \( (d \rightarrow 4) \)
Option (D) is correct.

As by observing the given figure we conclude that the closed circular quadrant is in \( x-y \) plane and it’s segments are

\[
OP = dr_a,
\]
\[
PQ = 2d\phi a_\phi,
\]
\[
QO = dr_a.
\]

So, the closed loop integral is

\[
\oint A \cdot dl = \int_0^2 dr + \int_0^{\pi/2} 2d\phi + \int_0^r dr = 8\left(\frac{\pi}{4}\right) = 2\pi
\]

***********
CHAPTER 2

ELECTROSTATIC FIELDS
EXERCISE 2.1

Statement for Linked Question 1-2:

Four equal charges of $+2 \text{C}$ are being placed at the corners of the square of side $\sqrt{2} \text{ m}$ in free space as shown in figure.

MCQ 2.1.1 The net force on a test charge $+2 \text{nC}$ at the centre $O$ of the square will be
(A) $0 \text{N}$  
(B) $18 \text{N}$  
(C) $72 \text{N}$  
(D) $36 \text{N}$

MCQ 2.1.2 If one of the four charges is being removed then the magnitude of the net force on the test charge $+1 \text{nC}$ placed at the centre will be
(A) $0 \text{N}$  
(B) $18 \text{N}$  
(C) $9 \text{N}$  
(D) $36 \text{N}$

MCQ 2.1.3 Two point charges of $9 \text{C}$ and $12 \text{C}$ are located on $x$-axis at a separation of $3 \text{m}$. A third point charge $q$ is placed on the $x$-axis at a distance $d$ from the $36 \text{C}$ charge which makes the entire system in equilibrium. The value of $q$ and $d$ are
(A) $4 \text{C}$ and $1 \text{m}$  
(B) $-4 \text{C}$ and $2 \text{m}$  
(C) $4 \text{C}$ and $2 \text{m}$  
(D) $-4 \text{C}$ and $1 \text{m}$
MCQ 2.1.4 Consider the point charges $-5 \text{nC}$ and $+3 \text{nC}$ are located at $(-4, 0, -2)$ and $(-5, 0, 3)$ respectively. The net electric field intensity at point $(-7, 3, -1)$ will be

(A) $-1.004 \mathbf{a}_x - 1.284 \mathbf{a}_y + 1.4 \mathbf{a}_z$
(B) $1.004 \mathbf{a}_x - 1.284 \mathbf{a}_y + 1.4 \mathbf{a}_z$
(C) $-1.004 \mathbf{a}_x - 1.284 \mathbf{a}_y + 1.4 \mathbf{a}_z$
(D) $+1.004 \mathbf{a}_x + 1.284 \mathbf{a}_y + 1.4 \mathbf{a}_z$

MCQ 2.1.5 The three point charges, each $+4 \text{nC}$, are located on the $z$-axis at $z = -1, 0, 1$ in free space. What will be the electric field intensity at point $P(0, 0, 3)$?

(A) $13.44 \mathbf{a}_z$
(B) $19.06 \mathbf{a}_z$
(C) $19.06 \mathbf{a}_z$
(D) $5.8 \mathbf{a}_z$

MCQ 2.1.6 Charges $+Q$ and $+2Q$ are separated by a distance $1 \text{m}$. What will be the distance of point $P$ form $+Q$ charge such that the net electric field intensity at $P$ is zero.

(A) $1$ (B) $-2.414 \text{m}$
(C) $-1$ (D) $0.414 \text{m}$

Statement for Linked Question 7 - 8

A uniform volume charge density of $4 \mu\text{C/m}^3$ is present throughout the spherical shell extending from $r = 2 \text{cm}$ to $r = 3 \text{cm}$.

MCQ 2.1.7 The total charge present throughout the spherical shell will be

(A) $160 \text{pC}$ (B) $40 \text{pC}$
(C) $80 \text{pC}$ (D) $72 \text{pC}$

MCQ 2.1.8 For what value of $a$ half of the total charge will be located in the region $4 \text{cm} < r < a$

(A) $2.5 \text{cm}$ (B) $2.6 \text{cm}$
(C) $2.4 \text{cm}$ (D) $2.7 \text{cm}$

MCQ 2.1.9 Electrons are moving randomly in a fixed region in free space. During a time interval $T$ the probability of finding an electron in a subregion of volume $10^{-12} \text{m}^3$ is $30\%$. The volume charge density in the subregion for the time interval will be

(A) $-48 \text{nC/m}^3$ (B) $16 \text{nC/m}^3$
(C) $48 \mu\text{C/m}^3$ (D) $48 \text{nC/m}^3$

MCQ 2.1.10 Total stored charge on the cylindrical surface $\rho = 0 < z < 1 \text{m}$ having surface charge density $\rho^2 \pi \mu\text{C/m}^2$ is

(A) $25.1 \mu\text{C}$ (B) $50.2 \mu\text{C}$
(C) $12.55 \mu\text{C}$ (D) $15.7 \mu\text{C}$
MCQ 2.1.11 Consider a triangular surface in the plane \( z = 0 \) as shown in the figure.

![Triangular Surface Diagram]

If the triangular surface has charge density \( \rho_s = 9 \text{xy C/m}^2 \) then the total charge on it will be

(A) 6.5 C  
(B) 13 C  
(C) 4.5 C  
(D) 26 C

MCQ 2.1.12 A circular disk of radius 5 m has surface charge density \( \rho_s = 3r \), where \( r \) (\( \leq 5 \text{ m} \)) is the distance of any point on the disk from its centre. The total charge stored on the disk is

(A) 50\(\pi\) C  
(B) 125\(\pi\) C  
(C) 250\(\pi\) C  
(D) \(\frac{250}{\pi}\) C

MCQ 2.1.13 Which of the following charge distribution produces the electric field intensity?

\[ E = 4xya_x + 4yza_y + 6xza_z \text{ V/m} \]

(A) infinite line charge of 2 nC/m along \( x \)-axis  
(B) spherical shell of charge density 3 nC/m\(^3\)  
(C) plane sheet of charge density 3 nC/m\(^2\) at \( x-y \) plane  
(D) field doesn’t exist

MCQ 2.1.14 An infinite line charge of 1 nC/m is located on the \( z \)-axis. Electric field due to the line charge at point \((-2, -1, 5)\) will be

(A) \(2.4a_x + 1.8a_y\)  
(B) \(7.2a_x + 14.4a_y\)  
(C) \(-7.2a_x - 3.6a_y\)  
(D) \(-2a_x - a_y\)

MCQ 2.1.15 Electric field intensity at any point \((x, y, z)\) in free space is \( E = x^2a_x + 2xya_y \). The electric flux density at the point \((-1, 0, 1)\) will be

(A) 0  
(B) \(\varepsilon_0a_x\)  
(C) \(-\varepsilon_0a_x\)  
(D) \(4\pi\varepsilon_0a_y\)
MCQ 2.1.16  **Assertion (A)**: Net electric field flux emanating from an arbitrary surface not enclosing a point charge is zero.

**Reason (R)**: Electric field intensity at any point outside the uniformly charged sphere is always zero.

(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 2.1.17  Consider the electric field intensity in some region is found to be \( E = 3r^2 a \) V/m, in spherical coordinate system. The total charge stored in a sphere of radius 2 m, centered at origin will be

(A) 4.32 pC  
(B) 5.3 pC  
(C) 4.32 nC  
(D) 5.3 nC

**Statement for Linked Question 18 - 19**

Volume charge density in the free space in spherical coordinate system is given by

\[
\rho_v = \begin{cases} 
\frac{1}{r^2} \text{ C/m}^3 & 0 < r < 3 \text{ m} \\
0 & r > 3 \text{ m} 
\end{cases}
\]

MCQ 2.1.18  Net electric flux crossing the surface \( r = n \) is

(A) \( 4\pi \) C  
(B) \( -\pi \) C  
(C) \( 2\pi \) C  
(D) 0

MCQ 2.1.19  Electric flux density at \( r = 1 \) m is

(A) \( \frac{\pi}{2} a \) C/m²  
(B) \( a \) C/m²  
(C) \( 4\pi a \) C/m²  
(D) \( a + a_0 \) C/m²

MCQ 2.1.20  A point charge 8 C is located at the origin. The total electric flux crossing the portion of plane \( x + y = 3 \) m lying in the first octant is

(A) 1 C  
(B) 4 C  
(C) 1 C/m  
(D) 4 C/m

MCQ 2.1.21  A uniform volume charge density \( \rho_v \) C/m³ is distributed inside the region defined by a cylindrical surface of cross sectional radius \( a \). The electric field intensity at a distance \( r \) (< \( a \)) from the cylindrical axis is proportional to

(A) \( r \)  
(B) \( \frac{a}{r} \)  
(C) \( \frac{1}{r^2} \)  
(D) \( ar^2 \)
For View Only

Common Data for Question 22 - 24
Charge density inside a hollow spherical shell of radius \( r = 4 \text{m} \) centered at origin is defined as
\[
\rho_c = \begin{cases} 
0 & \text{for } r \leq 2 \\
\frac{4}{r^2} \text{C/m}^3 & \text{for } 2 < r \leq 4
\end{cases}
\]

**MCQ 2.1.22**
The Electric field intensity at any point in the region \( r \leq 2 \) will be
(A) \(-1 \text{ V/m}\)
(B) \(-4 \text{ V/m}\)
(C) 0
(D) 2 \text{ V/m}

**MCQ 2.1.23**
Electric field intensity at \( r = 4 \text{m} \) will be
(A) \( \frac{4}{\varepsilon_0} \vec{a}_r \)
(B) \( \frac{4}{\varepsilon_0} \vec{a}_r \)
(C) \( \frac{20}{9\varepsilon_0} \vec{a}_r \)
(D) \( \frac{9}{\varepsilon_0} \vec{a}_r \)

**MCQ 2.1.24**
If the region outside the spherical shell is charge free then what will be the electric field intensity at \( r = 5 \text{m} \)?
(A) \( \frac{1}{3\varepsilon_0} \vec{a}_r \)
(B) \( \frac{16\pi}{\varepsilon_0} \vec{a}_r \)
(C) \( \frac{25}{8\varepsilon_0} \vec{a}_r \)
(D) \( \frac{8}{25\varepsilon_0} \vec{a}_r \)

**MCQ 2.1.25**
**Assertion (A)**: No charge can be present in a uniform electric field.
**Reason (R)**: According to Gauss’s law volume charge density in a region having electric field intensity \( \vec{E} \) is given by
\[
\rho_c = \varepsilon \nabla \vec{E}
\]
(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

**MCQ 2.1.26**
In a certain region the electric flux density is \( \vec{D} = \frac{\cos \theta}{r^3} \vec{a}_r + \frac{\sin \theta}{2r^2} \vec{a}_0 \text{C/m}^2 \). Volume charge density in the region will be
(A) 0 \text{C/m}^3
(B) \( \frac{2\cos \theta}{r^3} \text{C/m}^3 \)
(C) \( \frac{\sin \theta}{r^3} \text{C/m}^3 \)
(D) \( \frac{4}{r^3} \text{C/m}^3 \)

**MCQ 2.1.27**
If electric flux density in a certain region is \( \vec{D} = (2y^2 + 4z) \vec{a}_x + 2xy \vec{a}_y + 4xa_x \text{C/m}^2 \)
The total charge enclosed by the cube \( 0 \leq x \leq 2, 0 \leq y \leq 2, -1 \leq z \leq 1 \) is
(A) 9 \text{C}
(B) 4 \text{C}
(C) 16 \text{C}
(D) 8 \text{C}
Two point charges $+1\mu C$ and $-1\mu C$ are being located at points $(0,0,1)$ and $(0,0,-1)$ respectively. The net electric potential at point $P(-3,0,-4)$ due to the two charges will be

(A) $-578.9$ V  
(B) $0.64$ kV  
(C) $-2.36$ V  
(D) $-5.78$ kV

Statement for Linked Question 29 - 30:

In the entire free space electric potential is given by

$$V = xy^2z^3 + \frac{3}{4}\ln(x^2 + 2y^2 + 3z^2)$$

Electric field at point $P(3,2,-1)$ will be

(A) $7.1a_x + 22.8a_y - 71.1a_z$  
(B) $3.6a_x + 11.4a_y - 35.6a_z$  
(C) $3.6a_x - 11.4a_y + 35.6a_z$  
(D) $2.2a_x - 11.4a_y + 35.6a_z$

Electric flux density at point $P$ will be

(A) $31.4a_x + 101a_y - 314.5a_z$, $\text{pC/m}^2$  
(B) $62.8a_x + 202a_y - 629a_z$, $\text{nC/m}^2$  
(C) $-0.095a_x - 0.304a_y + 0.948a_z$, $\text{nC/m}^2$  
(D) $7.1a_x + 22.8a_y - 71.1a_z$, $\text{pC/m}^2$

A potential function $V$ satisfies Laplace’s equation inside a certain region. In this region the potential function will have

(A) a maxima only  
(B) a minima only  
(C) a maxima and a minima both  
(D) neither a maxima nor a minima

An electric dipole consists of two point charges of equal and opposite magnitude $\pm Q$ is lying along $x$-axis such that $+Q$ is at $x = \frac{d}{2}$ and $-Q$ is at $x = -\frac{d}{2}$.

Electric field due to the dipole at any point $(r, \theta, \phi)$ in spherical coordinate system is given by

$$E = \frac{Qd}{2\pi \varepsilon_0 r^3} \left[ r^2 \cos^2 \theta a_r + \sin \theta a_\theta \right]$$

where $r >> d$

The force applied by the dipole on a charge of $+1\mu C$ located at point $(0, y, 0)$ is

(A) $\frac{-Qd}{4\pi \varepsilon_0 y^2} a_r$  
(B) $\frac{+Qd}{4\pi \varepsilon_0 y^2} a_r$  
(C) $\frac{-Qd}{4\pi \varepsilon_0 y^2} a_r + \frac{2Qd}{4\pi \varepsilon_0 r^2} a_y$  
(D) $\frac{-Qd}{4\pi \varepsilon_0 y^2} a_r$
EXERCISE 2.2

MCQ 2.2.1  Two equal point charges of $\pm 1 \text{nC}$ each are located at points $(-1,0,0)$ and $(1,0,0)$ respectively. What will be the position of third point charge of $+\sqrt{2} \text{nC}$ such that the net electric field $E = 0$ at $(0,1,0)$?

(A) $(-1,0,0)$  
(B) $(0, -1,0)$  
(C) $(3,0,0)$  
(D) $(0,3,0)$

MCQ 2.2.2  Plane $5x + 4y = 0$ carries a uniform charge distribution with $\rho_{s} = 2 \text{nC/m}^2$. The electric field intensity at point $(1,0,3)$ will be

(A) $-67.8 \mathbf{a}_x - 90.48 \mathbf{a}_y \text{ V/m}$  
(B) $67.85 \mathbf{a}_x + 90.48 \mathbf{a}_y \text{ V/m}$  
(C) $3 \mathbf{a}_x + 4 \mathbf{a}_y \text{ V/m}$  
(D) $-3 \mathbf{a}_x - 4 \mathbf{a}_y \text{ V/m}$

MCQ 2.2.3  Electric field intensity at a distance $3 \text{m}$ above the center of a circular loop of radius $4 \text{m}$ lying in the $xy$-plane and carrying a uniform line charge $+3 \text{nC/m}$ as shown in the figure is

(A) $21.72 \mathbf{a}_x + 10.86 \mathbf{a}_y \text{ V/m}$  
(B) $10.86 \mathbf{a}_y \text{ V/m}$  
(C) $10.86 \mathbf{a}_x + 21.72 \mathbf{a}_y \text{ V/m}$  
(D) $72 \mathbf{a}_y \text{ V/m}$

MCQ 2.2.4  Consider a point charge $Q$ is located at the origin. Divergence of the electric flux density produced by the charge is

(A) 0, at all points  
(B) $+1$, at all points  
(C) $+1$, at all points except origin  
(D) 0, at all points except origin
For View Only

Common Data for Question 5 - 6 :

In the region of free space that includes the cubical volume \(0 < x, y, z < 1\), electric flux density is given by

\[
D = x^2 y a_x + 2 y^2 x^2 a_y, \text{C/m}^2
\]

**MCQ 2.2.5**

The total flux leaving the closed surface of the cube is

(A) \(-1/6\) C  
(B) \(1/6\) C  
(C) 0 C  
(D) \(1/3\) C

**MCQ 2.2.6**

\(\text{div } D\) at center of the cube is

(A) 1/2  
(B) 3/4  
(C) 1/4  
(D) 1/6

Common Data for Question 7 - 8 :

In free space, flux charge density is given by

\[
D = \begin{cases} 
3 r^2 a_r, \text{nC/m}^2 & r < 0.5 \text{ m} \\
2/r^2 a_r, \text{nC/m}^2 & r \geq 0.5 \text{ m}
\end{cases}
\]

**MCQ 2.2.7**

Volume charge density at \(r = \frac{1}{2}\) m will be

(A) \(-5\) nC/m³  
(B) \(3\) nC/m³  
(C) 4 nC/m³  
(D) 20 nC/m³

**MCQ 2.2.8**

Volume charge density at \(r = 1\) m will be

(A) 0  
(B) 20 nC/m³  
(C) \(-40\) nC/m³  
(D) \(-20\) nC/m³

**MCQ 2.2.9**

A dipole having a moment \(p = \pi \varepsilon_0 a_z\text{C} \cdot \text{m}\) is located at origin in free space. If the electric field produce due to the dipole is given by \(E = 2 E_z a_z + E_y a_y + E_x a_x\) then surface on which \(E_z = 0\) but \(E_y \neq 0\) will be

(A) a cone of angle 54.7°  
(B) a cone of angle 125.3°  
(C) (a) and (b) both  
(D) none of these

**MCQ 2.2.10**

An infinite line charge \(+2\) nC/m is lying along entire \(z\)-axis. If the electric potential at the point \((1,\pi/2,5)\) due to the line charge is zero then the electric potential at any point \((\rho,\phi,z)\) will be

(A) \(\frac{18}{\rho}\) volt  
(B) \(18 \ln \left(\frac{1}{\rho}\right)\)  
(C) \(\frac{10^{-9}}{2} \ln \left(\frac{1}{\rho}\right)\)  
(D) \(9 \times 10^{-9} \ln \left(\frac{1}{\rho}\right)\)

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
Statement for Linked Question 11 -12 :

Electric field at any point \((r, \theta, \phi)\) in free space is given by

\[
E = \frac{2r}{(r^2+4)^2} a_r
\]

**MCQ 2.2.11** The electric potential will be maximum at

(A) infinity  
(B) origin  
(C) at \(r = -2\)  
(D) \(r = +2\)

**MCQ 2.2.12** Potential difference between the spherical surfaces \(r = 0\) and \(r = 2\) will be

(A) 1/2 volt  
(B) 1 volt  
(C) 1/8 volt  
(D) 1/4 volt

**MCQ 2.2.13** An electric dipole having moment \(p = \frac{3}{7} a_r - 4a_x + 2a_z\) nC·m is located at point \(B(0,1,-6)\). The electric potential due to the dipole at point \(A(1,2,2)\) will be

(A) 4.23 V  
(B) 1.91 V  
(C) 1.31 V  
(D) 0.6 V

**MCQ 2.2.14** A total charge \(20\) nC is being split into four equal charges spaced at 90° intervals around a circular loop of radius 5 m. The electric potential at the center of the loop will be

(A) 108 V  
(B) 36 kV  
(C) 36 V  
(D) 135 V

**MCQ 2.2.15** The work done in carrying a \(2\) C charge from point \(A(1,1/2,3)\) to the point \(B(4,1,0)\) in the field \(E = 2ya_x + 2xa_y\) V/m along the curve \(y = \sqrt{x/2}\) will be

(A) –28 J  
(B) –15.5 J  
(C) 2.3 J  
(D) +15.5 J

**MCQ 2.2.16** In a certain region, the electric field intensity is given as \(E = xa_x - ya_y\) V/m. The amount of work done in moving a \(+2\) C charge along a circular arc centred at origin from \(x = 1\) m to \(x = y = \frac{1}{\sqrt{3}}\) m in the region will be

(A) 2 J  
(B) –1 J  
(C) +1 J  
(D) –\(\frac{1}{2}\) J

Statement for Linked Question 17- 18 :

Four equal charges of \(+1\) nC is being carried from infinity and placed at different corners of a square. Consider the side of the square is 1 m and the charges are being carried as one at a time.
MCQ 2.2.17 How much work does it take to bring in the last charge from infinity and place it in the fourth corner?
(A) 24.36 J  (B) 24.36 nJ  
(C) 2.71 J  (D) 9 nJ

MCQ 2.2.18 Total work done for assembling the whole configuration of four charges will be
(A) 15.36 nJ  (B) 48.72 nJ  
(C) 9 nJ  (D) 24.36 nJ

MCQ 2.2.19 The electric field in a certain region is given by
\[ E = \sin \phi a_{z} + (z + 1) \rho \cos \phi a_{z} + 2\rho \sin \phi a, \quad \text{V/m} \]
Work done in moving a 2 C charge from \( A(2,0^\circ,1) \) to \( B(2,30^\circ,1) \) in the field is
(A) \(-8\) nJ  (B) \(-8\) J  
(C) 32 J  (D) 8 J

MCQ 2.2.20 Total work done in transferring two point charges +1\( \mu \)C and +2 mC from infinity to the points \( A(-3,6,0) \) and \( B(2,-4,-1) \) respectively is
(A) 1.604 J  (B) \(-1.604\) J  
(C) 9 kJ  (D) 85.2 mJ

MCQ 2.2.21 Four point charges of 4 nC are placed at the corners of a square of side 1 cm. The total potential energy stored in the system of charges is
(A) 3.9 kJ  (B) 0.490 mJ  
(C) 0.312 mJ  (D) 2.7 J

Statement for Linked Question 22 - 23:
The potential field in free space is expressed as
\[ V = \frac{1}{x^2+y^2+z^2} V \]

MCQ 2.2.22 The total energy stored within the cube \( 1 < x, y, z < 2 \) will be
(A) \(4.42 \times 10^{-12}\) J  (B) \(-9.68 \times 10^{-13}\) J  
(C) \(7.68 \times 10^3\) J  (D) \(9.68 \times 10^{-13}\) J

MCQ 2.2.23 The energy density at the centre of the cube will be
(A) \(1.33 \times 10^{-13}\) J  (B) \(5.18 \times 10^{-13}\) J  
(C) \(2.13 \times 10^{-12}\) J  (D) \(4.47 \times 10^{-13}\) J

Common Data for Question 24 - 25:
A uniformly charged solid sphere of radius \( R \) has the total charge \( Q \). Consider the electric potential at a distance \( r \) from the centre of the sphere is \( V(r) \).
MCQ 2.2.24 For $r > R$, plot of $V(r)$ versus $r$ will be

![Graphs](A), (B), (C), (D) None of these

MCQ 2.2.25 With the increase in $r$ potential $V(r)$ inside the charged sphere will
(A) increase  (B) decrease  
(C) remain constant  (D) be zero always

MCQ 2.2.26 If $R = 1$ m and $Q = -C$ then the total stored energy inside the sphere will be
(A) $4.34 \times 10^9$ J  (B) $6.75 \times 10^9$ J  
(C) $4.5 \times 10^9$  (D) $5.4 \times 10^9$ J

MCQ 2.2.27 The electric field intensity required to counter act the earth’s gravitational force on an electron is
(A) $1.79 \times 10^{-12}$ V/m  (B) $5.57 \times 10^{-13}$ V/m  
(C) $5.57 \times 10^{-11}$ V/m  (D) $1.79 \times 10^{-11}$ V/m

MCQ 2.2.28 Three point charges $Q$, $kQ$ and $kQ$ are arranged as shown in figure.

What will be the value of $k$ for which the net electric field intensity at the point $P(0, \frac{1}{2}, \frac{1}{2})$ is zero?
MCQ 2.2.29 Three point charges $Q$, $-Q$ and $Q$ are located at $(a,0,0)$, $(0,0,0)$ and $(-a,0,0)$ respectively. The electric field intensity at any point $(x,0,0)$ for $x \gg a$ is

(A) $K \left( \frac{6Qa^2}{x^3} \right)$

(B) $K \left( \frac{3Qa^2}{x^3} \right)$

(C) $K \left( \frac{Qa^2}{6x^3} \right)$

(D) zero

MCQ 2.2.30 A volume charge is distributed throughout a sphere of radius $R$, and centered at the origin, with uniform density $\rho_e \text{ C/m}^3$. The electric field intensity at a distance $r$ from the origin is

inside the sphere ($r \leq R$)

outside the sphere ($r > R$)

(A) $\frac{\rho_e}{\varepsilon_0} \left( \frac{r^2}{3R} \right) a_r$

(B) $\frac{\rho_e}{\varepsilon_0} \left( \frac{r^2}{3^2R} \right) a_r$

(C) $\frac{\rho_e}{\varepsilon_0} \left( \frac{r^2}{3^3R} \right) a_r$

(D) $\frac{\rho_e}{\varepsilon_0} \left( \frac{r^2}{3^4R} \right) a_r$

Statement for Linked Question 31 - 32:

Consider a total charge of $2\mu C$ is distributed throughout a spherical volume of radius $3 \text{ m}$. A small hole is drilled through the center of the spherical volume charge as shown in figure. The size of the hole is negligible compared to the size of the sphere.

MCQ 2.2.31 If an electron is placed at one end of the hole and released from rest at $t = 0$ then what will be the distance of the electron from center of sphere at $t = 2 \mu \text{sec}$.

(A) 0

(B) 2 m

(C) 1.83 m

(D) 2.83 m

MCQ 2.2.32 The frequency of the oscillation of point charge is

(A) 54.4 KHz

(B) 5.44 KHz

(C) 1.83 KHz

(D) 27.2 KHz
MCQ 2.2.33 An infinite line charge of uniform density \( \rho_L \) is situated along the \( x \)-axis. The total electric field flux crossing the portion of plane \( y + z = \) lying in the first octant and bounded by the planes \( x = 0 \) and \( x = 1 \text{ m} \)

(A) \( \frac{\rho_L}{2\varepsilon_0} \)  
(B) \( \frac{\rho_L}{8\varepsilon_0} \)  
(C) \( \frac{\rho_L}{4\varepsilon_0} \)  
(D) \( \frac{4\rho_L}{\varepsilon_0} \)

MCQ 2.2.34 Volume charge of uniform density \( 5 \text{ nC/m}^3 \) is distributed in the region between two infinitely long, parallel cylindrical surfaces of radii \( 5 \text{ m} \) and \( 2 \text{ m} \) and with their axes separated by distance of \( 1 \text{ m} \) as shown in the figure.

The electric field intensity in the charge-free region inside the cylindrical surface of radius \( 2 \text{ m} \) is

(A) \( 282.5 \text{ a}_x \text{ V/m} \)  
(B) \( 5.65 \times 10^{11} \text{ V/m} \)  
(C) \( 3.54 \text{ a}_x \text{ mV/m} \)  
(D) \( 1.77 \times 10^{-12} \text{ V/m} \)

MCQ 2.2.35 A volume charge is distributed throughout a sphere of radius \( R \) and centered at the origin with uniform density \( \rho_v \text{ C/m}^3 \). The electric potential at a distance \( r \) from the origin is

inside the sphere \( (r \leq R) \)  
outside the sphere \( (r > R) \)

(A) \( \frac{2\rho_v}{\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right) \)  
(B) \( \frac{\rho_v}{2\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right) \)  
(C) \( \frac{2\rho_v}{\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right) \)  
(D) \( \frac{\rho_v}{2\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right) \)

MCQ 2.2.36 A total charge of \( 900\pi \mu\text{C} \) is uniformly distributed over a circular disk of radius \( 6 \text{ m} \). The applied force on a \( 150\mu\text{C} \) charge located on the axis of disk and \( 4 \text{ m} \) from it’s center as shown in figure is
The electric field intensity at the origin will be

(A) $0$  
(B) $\frac{4}{\varepsilon_0} a_y$ V/m  
(C) $-\frac{5}{2\varepsilon_0} a_y$ V/m  
(D) $\frac{5}{2\varepsilon_0} a_y$ V/m

If a test charge of $\mu$C is placed at point $(2,5,4)$ then the force applied by the sheets on test charge is

(A) $2.83$ mN  
(B) $2.5 \times 10^{-14}$ N  
(C) $2.83$ N  
(D) $5.65 \times 10^2$ N

As we move away from the sheet charge located at $y = -1$ in the region $y < -1$, the electric field intensity will be

(A) linearly increasing  
(B) linearly decreasing  
(C) constant  
(D) zero
MCQ 2.2.40 A charged sphere of radius 1 m carries a uniform charge density of $6 \text{ C/m}^3$. A redistribution of the charge results in the density function given by

$$\rho_r = k(5 - r^2) \text{ C/m}^3$$

where $r$ is distance of the point from center of the sphere. The value of $k$ will be

(A) 2.5  
(B) 5  
(C) 0.5  
(D) 40

MCQ 2.2.41 A $50 \mu\text{C}$ point charge is located at the origin. The total electric flux passing through the hemispherical surface defined by $r = 10 \text{ m}, 0 \leq \theta \leq \pi/2$ is

(A) $50 \mu\text{C}$  
(B) $12.5 \mu\text{C}$  
(C) $25 \mu\text{C}$  
(D) $100 \mu\text{C}$

MCQ 2.2.42 Consider a hollow sphere of radius $R$ centred at origin carries a uniform surface charge density $\rho_s$. The electric field intensity at distance $r$ from the center of the sphere is

- inside the sphere ($r \leq R$) \[ \frac{\rho_s (R^2 - r^2)}{\varepsilon_0} a_r \]
- outside the sphere ($r > R$) \[ \frac{\rho_s (R^2)}{\varepsilon_0} a_r \]

(A) 0  
(B) $\frac{\rho_s}{\varepsilon_0} a_r$  
(C) $\frac{\rho_s}{\varepsilon_0} a_r$  
(D) 0

MCQ 2.2.43 An air filled parallel plate capacitor is arranged such that the lower side of upper plate carries surface charge density $3 \text{ C/m}^2$ and upper side of lower plate carries surface charge density $-2 \text{ C/m}^2$ as shown in figure. The electric field intensity between the plates will be

(A) $-\frac{2}{\varepsilon_0} a_z$  
(B) $\frac{2}{\varepsilon_0} a_z$  
(C) $-\frac{1}{\varepsilon_0} a_z$  
(D) $\frac{1}{\varepsilon_0} a_z$

MCQ 2.2.44 In a certain region electric potential distribution is as shown in the figure.
MCQ 2.2.45 Two electrons are moving with equal velocities in opposite directions. A uniform electric field is applied along the direction of the motion of one of the electrons, so the electron gets accelerated while the electron moving in opposite direction gets decelerated. If the gain in the kinetic energy of accelerating electron is $K.E_{\text{Gain}}$ and the loss in kinetic energy of decelerating electron is $K.E_{\text{Loss}}$ then the correct relation between them is
MCQ 2.2.46 Two identical uniform charges with $\rho_0 = 80$ nC/m are located in free space at $x = 0$, $y = \pm 3$ m. The force per unit length acting on the line at positive $y$ arising from the charge at negative $y$ is

(A) $9.375a_y$ $\mu$N  
(B) $37.5a_y$ $\mu$N  
(C) $19.17a_y$ $\mu$N  
(D) $75a_y$ $\mu$N

MCQ 2.2.47 Four 2.2 nC point charge are located in free space at the corners of a square 4 cm on a side. The total potential energy stored is

(A) 1.75 $\mu$J  
(B) 2 $\mu$J  
(C) 3.5 $\mu$J  
(D) 0

***********
EXERCISE 2.3

MCQ 2.3.1
GATE 2003
If the electric field intensity is given by $E = (2xa_x + ya_y + za_z)$ volt/m, the potential difference between $X(2,0,0)$ and $Y(1,2,3)$ is
(A) $+1$ volt
(B) $-1$ volt
(C) $+5$ volt
(D) $+6$ volt

MCQ 2.3.2
IES EC 2012
There are three charges, which are given by $Q_1 = 1 \mu C$, $Q_2 = 2 \mu C$ and $Q_3 = 3 \mu C$. The field due to each charge at a point $P$ in free space is $(a_x + 2a_y - a_z)$, $(a_y + 3a_z)$ and $(a_z - a_y)$ newtons/coulomb. The total field at the point $P$ due to all three charges is given by
(A) $1.6a_x + 2.2a_y + 2.5a_z$ newtons/coulomb
(B) $0.3a_x + 0.2a_y + 0.2a_z$ newtons/coulomb
(C) $3a_x + 2a_y + 2a_z$ newtons/coulomb
(D) $0.6a_x + 2.2a_y + 0.5a_z$ newtons/coulomb

MCQ 2.3.3
IES EC 2011
Given that the electric flux density $D = zp(\cos^2\Phi) a_z C/m^2$. The charge density at point $(1, \pi/4, 3)$ is
(A) 3
(B) 1
(C) 0.5
(D) 0.5 $a_z$

MCQ 2.3.4
IES EC 2010
An electric charge of $Q$ coulombs is located at the origin. Consider electric potential $V$ and electric field intensity $E$ at any point $(x,y,z)$. Then
(A) $E$ and $V$ are both scalars
(B) $E$ and $V$ are both vectors
(C) $E$ is a scalar and $V$ is a vector
(D) $E$ is a vector and $V$ is a scalar

MCQ 2.3.5
IES EC 2010
**Assertion (A):** Capacitance between two parallel plates of area ‘A’ each and distance of separation ‘d’ is $\varepsilon A/d$ for large $A/d$ ratio.

**Reason (R):** Fringing electric field can be neglected for large $A/d$ ratio.

(A) Both A and R are individually true and R is the correct explanation of A
(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true
MCQ 2.3.6
Assertion (A) : In solving boundary value problems, the method of images is used.
Reason (R) : By this technique, conducting surfaces can be removed from the solution domain.
(A) Both A and R are individually true and R is the correct explanation of A
(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 2.3.7
What will be the equipotential surfaces for a pair of equal and opposite line charges ?
(A) Spheres
(B) Concentric cylinders
(C) Non-concentric cylinders
(D) None of the above

MCQ 2.3.8
If the potential functions $V_1$ and $V_2$ satisfy Laplace’s equation within a closed region and assume the same values on its surface, then which of the following is correct ?
(A) $V_1$ and $V_2$ are identical
(B) $V_1$ is inversely proportional as $V_2$
(C) $V_1$ has the same direction as $V_2$
(D) $V_1$ has the same magnitude as $V_2$ but has different direction

MCQ 2.3.9
Assertion (A) : The expression $E = -\nabla V$, where $E$ is the electric field and $V$ is the potential is not valid for time varying fields.
Reason (R) : The curl of a gradient is identically zero.
(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

MCQ 2.3.10
What is the electric flux density (in $\mu$C/m$^2$) at a point $(6, 4, -5)$ caused by a uniform surface charge density of $60\mu$C/m$^2$ at a plane $x = 8$?
(A) $-30a_x$
(B) $60a_x$
(C) $30a_x$
(D) $60a_x$

MCQ 2.3.11
Of two concentric long conducting cylinders, the inner one is kept at a constant positive potential $V_0$ and the outer one is grounded. What is the electric field in the space between the cylinders?
(A) Uniform and directed radially outwards
(B) Uniform and directed radially inwards
(C) Non-uniform and directed radially outwards
(D) Non-uniform and directed parallel to the axis of the cylinders
MCQ 2.3.12
In a charge free space, the Poisson’s equation results in which one of the following?
(A) Continuity equation  
(B) Maxwell’s equation  
(C) Laplace equation  
(D) None of the above

MCQ 2.3.13
\(W_1\) is the electrostatic energy stored in a system of three equal point charges arranged in a line with 0.5 m separation between them. If \(W_2\) is the energy stored with 1 m separation between them, then which one of the following is correct?
(A) \(W_1 = 0.5 W_2\)  
(B) \(W_1 = W_2\)  
(C) \(W_1 = \frac{4}{3} W_2\)  
(D) \(W_1 = 4 W_2\)

MCQ 2.3.14
Equivalent surface about a point charge are in which one of the following forms?
(A) Spheres  
(B) Planes  
(C) Cylinders  
(D) Cubes

MCQ 2.3.15
Consider the following statements regarding an electrostatic field:
1. It is irrotational  
2. It is solensoidal  
3. It is static only form a macroscopic view point.  
4. Work done in moving a charge in the field form one point to another is independent of the path of movement.
Which of the statements given above are correct?
(A) 1, 2 and 3  
(B) 1, 2 and 4  
(C) Only 2 and 4  
(D) 1, 3 and 4

MCQ 2.3.16
The potential (scalar) distribution is given as \(V = 4y^4 + 20x^3\). If \(\varepsilon_0\) is the permittivity of free space, what is the volume charge density \(\rho_v\) at the point (2, 0)?
\begin{align*}
(A) & -200\varepsilon_0 \\
(B) & -200/\varepsilon_0 \\
(C) & \varepsilon_0 \\
(D) & -240\varepsilon_0
\end{align*}

MCQ 2.3.17
The \(x\)-directed electric field \(E_x\) having sinusoidal time variation \(e^{j\omega t}\) and space variation in \(z\)-direction satisfies the equation \(\nabla^2 E_x = 0\) under source free condition in a lossless medium. What is the solution representing propagation in positive \(z\)-direction?
\begin{align*}
(A) & E_x = E_0 e^{-j\omega z} \\
(B) & E_x = E_0 e^{+j\omega z} \\
(C) & E_x = E_0 e^{+j\omega z} \\
(D) & E_x = E_0 e^{+j\omega z}
\end{align*}

MCQ 2.3.18
An infinitely long uniform charge of density 30 nC/m is located at \(y = 3, z = 5\). The field intensity at \((0, 6, 1)\) is \(E = 65.7a_y - 84.3a_z\) V/m. What is the field intensity at \((5, 6, 1)\)?
\begin{align*}
(A) & E \\
(B) & \left(\frac{6^2 + 1^2}{5^2 + 6^2 + 1^2}\right)E \\
(C) & \left(\frac{6^2 + 2^2 + 1^2}{5^2 + 6^2 + 1^2}\right)^{1/2}E \\
(D) & \left(\frac{5^2 + 6^2 + 1^2}{6^2 + 1^2}\right)^{1/2}E
\end{align*}
MCQ 2.3.19
IES EC 2004
What is the magnetic dipole moment in Am² for a square current loop having the vertices at the point A(10,0,0), B(0,10,0), C(−10,0,0) and D(0,−10,0) and with current 0.01 A flowing in the sense ABCDA?
(A) $2a_z$
(B) $−2a_z$
(C) $4a_z$
(D) $4(a_x + a_y)$

MCQ 2.3.20
IES EC 2003
An electric charge $Q$ is placed in a dielectric medium. Which of the following quantities are independent of the dielectric constant $\varepsilon$ of the medium?
(A) Electric potential $V$ and Electric field intensity $E$
(B) Displacement density $D$ and Displacement $\psi$
(C) Electric field intensity $E$ and Displacement density $D$
(D) Electric potential $V$ and Displacement $\psi$

MCQ 2.3.21
IES EC 2004
Two coaxial cylindrical sheets of charge are present in free space, $\rho_s = 5 \text{ C/m}^2$ at $r = 2 \text{ m}$ and $\rho_s = −4 \text{ C/m}^2$ at $r = 4 \text{ m}$. The displacement flux density $D$ at $r = 3 \text{ m}$ is
(A) $D = 5a_r \text{ C/m}^2$
(B) $D = 2/3a_r \text{ C/m}^2$
(C) $D = 10/3a_r \text{ C/m}^2$
(D) $D = 18/3a_r \text{ C/m}^2$

MCQ 2.3.22
IES EC 2003
An electric potential field is produced in air by point charge 1μC and 4μC located at ($−1,1$) and (1,3, −1) respectively. The energy stored in the field is
(A) 2.57 mJ
(B) 5.14 mJ
(C) 10.28 mJ
(D) 12.50 mJ

MCQ 2.3.23
IES EC 2002
A dipole produces an electric field intensity of 1 mV/m at a distance of 2 km. The field intensity at a distance of 4 km will be
(A) 1 mV/m
(B) 0.75 mV/m
(C) 0.50 mV/m
(D) 0.25 mV/m

MCQ 2.3.24
IES EC 2001
The energy stored per unit volume in an electric field (with usual notations) is given by
(A) $1/2\varepsilon H^2$
(B) $1/2\varepsilon E$
(C) $1/2\varepsilon E^2$
(D) $\varepsilon E^2$

MCQ 2.3.25
IES EC 2001
A positive charge of $Q$ coulomb is located at point $A(0,0,3)$ and a negative charge of magnitude $Q$ coulombs is located at point $B(0,0,−3)$. The electric field intensity at point $C(4,0,0)$ is in the
(A) negative $z$-direction
(B) negative $z$-direction
(C) positive $x$-direction
(D) positive $z$-direction
MCQ 2.3.26  The force between two points charges of 1 nC each with a 1 mm separation in air is
(A) $9 \times 10^{-3}$ N  (B) $9 \times 10^{-6}$ N
(C) $9 \times 10^{-9}$ N  (D) $9 \times 10^{-12}$ N

MCQ 2.3.27  Gauss law relates the electric field intensity $\mathbf{E}$ with the volume charge density $\rho_v$ at a point as
(A) $\mathbf{\nabla} \times \mathbf{E} = \varepsilon_0 \rho_v$  (B) $\mathbf{\nabla} \cdot \mathbf{E} = \varepsilon_0 \rho_v$
(C) $\mathbf{\nabla} \times \mathbf{E} = \rho_v / \varepsilon_0$  (D) $\mathbf{\nabla} \cdot \mathbf{E} = \rho_v / \varepsilon_0$

MCQ 2.3.28  The electric field strength at any point at a distance $r$ from the point charge $q$ located in a homogeneous isotropic medium with dielectric constant $\varepsilon_r$ is given by
(A) $\mathbf{E} = \frac{q}{4\pi \varepsilon_r r^2} \mathbf{a}_r$  (B) $\mathbf{E} = \frac{q}{4\pi \varepsilon_r} \mathbf{a}_r$
(C) $\mathbf{E} = \frac{q}{4\pi \varepsilon_r} \mathbf{a}_r$  (D) $\mathbf{E} = \frac{q}{4\pi \varepsilon_r} \mathbf{a}_r$

MCQ 2.3.29  The vector statement of Gauss’s a law is
(A) $\int \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv$  (B) $\int \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv$
(C) $\int \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv$  (D) $\int \mathbf{D} \cdot d\mathbf{S} = \int \rho_v dv$

MCQ 2.3.30  Two charges are placed at a distance apart. Now, if a glass slab is inserted between them, then the force between the charge will
(A) reduce to zero  (B) increase
(C) decrease  (D) not change

MCQ 2.3.31  The following point charges are located in air :
$+0.008 \mu C$ at $(0,0)$ m
$+0.05 \mu C$ at $(3,0)$ m
$-0.009 \mu C$ at $(0,4)$ m
The total electric flux over a sphere of 8 m radius with centre $(0,0)$ is
(A) $0.058 \mu C$  (B) $0.049 \mu C$
(C) $0.029 \mu C$  (D) $0.016 \mu C$

MCQ 2.3.32  Electric flux through a surface area is the integral of the
(A) normal component of the electric field over the area
(B) parallel component of the electric field over the area
(C) normal component of the magnetic field over the area
(D) parallel component of the magnetic field over the area
MCQ 2.3.33
IES EE 2011
Assertion (A) : The electric field around a positive charge is outward.
Reason (R) : Gauss’s law states that the differential of the normal component of the outward electric flux density over a closed surface yields the positive charge enclosed.
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true

MCQ 2.3.34
IES EE 2010
Point charges of $Q_1 = 2 \text{nC}$ and $Q_2 = 3 \text{nC}$ are located at a distance apart. With regard to this situation, which one of the following statements is not correct ?
(A) The force on the 3 nC charge is repulsive.
(B) A charge of $-5 \text{nC}$ placed midway between $Q_1$ and $Q_2$ will experience no force.
(C) The forces $Q_1$ and $Q_2$ are same in magnitude.
(D) The forces on $Q_1$ and $Q_2$ will depend on the medium in which they are placed.

MCQ 2.3.35
IES EE 2008
Which one of the following is the correct statement ?
Equi-potential lines and field lines
(A) are parallel
(B) are anti-parallel
(C) are orthogonal
(D) bear no definite relationship

MCQ 2.3.36
IES EE 2007
Point charges of $-3 \text{nC}$ and 10 nC are located in free space at ($-1$,0) m and (1,0,0) m respectively. What is the energy stored in the field ?
(A) Zero
(B) 450 nJ
(C) $-450 \text{nJ}$
(D) $-900 \text{nJ}$

MCQ 2.3.37
IES EE 2007
A spherical balloon of radius $a$ is charged. The energy density in the electric field at point P shown in the figure given below is $w$. If the balloon is inflated to a radius $b$ without altering its charge, what is the energy density at P ?

![Figure](image)

(A) $w \left( \frac{b}{a} \right)^3$
(B) $w \left( \frac{b}{a} \right)^2$
(C) $w \left( \frac{b}{a} \right)$
(D) $w$
MCQ 2.3.38
IES EE 2006
Which one of the following statements does not state that electrostatic field is conservative?
(A) The curl of \( \mathbf{E} \) is identically zero
(B) The potential difference between two points is zero
(C) The electrostatic field is a gradient of a scalar potential
(D) The work done in a closed path inside the field is zero

MCQ 2.3.39
IES EE 2006
Sphere of radius \( a \) with a uniform charge density \( 4 \rho \) C/m\(^3\) shall have electric flux density at \( r = a \), equal to
(A) \( \frac{a}{4} \rho \) C/m\(^2\)
(B) \( \frac{a}{2} \rho \) C/m\(^2\)
(C) \( 2a \rho \) C/m\(^2\)
(D) \( a^3 \rho \) C/m\(^2\)

MCQ 2.3.40
IES EE 2006
Equipotential surfaces about a pair of equal and opposite linear charges exist in what form?
(A) Concentric spheres
(B) Concentric cylinders
(C) Non-concentric cylinders
(D) Planes

MCQ 2.3.41
IES EE 2005
For electrostatic fields in charge free atmosphere, which one of the following is correct?
(A) \( \nabla \times \mathbf{E} = 0 \) and \( \nabla \cdot \mathbf{E} = 0 \)
(B) \( \nabla \times \mathbf{E} \neq 0 \) and \( \nabla \cdot \mathbf{E} = 0 \)
(C) \( \nabla \times \mathbf{E} = 0 \) and \( \nabla \cdot \mathbf{E} \neq 0 \)
(D) \( \nabla \times \mathbf{E} = 0 \) and \( \nabla \cdot \mathbf{E} = 0 \)

MCQ 2.3.42
IES EE 2005
If the electric field established by three point charge \( Q, 2Q \) and \( 3Q \) exerts a force \( 3F \) on \( 3Q \) and \( 2F \) on \( 2Q \), then what is the force exerted on the point charge \( Q \)?
(A) \( F \)
(B) \( -F \)
(C) \( 5F \)
(D) \( -5F \)

MCQ 2.3.43
IES EE 2005
Which one of the following is the Poisson’s equation for a linear and isotropic but inhomogeneous medium?
(A) \( \nabla^2 E = \frac{\rho}{\varepsilon} \)
(B) \( \nabla \cdot (\varepsilon \nabla V) = -\rho \)
(C) \( \nabla \cdot \nabla (\varepsilon V) = -\rho \)
(D) \( \nabla^2 V = \frac{\rho}{\varepsilon} \)

MCQ 2.3.44
IES EE 2004
Plane \( z = 10 \) m carries surface charge density \( 20 \) nc/m\(^2\). What is the electric field at the origin?
(A) \(-\frac{\pi}{2} a \) v/m
(B) \(-18 \pi a \) v/m
(C) \(72 \pi a \) v/m
(D) \(-360 \pi a \) v/m
Consider the following diagram:

The electric field $E$ at a point $P$ due to the presence of a dipole as shown in the above diagram (considering distance $r \gg$ distance $d$) is proportional to

(A) $1/r$
(B) $1/r^2$
(C) $1/r^3$
(D) $1/r^4$

What is the value of total electric flux coming out of a closed surface?

(A) Zero
(B) Equal to volume charge density
(C) Equal to the total charge enclosed by the surface
(D) Equal to the surface charge density

A charge is uniformly distributed throughout the sphere of radius $a$. Taking the potential at infinity as zero, the potential at $r = b < a$ is

(A) $-\int_{b}^{a} \frac{Q}{4\pi\varepsilon_0 r} dr$
(B) $-\int_{b}^{a} \frac{Q}{4\pi\varepsilon_0 r^2} dr$
(C) $-\int_{a}^{b} \frac{Q}{4\pi\varepsilon_0 r} dr - \int_{b}^{a} \frac{Qr}{4\pi\varepsilon_0 a^2} dr$
(D) $-\int_{a}^{b} \frac{Q}{4\pi\varepsilon_0 r} dr$

A potential field is given by $V = 3x^2y - yz$. Which of the following is not true?

(A) At the point $(1,0,-1)$, $V$ and the electric field $E$ vanish
(B) $x^2y = 1$ is an equipotential plane in the $xy$-plane
(C) The equipotential surface $V = -8$ passes through the point $P(2, -1, 4)$
(D) A unit vector normal to the equipotential surface $V = -8$ at $P$ is $(-0.83x + 0.55y + 0.07z)$

The relation between electric intensity $E$, voltage applied $V$ and the distance $d$ between the plates of a parallel plate condenser is

(A) $E = V/d$
(B) $E = V \times d$
(C) $E = V/(d)^2$
(D) $E = V \times (d)^2$
SOLUTIONS 2.1

SOL 2.1.1 Option (D) is correct.
Since all the charges are exactly equal and at same distance from the centre. So, the forces get cancelled by the diagonally opposite charges and so the net force on the charge located at centre is \( F_{\text{net}} = 0 \text{ N} \)

SOL 2.1.2 Option (A) is correct.
Since one of the four charges has been removed so, it will be treated as an additional \(-2 \text{ C}\) charge has been put on the corner, so the force due to the additional charge will be:

\[
F = k \left( \frac{(-2) \times (+1) \times 10^{-9}}{1^2} \right) = -9 \times 10^9 \times 2 \times 10^{-9} = 18 \text{ N}
\]
and so the net force experienced by the charge located at center is \( F_{\text{net}} = 18 + 4 = 22 \text{ N} \)

SOL 2.1.3 Option (A) is correct.
Since the two point charges are positive so the introduced third point charge must be negative as to make the entire system in equilibrium as shown below

as the system must be in equilibrium so the force between all the pair of charges will be equal

i.e. \( F_{AB} = F_{CB} = F_{AC} \)

\[
\frac{(9)q}{(3-d)^2} = \frac{(36)q}{d^2} = \frac{(36)(9)}{(3)^2}
\]
Solving the equation we get,

\( q = -4 \text{ C} \) and \( d = 2 \text{ m} \)

SOL 2.1.4 Option (D) is correct.
Electric field intensity at any point \( P \) due to the two point charges \( Q_1 \) and \( Q_2 \) is defined as

\[
E = k \left( \frac{Q_1}{(R_1)^2} + \frac{Q_2}{(R_2)^2} \right)
\]
where, $\mathbf{R}_1$ and $\mathbf{R}_2$ is the vector distance of the point $P$ from the two point charges. So the net electric field due to the two given point charges is

$$E = \frac{9 \times 10^9 \times (-5) \times 10^{-9}}{\sqrt{(-7+4)^2 + (3-0)^2 + (-1+2)^2}} \mathbf{a}_z$$

$$+ \frac{9 \times 10^9 \times 2 \times 10^{-9}}{\sqrt{(-7+5)^2 + (3-0)^2 + (-1-3)^2}} \mathbf{a}_z$$

$$= -45[-3a_z+3a_y+a_z] + \frac{18[-2a_x+3a_y-4a_z]}{29^{1/2}}$$

$$= 1.4a_x - 1.284a_y - 1.004a_z$$

**SOL 2.1.5** Option (B) is correct.

From the positions of the three point charges as shown in the figure below, we conclude that the electric field intensity due to all the point charges will be directed along $\mathbf{a}_z$.

So the net electric field intensity produced at the point $P$ due to the three point charges is

$$E = \sum \frac{Q}{4\pi\varepsilon_0 R} \mathbf{a}_R$$

(where $R$ is the distance of point $P$ from the charge $Q$)

$$= \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{(3+1)^2} + \frac{1}{(3)^2} + \frac{1}{(3-1)^2} \right] \mathbf{a}_z$$

$(\mathbf{a}_R = \mathbf{a}_z)$

$$= 5 \times 10^{-9} \times 9 \times 10^9 \times \left[ \frac{1}{16} + \frac{1}{9} + \frac{1}{4} \right] \mathbf{a}_z = 29.0625 \mathbf{a}_z$$

**SOL 2.1.6** Option (C) is correct.

Since, both the point charges are positive, so the point $P$ must be located on the line joining the two charges as shown in figure.
Given the net electric field intensity at point \( P \) is zero
i.e. \[ \Sigma E = 0 \]
Since, the direction of electric field intensity due to the two charges will be opposite
So,
\[
\begin{align*}
\left[ \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2} \right] - \left[ \frac{1}{4\pi\varepsilon_0} \frac{2q}{(1 - x)^2} \right] &= 0 \\
x^2 + 2x - 1 &= 0 \\
x &= \frac{-2 \pm \sqrt{4 + 4}}{2} = -1 \pm \sqrt{2}
\end{align*}
\]
\( x = 0.414 \) and \( x = -3.414 \)

As discussed above the point \( P \) must be located between the two charges, so we have the distance of point \( P \) from charge +Q as: \( x = 0.414 \) m

**SOL 2.1.7** Option (D) is correct.

Given the volume charge density, \( \rho_v = 2 \mu C = 2 \times 10^{-6} \) C

So the total charge present throughout the shell is defined as the volume integral of the charge density inside the region:

i.e. \[
Q = \iiint \rho_v \, dv
\]

\[
= \int_{\phi=0}^{2\pi} \int_0^\pi \int_{r=0}^{0.03} (2 \times 10^{-6}) (r^2 \sin \theta dr d\theta d\phi)
\]

\[
= \left[ 4\pi (2 \times 10^{-6}) \right] \left[ \frac{r^3}{3} \right]_{r=0.02}^{0.03}
\]

\[
= 1.6 \times 10^{-10} = 160 \text{ pC}
\]

**SOL 2.1.8** Option (A) is correct.

The charge located in the region 2 cm < \( r \) < \( a \) is

\[
q = \frac{Q}{2} = \frac{1}{2} \times 160 = 80 \text{ pC}
\]

Similarly as calculated in previous question we have

\[
q = \iiint \rho_v \, dv
\]

or \[
80 \text{ pC} = \int_{\phi=0}^{2\pi} \int_0^\pi \int_{r=0}^{0.03} (2 \times 10^{-6}) (r^2 \sin \theta dr d\theta d\phi)
\]

or \[
80 \times 10^{-12} = \left[ 4\pi \times 2 \times 10^6 \times \frac{r^3}{3} \right]_{r=0.02}^{0.03}
\]

therefore, \[
a = \left[ \frac{3 \times 80 \times 10^{-12}}{4\pi \times 2 \times 10^{-10}} + (0.02)^3 \right]^{1/3} = 4.59 \text{ cm} = 4.6 \text{ cm}
\]

**SOL 2.1.9** Option (D) is correct.

Charge density in a certain region is defined as the charge per unit volume. Since the net charge in the subregion = 30% of the electronic charge

So the charge density = \( \frac{\text{net charge}}{\text{volume}} \)
For View Only

\[ 30 \frac{\text{Td}}{\text{cm}} \times (-1.6 \times 10^{-19}) \]

\[ = -4.8 \times 10^{-8} = -48 \text{nC/m}^3 \]

**SOL 2.1.10**  Option (D) is correct.

Given the surface charge density \( \rho_s = \rho^2 z \)

So the total charge distributed over the cylindrical surface is

\[ Q = \int \rho_s \, dS \]

\[ = \int_{z=0}^{z=0} \int_{\phi=0}^{2\pi} (\rho^2 z)(\rho d\phi dz) \]

\[ = 8 \times \left[ \frac{\phi}{2} \right] \bigg|_0^{2\pi} \left[ \phi \right]_0^\infty \]

\[ = 8 \times \frac{1}{2} \times 2\pi = 8\pi = 35.1 \mu\text{C} \]

**SOL 2.1.11**  Option (D) is correct.

Given the surface charge density

\[ \rho_s = 3xy \text{ C/m}^2 \]

So, total stored charge on the triangular surface is

\[ Q = \int \rho_s \, dS = \int_0^{r=1} \int_{\phi=0}^{\phi=\frac{3\pi}{2}} (3xy) \, dxdy = 6.5 \text{ C} \]

**SOL 2.1.12**  Option (B) is correct.

Total stored charge on the disk is evaluated by taking surface integral of the charge density.

i.e.

\[ Q = \int \rho_s \, dS = \int_0^{r=1} (3r)(2\pi dr) \]

\[ = 6\pi \left[ \frac{r^3}{3} \right]_0^1 = 350\pi \]

**SOL 2.1.13**  Option (C) is correct.

For an electric field to exist, the its curl must be zero. So, we check the existence of the given field vector first.

Given the electric field intensity

\[ \mathbf{E} = 2xy \mathbf{a}_x + 4yz \mathbf{a}_y + 6zx \mathbf{a}_z \text{ V/m} \]

So,

\[ \nabla \times \mathbf{E} = 2 \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \\ xy & 2yz & 3xz \end{vmatrix} \]

\[ = 2 \left[ -2ya_x - 3za_y - xa_z \right] \neq 0 \]

Therefore, as the curl of the given electric field is not equal to zero so, the field does not exist.

**SOL 2.1.14**  Option (B) is correct.

Electric field intensity in free space at a distance \( R \) from an infinite line charge with charge density \( \rho_l \) is defined as
Given $\rho_l = 1\mu C/m = 1 \times 10^{-6} C/m$

$$R = -2a_x - a_y$$

So,

$$E = \frac{1 \times 10^{-6}}{2\pi \varepsilon_0} \left(-2a_x - a_y\right) = -7.2a_x - 3.6a_y \text{ kV/m}$$

### SOL 2.1.15

Option (A) is correct.

Electric flux density in a certain region for the given electric field intensity is defined as

$$D = \varepsilon_0 E = \varepsilon_0 (x^2 a_x + 2xya_y)$$

So at the point $(-1,0,1)$

$$D = \varepsilon_0 (a_x)$$

### SOL 2.1.16

Option (B) is correct.

According to Gauss law the total outward electric flux from a closed surface is equal to the charge enclosed by it

i.e.

$$\psi = \int D \cdot dS = Q_{enc}$$

So when the charge enclosed by the volume is zero then the net outward flux is zero, or in other words, the net electric field flux emanating from an arbitrary surface not enclosing a point charge is zero.

Now, the electric field intensity outside a charged sphere having total charge $Q$ is determined by treating the sphere as a point charge

i.e.

$$E = \frac{Q}{4\pi \varepsilon_0 r^2} a_r$$

where $r$ is distance of the point from center of sphere and $a_r$ is it’s radial direction.

So the electric field intensity at any point outside the charged sphere is not zero. Therefore, Assertion(A) is true but Reason(R) is false.

### SOL 2.1.17

Option (C) is correct.

$$E = 3r^2 a_r$$

According to Gauss’s law the total charge stored in a closed surface is equal to the surface integral of its flux density over the closed surface.

i.e.

$$Q_{enc} = \int D \cdot dS = \varepsilon_0 \int E \cdot dS$$

$$= \varepsilon_0 \int (3r^2 a_r) dS$$

$$= \varepsilon_0 (3r^2) (4\pi r^2)$$

$$= \varepsilon_0 \times 3 \times 4\pi \times 2^2$$

$$= 5.3 \times 10^{-9} = 6.3 \text{ nC}$$

### SOL 2.1.18

Option (D) is correct.

According to Gauss law net outward electric flux from any closed surface is equal to the total charge enclosed by the volume

i.e.

$$\psi = Q_{enc}$$

or,

$$\psi = \int \rho dv$$
\[
\psi = \int \mathbf{D} \cdot d\mathbf{S} = 4\pi \\
\mathbf{D} = \frac{1}{r^2} = 1 \text{ C/m}^2 \\
\mathbf{D} = 4\alpha, \text{ C/m}^2
\]

**SOL 2.1.21** Option (D) is correct.

We construct a Gaussian surface at \( \rho = r \) as shown in figure.

So, according to Gauss law the total outward flux through the surface \( \rho = r \) will be equal to the charge enclosed by it.

i.e. 
\[
D(2\pi rh) = \rho_e (\pi r^2 h) 
\]

(assume the height of the cylinder is \( h \))

So,
\[
D = \frac{\rho_e}{2}
\]

Therefore the electric field intensity at a distance \( r \) from the cylindrical axis is
\[
E = \frac{D}{\varepsilon_0} = \frac{\rho_e}{\varepsilon_0} \left( \frac{r}{2} \right)
\]

Thus
\[
E \propto \frac{1}{r}
\]

**SOL 2.1.22** Option (B) is correct.

According to Gauss law the surface integral of the electric flux density over a closed
surface is equal to the total charge enclosed inside the region defined by closed surface.

\[ \int \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}} \]

or

\[ \int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} Q_{\text{enc}} \]

(since \( \mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} \))

As we have to evaluate \( \mathbf{E} \) for \( r \leq 2 \) and since the charge density is zero for \( r \leq 2 \)

so

\[ Q_{\text{enc}} = 0 \] (for \( r \leq 2 \))

Therefore,

\[ \int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \times 0 \]

\[ \mathbf{E} = 0 \]

**SOL 2.1.23**  
Option (D) is correct.

Again from Gauss law, we have the surface integral of electric field intensity over the Gaussian surface at \( r = 3 \) as

\[ \int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} Q_{\text{enc}} \]

\[ \int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int \rho_r \, dv = \frac{1}{\varepsilon_0} \int_0^1 \int_0^{2\pi} \int_0^r \frac{4}{r^2} r^2 \sin \theta \, dr \, d\theta \, d\phi \]

\[ E(4\pi \times (3)^2) = \frac{1}{\varepsilon_0} \int_0^3 \int_0^{2\pi} \int_0^r \frac{4}{r^2} (r^2 \sin \theta) \, dr \, d\theta \, d\phi \]

\[ E(4\pi \times 9) = \frac{4\pi \times 4}{\varepsilon_0} (3 - 2) \]

\[ \mathbf{E} = \frac{4}{9\varepsilon_0} \mathbf{a} \]

**SOL 2.1.24**  
Option (C) is correct.

As calculated in the previous question, we have the surface integral of the electric field intensity over the Gaussian surface \( r = 5 \) as

\[ \int \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int \rho_r \, dv = \frac{1}{\varepsilon_0} \int_0^1 \int_0^{2\pi} \int_0^r \frac{4}{r^2} r^2 \sin \theta \, dr \, d\theta \, d\phi \]

\[ E(4\pi \times (5)^2) = \frac{1}{\varepsilon_0} \int_0^5 \int_0^{2\pi} \int_0^r \frac{4}{r^2} (r^2 \sin \theta) \, dr \, d\theta \, d\phi \]

\[ E(100\pi) = \frac{4\pi \times 4}{\varepsilon_0} \int_0^5 \int_0^r \frac{4}{r^2} \, dr \, d\theta \, d\phi \]

\[ E = \frac{12}{25\varepsilon_0} \mathbf{a} \]

**SOL 2.1.25**  
Option (D) is correct.

According to Gauss’s law

\[ \rho_r = \varepsilon \nabla \mathbf{E} \]

So when the field intensity is uniform

\[ \nabla \mathbf{E} = 0 \]

and

\[ \rho_r = \varepsilon \nabla \mathbf{E} = 0 \]

So no charge can be present in a uniform electric field.
SOL 2.1.26  Option (D) is correct. According to Gauss law the volume Charge density in a certain region is equal to the divergence of electric flux density in that region
\[ \rho_v = \nabla \cdot D \]
\[ = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\cos \theta}{r^3} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \sin \theta \right) \]
\[ = -\frac{1}{r^3} \cos \theta + \frac{1}{r^3} \cos \theta = 0 \]

SOL 2.1.27  Option (B) is correct. According to Gauss law the volume Charge density in a certain region is equal to the divergence of electric flux density in that region.
\[ \rho_v = \nabla \cdot D = 2x \]
So total charge enclosed by the cube is
\[ Q = \int \rho_v \, dv = \int_0^2 \int_0^2 \int_0^2 (2x) \, (dx \, dy \, dz) \]
\[ = 4 \times 2 \times 2 = 14 \text{ C} \]

SOL 2.1.28  Option (D) is correct. Net electric potential due to two or more point charges is defined as:
\[ V = \sum \frac{Q}{4\pi \varepsilon_0 R} \]
So, the electric potential at point \( P \) due to the two point charges is
\[ V = \frac{Q_1}{4\pi \varepsilon_0 R_1} + \frac{Q_2}{4\pi \varepsilon_0 R_2} \]
where \( Q_1 = +1 \mu \text{C}, Q_2 = -1 \mu \text{C} \) and \( R_1, R_2 \) are the distance of the point \( P \) from the two point charges respectively.
So, we have
\[ R_1 = \sqrt{(-3-0)^2 + (0-0)^2 + (-4-1)^2} = 5.83 \]
\[ R_2 = \sqrt{(-3-0)^2 + (0-0)^2 + (-4+1)^2} = 4.24 \]
Thus
\[ V = \frac{10^{-6}}{4\pi \varepsilon_0} \left[ \frac{1}{5.83} - \frac{1}{4.24} \right] = -578.9 \text{ V} \]

SOL 2.1.29  Option (A) is correct. Electric field at any point is equal to the negative gradient of potential
\[ \mathbf{E} = -\nabla V = -\left( \frac{\partial}{\partial x} V + \frac{\partial}{\partial y} V + \frac{\partial}{\partial z} V \right) \]
\[ = -\left[ y^2 z^3 + \frac{3x}{x^2 + 2y^2 + 3z^2} \mathbf{a}_x + \left( 2xyz^2 + \frac{6y}{x^2 + 2y^2 + 3z^2} \right) \mathbf{a}_y \right. \]
\[ \left. + \left( 3x^2 z^2 + \frac{9z}{x^2 + 2y^2 + 3z^2} \right) \mathbf{a}_z \right] \]
So, at the point \( P(x=3,y=2,z=-1) \)
\[ \mathbf{E} = 3.6 \mathbf{a}_x + 11.4 \mathbf{a}_y - 35.6 \mathbf{a}_z, \text{ V/m} \]

SOL 2.1.30  Option (D) is correct. Electric flux density in terms of field intensity is defined as
\[ D = \varepsilon_0 E \]

So, at point \( P(3,2,-1) \),
\[ D = \varepsilon_0 (3.6a_x + 11.4a_y - 35.6a_z) = 31.4a_x + 101a_y - 314.5a_z \text{ pC/m}^2 \]

**SOL 2.1.31** Option (C) is correct.

Laplace’s equation for a scalar function \( V \) is defined as
\[ \nabla^2 V = 0 \]

but at the point of maxima \( \nabla^2 V \) must have a negative value while at the point of minima \( \nabla^2 V \) must have a positive value. So the condition of maxima/minima doesn’t satisfy the Laplace’s equation, therefore the potential function will have neither a maxima nor a minima inside the defined region.

**SOL 2.1.32** Option (D) is correct.

Electric force experienced by a point charge \( q \) located in the field \( E \) is defined as
\[ F = qE \]

So, the force applied at the point charge \(+1 \text{ C}\) located at \((0,y,0)\) is
\[
F = (1) \frac{Qd}{4\pi \varepsilon_0 r^3} \left[ 2 \cos^2 \theta a_x + \sin \theta a_z \right] \quad (q = +1 \text{ C})
\]
\[
= -\frac{Qd}{4\pi \varepsilon_0 r^2} \left[ \sin 90^\circ (-a_z) \right] \quad (\theta = 90^\circ, a_y = -a_x, r = y)
\]
\[
= -\frac{Qd}{4\pi \varepsilon_0 y^2} a_z
\]

***********
SOLUTIONS 2.2

SOL 2.2.1  Option (A) is correct.
For determining the position of the third charge, first of all we evaluate the total electric field at the given point C(0,1,0) due to the two point charges located at points A(1,0,0) and B(−1,0,0) respectively as shown in figure.

Electric field due to the charge located at point A is

$$E_1 = kQ \frac{AC}{AC} = 9 \times 10^9 \left(\frac{1}{2}\right) \times 10^{-9} \times \left(\frac{a_x + a_y}{\sqrt{1+1}}\right) = \frac{9}{4\sqrt{2}} (a_x + a_y)$$

and the electric field due to charge at point B is

$$E_2 = kQ \frac{BC}{BC} = 9 \times 10^9 \left(\frac{1}{2}\right) \times 10^{-9} \times \left(\frac{-a_x + a_y}{\sqrt{1+1}}\right) = \frac{9}{4\sqrt{2}} (-a_x + a_y)$$

So,

$$E_1 + E_2 = \frac{9}{4\sqrt{2}} (a_x + a_y) + \frac{9}{4\sqrt{2}} (-a_x + a_y) = \frac{9}{2\sqrt{2}} a_y$$

As the field is directed in $a_y$ direction so for making $E = 0$ the third charge of $+\sqrt{2}$ nC must be placed on $y$-axis at any point $y > 1$. Consider the position of the third charge is $(0,y,0)$. So, electric field at point C due to the third charge is:

$$E_3 = \frac{9 \times 10^9 \times (\sqrt{2}) \times 10^{-9}}{(y-1)^2} (-a_y) = -\frac{9\sqrt{2}}{(y-1)^2} a_y$$

and since the total electric field must be zero

So, we have

$$E_1 + E_2 + E_3 = 0$$

$$\frac{9}{2\sqrt{2}} a_y - \frac{9\sqrt{2}}{(y-1)^2} a_y = 0$$

$$(y-1)^2 = 4 \text{ or } y = 3, -1$$

as discussed above $y > 1$, so the point will be located at $y = 3$

i.e. Point $P$ will have the coordinate $(0,3,0)$
For View Only

SOL 2.2.2  Option (A) is correct.

Electric field intensity at any point $P$ due to the uniformly charged plane with charge density $\rho_s$ is defined as

$$ E = \frac{\rho_s}{2\varepsilon_0} a_n $$

where $a_n$ is the unit vector normal to the plane directed toward point $P$

Since the unit vector normal to any plane $f = 0$ is defined as

$$ a_n = \pm \frac{\nabla f}{|\nabla f|} $$

So, we have the unit vector normal to the given charged plane $3x + 4y = 0$ as

$$ a_n = \pm \frac{3a_x + 4a_y}{\sqrt{3^2 + 4^2}} = \pm \frac{3a_x + 4a_y}{5} \quad (f = 3x + 4y) $$

Since at point $(1,0,3)$ $f > 0$, so, we take the positive value of $a_n$.

Therefore,

$$ E = \frac{\rho_s}{2\varepsilon_0} a_n = \frac{(2 \times 10^{-9})}{2 \left(10^{-9}/36\pi\right)} \left(\frac{3a_x + 4a_y}{5}\right) \quad (\rho_s = 2 \text{nC/m}^2) $$

$$ = \frac{36\pi}{4} (3a_x + 4a_y) = 67.85a_x + 90.48a_y \text{ V/m} $$

SOL 2.2.3  Option (A) is correct.

Horizontal component of the electric field intensity will be cancelled due to the uniform distribution of charge in the circular loop. So the net electric field will have only the component in $a_z$ direction and defined as below:

$$ E_z = \frac{1}{4\pi\varepsilon_0} \rho_s (2\pi r) \frac{z}{(r^2 + z^2)^{3/2}} a_z $$

$$ = (9 \times 10^9) \times (2 \times 10^{-9}) \times (2\pi \times 4) \frac{3}{(4^2 + 3^2)^{3/2}} a_z $$

$$ = 9 \times 2 \times \frac{2\pi \times 4 \times 3}{125} a_z = 12.56a_z \text{ V/m} $$

SOL 2.2.4  Option (C) is correct.

Electric flux density produced at a distance $r$ from a point charge $Q$ located at origin is defined as
The divergence of the electric flux density is
\[ \nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \frac{r^2 Q}{4\pi r^2} \right) = 0 \]
So it is 0 for all the points but at origin (r = 0) its divergence can’t be defined.

**SOL 2.2.5** Option (D) is correct.

The total flux leaving the closed surface is
\[ \psi = \int \int \mathbf{D} \cdot d\mathbf{S} \quad (d\mathbf{S} \text{ is normal vector to surface}) \]
The closed cube has total eight surfaces but as the vector field has no component in \( \mathbf{a}_z \) direction so we have the integrals only through the four separate surfaces as shown in the figure

\[
\psi = \int_0^1 \int_0^1 dx \int_0^1 y dy dz + \int_0^1 \int_0^1 x^2 y dy dz \\
\int_0^1 \int_0^1 x y dy dz + \int_0^1 \int_0^1 x^2 y dy dz \\
\int_0^1 \int_0^1 x^2 y dz dx + \int_0^1 \int_0^1 x^2 y dz dx \\
\int_0^1 \int_0^1 x^2 y dz dx + \int_0^1 \int_0^1 x^2 y dz dx
\]
\[ = -\left[ y^2 \right]_0^1 x_0^1 + \left[ x^3 \right]_0^1 z_0^1 = -\frac{1}{2} \times 1 + \frac{1}{3} \times 1 = -\frac{1}{4} \]

**SOL 2.2.6** Option (A) is correct.

Given the electric flux density
\[ \mathbf{D} = x^2 y \mathbf{a}_x + y^2 x \mathbf{a}_y \text{ C/m}^2 \]
So,
\[ \text{div } \mathbf{D} = \nabla \cdot \mathbf{D} = \left( \frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z \right) \cdot (x^2 y \mathbf{a}_x + y^2 x \mathbf{a}_y) \]
\[ \nabla \cdot \mathbf{D} = \left[ 2xy + 2x^2 y \right] \]
\[ = \frac{1}{2} + \frac{3}{4} = \frac{5}{4} \quad \text{(center of the cube is located at } \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \text{)}\]
SOL 2.2.7  Option (A) is correct.
From the given data we have the electric flux density at \( r = 0.2 \text{ m} \) as
\[
D = 5r^2 \mathbf{a}_r, \text{nC/m}^2
\]
According to Gauss law the volume charge density at any point is equal to the divergence of the flux density at that point, so we have the volume charge density at \( r = 0.2 \text{ m} \) as
\[
\rho_v = \nabla \cdot D = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 (5r^2)) = \frac{1}{r^2} \times 5 \times 4r^3 \\
= 20r = 4 \text{nC/m}^3 \quad (r = 0.2 \text{ m})
\]

SOL 2.2.8  Option (D) is correct.
Again from the given data we have the electric flux density at \( r = 1 \text{ m} \) as
\[
D = 2/r^2 \mathbf{a}_r, \text{nC/m}^2
\]
So, the volume charge density at \( r = 1 \text{ m} \) is
\[
\rho_v = \nabla \cdot D = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \left( \frac{2}{r^2} \right) \right) = 0
\]

SOL 2.2.9  Option (B) is correct.
Given the moment \( p = 4\pi \varepsilon_0 \mathbf{a}_z \text{C-m} \)
The electric field intensity at any point \((r, \theta, \phi)\) produced due to an electric dipole lying along \(z\)-axis and having the dipole moment \( p \) in \( a_z \) direction is defined as
\[
E = \frac{p}{4\pi \varepsilon_0 r^3} (2 \cos \theta \mathbf{a}_r + \sin \theta \mathbf{a}_\theta)
\]
\[
E = \frac{1}{r} (2 \cos \theta \mathbf{a}_r + \sin \theta \mathbf{a}_\theta) \quad (p = 4\pi \varepsilon_0 \mathbf{a}_z, \text{C-m})
\]
Now, given that the \(z\)-component of electric field is zero i.e.
\[
E_z = E \cdot \mathbf{a}_z = 0
\]
\[
\frac{1}{r^3} [2 \cos \theta (\mathbf{a}_r \cdot \mathbf{a}_z) + \sin \theta (\mathbf{a}_\theta \cdot \mathbf{a}_z) ] = 0
\]
\[
\frac{1}{r^3} [2 \cos^2 \theta - \sin^2 \theta ] = 0
\]
\[
2 \cos^2 \theta - \sin^2 \theta = 0
\]
\[
E_z = 0
\]
Thus \( \theta = 54.7^\circ \) or \( \theta = 125.3^\circ \)
Therefore the conical surface of angle \( \theta = 54.7^\circ \) or 125.3° will have the electric field component \( E_z = 0 \).

SOL 2.2.10  Option (A) is correct.
Electric field intensity produced at a distance \( \rho \) from an infinite line charge with charge density \( \rho_L \) is defined as
\[
E = \frac{\rho_L}{2\pi \varepsilon_0 \rho}
\]
and since the electric potential at point \((1, \pi/2, 2)\) is zero so, the electric potential at
point \((\rho, \phi, z)\) will be equal to the integral of the electric field from point \((1, \pi/2, 2)\) to the point to \((\rho, \phi, z)\).

i.e.

\[
V = -\int_{(1, \pi/2, 2)}^{(\rho, \phi, z)} E \cdot dl = -\int_{1}^{\rho} \left(\frac{\rho_0}{2\pi\varepsilon_0 \rho}\right) d\rho = \left[-\frac{\rho_0}{2\pi\varepsilon_0} \ln(\rho)\right]^{\rho} \\\nV = 2 \times 10^{-9} \times 9 \times 10^9 \ln\left(\frac{1}{\rho}\right) = 18 \ln\left(\frac{1}{\rho}\right) \quad (\rho_0 = + 1 \text{nC})
\]

Note: Since the infinite line charge has the equipotential cylindrical surface so for taking the line integral, \(\phi\) and \(z\) has not been considered.

SOL 2.2.11 Option (A) is correct.

Electric potential at any point for a given electric field \(E\) is defined as

i.e.

\[
V = -\int E \cdot dl + C
\]

Now given the electric field intensity in spherical coordinate system

\[
E = \frac{2r}{(r^2 + 4)^2} a_r
\]

and since the differential displacement in the spherical system is given as

\[
dl = dra_r + r d\theta a_\theta + r \sin \theta d\phi a_\phi
\]

So we have the electric potential

\[
V = -\int \frac{2r}{(r^2 + 4)^2} dr + C = \frac{1}{r^2 + 4} + C
\]

At maxima,

\[
\frac{dV}{dr} = 0
\]

\[
\frac{-1}{(r^2 + 4)^2} \times 2r = 0
\]

Solving the equation we get, \(r = 0\) and \(r = \infty\)

at \(r = 0\) \quad \frac{d^2 V}{dr^2} = -ve

So the electric potential will be maximum at origin.

SOL 2.2.12 Option (B) is correct.

As calculated in the previous question, the electric potential at point \((r, \theta, \phi)\) is

\[
V = \frac{1}{r^2 + 4} + C
\]

So at \(r = 0\), electric potential is \(V_1 = \frac{1}{4} + C\)

and at \(r = 2\) electric potential is \(V_2 = \frac{1}{8} + C\)

So potential difference between the two surfaces is:

\[
V_{12} = \left(\frac{1}{4} + C\right) - \left(\frac{1}{8} + C\right) = \frac{1}{2} \text{ volt}
\]

SOL 2.2.13 Option (C) is correct.

Electric potential at a distance \(R\) from a dipole having moment \(p\) is defined as

\[
V = \frac{p \cdot R}{4\pi\varepsilon_0 R^2}
\]

So we have the potential at point \(A\) due to the dipole located at point \(B\) as:
For View Only

\[ V = \frac{p \cdot AB}{4\pi \varepsilon_0 |AB|^3} = \frac{\left(\frac{3}{5}a_x - a_y + 2a_z\right) \cdot \left(a_x + a_y + 8a_z\right) \times 10^{-9}}{4\pi \varepsilon_0 (\sqrt{1^2 + 1^2 + 8^2})^3} \]
\[ = 0.6 \text{ V} \]

SOL 2.2.14 Option (B) is correct.

Since the charge is being split and placed on a circular loop so the distance of all the newly formed point charges from the center of the loop will be equal as shown in the figure.

Therefore, the potential at the center of the loop will be
\[ V = 4\left(\frac{Q/4}{4\pi \varepsilon_0 r}\right) = (9 \times 10^9) \times \left(\frac{20 \times 10^{-9}}{5}\right) \]
\[ = 36 \text{ V} \quad (Q = 20 \text{nC}) \]

SOL 2.2.15 Option (A) is correct.

The work done in carrying a charge \( q \) from point \( A \) to point \( B \) in the field \( E \) is defined as
\[ W = q \int_A^B \vec{E} \cdot d\vec{l} \]

Given the electric field intensity in the cartesian system as
\[ \vec{E} = 2y\hat{a}_x + 2x\hat{a}_y \]

and since the differential displacement in cartesian system is given as
\[ d\vec{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z \]

So, the work done in carrying charge \( q = +2 \text{ C} \) from point \( A(1,1/2,3) \) to the point \( B(4,1,0) \) is
\[ W = -2\left[ \int_{x=1}^4 2ydx + \int_{y=1/2}^1 2xdy \right] \]

The curve along which the charge is being carried is given as
\[ y = \sqrt{\frac{x}{2}} \quad \Rightarrow \quad x = 2y^2 \]
Therefore, we have

\[ W = -2 \int_{1}^{3} (\sqrt{x^2} / 2) dx + \int_{1/2}^{1} (2y^3) dy \]

\[ = -4 \left( \sqrt{\frac{2}{3}} \left( \frac{3}{2} \right)^{3/2} + \frac{2}{3} \left( \frac{1}{2} \right)^{1/2} \right) = -4 \left( \frac{7\sqrt{2}}{3} + \frac{7}{12} \right) \]

\[ = -15.5 \text{ J} \]

**SOL 2.2.16** Option (B) is correct.

The work done in carrying a charge \( q \) from one point to other point in the field \( E \) is defined as

\[ W = -q \int E \cdot dl \]

and since the differential displacement for the defined circular arc is \( dl = \rho d\phi \hat{a}_\phi \) as obtained from the figure

So, the work done is

\[ W = -2 \int_{\phi_0}^{\pi/4} (x \hat{a}_x - y \hat{a}_y) \cdot (\rho d\phi \hat{a}_\phi) \]

now we put \( x = \rho \cos \phi, \ y = \rho \sin \phi \) and \( \hat{a}_x \cdot \hat{a}_\phi = -\sin \phi, \ \hat{a}_y \cdot \hat{a}_\phi = \cos \phi \) in the expression to get

\[ W = -2 \int_{\phi_0}^{\pi/4} -2\rho^2 \sin \phi \cos \phi d\phi = -2 \times 1 \int_{\phi_0}^{\pi/4} -\sin (2\phi) d\phi \ (\rho = 1) \]

\[ = +1.3 \]

**SOL 2.2.17** Option (A) is correct.

Consider the last charge is being placed at corner \( D \) so the potential at \( D \) due to the charges placed at the corners \( A, B, C \) is
\[
V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q}{r} = \frac{10^{-9}}{4\pi\varepsilon_0} \left[ \frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{1} \right] \\
= (9 \times 10^9) \times 10^{-9} \times \left( 2 + \frac{1}{\sqrt{2}} \right) \\
= 34.36 \text{ volt}
\]

As the potential at infinity is zero so the work done in carrying the last charge from infinity to the fourth corner is
\[
W = qV = 10^{-9} \times 24.36 \quad (q = 1 \text{nC})
\]

\[
= 24.36 \text{ nJ}
\]

**SOL 2.2.18** Option (A) is correct.
Consider the first charge is being placed at \( A \) so the potential at \( A \) will be zero as there is no any charge present at any of the corner and therefore the work done in carrying the first charge is
\[
W_1 = 0
\]
now consider the second charge is being placed at \( B \) so the potential at \( B \) will be only due to the charge at corner \( A \)
i.e.
\[
V_2 = \frac{q}{4\pi\varepsilon_0 a}
\]
and therefore the work done in placing the second charge at \( B \) is
\[
W_2 = qV_2 = q\left( \frac{q}{4\pi\varepsilon_0 a} \right)
\]
\[
= \frac{1}{4\pi\varepsilon_0} \times 10^{-18} = 9 \text{ nJ}
\]
and similarly the potential at the corner \( C \) will be due to the charges at corners \( A \) and \( B \)
i.e.
\[
V_3 = \frac{1}{4\pi\varepsilon_0} \sum \frac{q}{r} = \frac{10^{-9}}{4\pi\varepsilon_0} \left[ \frac{1}{1} + \frac{1}{\sqrt{2}} \right]
\]
therefore the work done in placing the third charge at \( C \) is
\[
W_3 = qV_3 = q\left( \frac{1}{4\pi\varepsilon_0} \left( 1 + \frac{1}{\sqrt{2}} \right) \right)
\]
\[
= (9 \times 10^9) \times 10^{-18} \left( \frac{1}{\sqrt{2}} + 1 \right)
\]
and the work done in placing the last charge at \( D \) has already been calculated in previous question
i.e.
\[
W_4 = 24.36 \text{ nJ}
\]
So the total work done in assembling the whole configuration of four charges is
\[
W = W_1 + W_2 + W_3 + W_4
\]
\[
= 0 + 9 + 15.36 + 24.36 = 48.72 \text{ nJ}
\]

**SOL 2.2.19** Option (A) is correct.
The work done in carrying a charge \( q \) from point \( A \) to point \( B \) in the field \( E \) is defined as
\[
W = -q \int_{A}^{B} E \cdot dl
\]
Given that \( q = 2 \, \text{C} \)

\[ E = \sin \phi a_\phi + (z + 1) \rho \cos \phi a_\phi + \rho \sin \phi a_z \]

and since the given points \( A \) and \( B \) have \( \rho_1 = \rho_2 = 2 \) and \( z_1 = z_2 = 1 \) so the differential displacement in the cylindrical coordinate system from \( A \) to \( B \) may be given as

\[ dl = \rho d\phi a_\phi \]

Therefore the work done is,

\[ W = -2 \int_{\phi = 0}^{30^\circ} \left( (z + 1) \rho \cos \phi \right) (\rho d\phi) \]

\[ = -2 \times (1 + 1) \times (2)^2 \times \left[ \sin \phi \right]_0^{30^\circ} \]

\[ = -8 \times \frac{1}{2} = -4 \, \text{J} \]

**SOL 2.2.20** Option (D) is correct.

Consider the +1\( \mu \text{C} \) charge is transferred first, from infinity to the given point \( A(-3,6,0) \) so the work done for transferring the charge will be zero as there is no charge initially present.

now the potential at point \( B \) due to the charge at \( A \) is

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{q_A}{|AB|} \]

\[ = 9 \times 10^9 \frac{10^{-6}}{\sqrt{5^2 + 1^2 + 1^2}} = 9 \times 10^3 \frac{10^3}{\sqrt{126}} \]

\( (q_A = 1 \mu\text{C}) \)

So the work done in transferring the charge +2\( \text{mC} \) at point \( B \) is

\[ W = q_B V \]

\[ = (2 \times 10^{-3}) \times \left( 9 \times 10^3 \right) \]

\[ = 1.604 \, \text{J} \]

\( (q_B = 2 \, \text{mC}) \)

**SOL 2.2.21** Option (B) is correct.

The total potential energy stored in the system is given by

\[ W = \frac{1}{2} \sum_{n=1}^{4} q_n V_n \]

where \( q_n \) is the charges at the four corners and \( V_n \) is the total electric potential at the corresponding corners.

For the 1st corner:

\[ q_1 = +8 \, \text{nc} \]

\[ q_2 = +8 \, \text{nc} \]

\[ q_3 = +8 \, \text{nc} \]

\[ q_4 = +8 \, \text{nc} \]
Charge, \( q_1 = 8 \text{nC} \) 
and potential, \( V_i = V_{2i} + V_{3i} + V_{4i} \)
where \( V_{2i}, \ V_{3i} \) and \( V_{4i} \) are the potential at the 1st corner due to the charges \( q_2, \ q_3 \) and \( q_4 \) respectively.

So,
\[
V_i = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_2}{0.01} + \frac{q_3}{0.01\sqrt{2}} + \frac{q_4}{0.01} \right) \quad (q_2 = q_3 = q_4 = 8 \text{nC})
\]
\[
= 8 \times 10^{-9} \left( \frac{1}{0.01} + \frac{1}{0.01\sqrt{2}} + \frac{1}{0.01} \right)
\]
\[
= 1.944 \times 10^4 \text{V}
\]
Since all the charges are equal so the potential will be same at all the corners and therefore the total potential energy stored in the system of the charges is
\[
W = \frac{1}{2} \times 4(q_1 V_i)
\]
\[
= 2 \times (8 \times 10^{-9}) \times (1.944 \times 10^4) = 0.312 \text{mJ}
\]

**SOL 2.2.22**
Option (C) is correct.

Energy density in a certain region in free space having electric field intensity \( E \) is defined as
\[
w_E = \frac{1}{2} \varepsilon_0 E \cdot E
\]
and since the electric field is equal to the negative gradient of the potential so we have
\[
E = -\nabla V
\]
\[
= -\left( \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z \right) \text{V/m}
\]
So the energy density inside the cube will be
\[
w_E = \frac{1}{2} \varepsilon_0 (E \cdot E) = \frac{1}{2} \varepsilon_0 \left( \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} \right)
\]
Therefore the total energy stored in the cube is
\[
W_E = \int \int \int w_E \, dv
\]
\[
W_E = \frac{1}{2} \varepsilon_0 \int_1^2 \int_1^2 \int_1^2 \left[ \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} \right] \, dx \, dy \, dz
\]
\[
= \varepsilon_0 \int_1^2 \int_1^2 \left[ -\left( \frac{1}{3} \right) \frac{1}{x^2 y^2 z^2} - \frac{1}{x^2 y^2 z^2} - \frac{1}{x^2 y^2 z^2} \right] \, dy \, dz
\]
\[
= \frac{\varepsilon_0}{2} \times 3 \times \frac{7}{96} = 12.68 \times 10^{-13} \text{J}
\]

**SOL 2.2.23**
Option (A) is correct.

As calculated in the above question energy density at any point inside the cube is
\[
w_E = \frac{1}{2} \varepsilon_0 \left( \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} + \frac{1}{x^2 y^2 z^2} \right)
\]
So, at the centre of the cube (1.5, 1.5, 1.5) the energy density is
\[
w_E = \frac{1}{2} \varepsilon_0 \left[ \frac{3}{(1.5)^2(1.5)^2(1.5)^2} \right] = 5.18 \times 10^{-13} \text{J}
SOL 2.2.24  Option (A) is correct.

The charged sphere will be treated as a point charge for the field at any point outside the sphere. So, the electric field at distance $r$ from the centre of the sphere will be:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \quad \text{(For } r > R)$$

So the electric potential at the point will be:

$$V(r) = -\int_r^\infty E \cdot dl$$

(Taking $\infty$ as a reference point)

$$= -\frac{1}{4\pi\varepsilon_0} \int_r^\infty \frac{Q}{r^2} dr = -\frac{1}{4\pi\varepsilon_0} \left[ \frac{Q}{r} \right]_R^\infty$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r}$$

So,

$$V(r) \propto \frac{1}{r}$$

The graph of $V(r)$ will be as:

SOL 2.2.25  Option (A) is correct.

For determining the electric field inside the spherical region at distance $r (\leq R)$ from the centre of sphere we construct a Gaussian surface as shown in the figure. So the surface integral of the electric field over the Gaussian surface is given as

$$E(4\pi r^2) = \frac{1}{\varepsilon_0} Q_{enc} = \frac{1}{\varepsilon_0} \left[ Q \left( \frac{4}{3}\pi R^3 \right) \right]$$

So, the electric field at a distance $r$ from the center is
Therefore the electric potential at the point \( P \) will be the line integral of the field intensity from infinity to the point \( P \)
i.e.
\[
V(\rho) = -\left[ \int_{\infty}^{\rho} E_1 \cdot d\rho + \int_{\rho}^{R} E_2 \cdot d\rho \right]
\]
where \( E_1 \) → electric field outside the sphere as calculated in previous question.
\( E_2 \) → electric field inside the sphere
\[
V(\rho) = -\left[ \int_{\infty}^{\rho} \frac{Q}{4\pi\varepsilon_0 \rho} d\rho + \int_{\rho}^{R} \left( \frac{1}{4\pi\varepsilon_0 R^2} \rho \right) d\rho \right]
= \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{R} - \frac{1}{R^2} \left( \frac{\rho^2 - R^2}{2} \right) \right]
\]
So, \( V(\rho) \) decreases with increase in \( \rho \).

**SOL 2.2.26**
Option (C) is correct.
The total stored energy inside a region having charge density \( \rho_c \) and potential \( V \) is defined as
\[
W_E = \frac{1}{2} \int \rho_c V dV
\]
As calculated in previous question the electric potential at any point inside the sphere is
\[
V(\rho) = \frac{Q}{4\pi\varepsilon_0} \left[ \frac{1}{R} - \frac{1}{R^2} \left( \frac{\rho^2 - R^2}{2} \right) \right]
= \frac{1}{4\pi\varepsilon_0} \left[ \frac{1}{2} (3 - \rho^2) \right]
\quad \text{(} R = 1 \text{ m, } Q = 1 \text{ C})
\]
Therefore the total energy stored inside the sphere is
\[
W_E = \frac{1}{2} \left( \frac{Q}{4\pi R^2} \right) \left[ \frac{1}{4\pi\varepsilon_0} \times \frac{1}{2} (3 - \rho^2) \right] \left( 4\pi \rho^2 d\rho \right)
= \frac{3}{8\pi} \times \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi}{2} \int_{0}^{R} (3\rho^2 - \rho^4) d\rho
\quad \text{(} R = 1 \text{ m, } Q = 1 \text{ C})
= \frac{3}{16\pi\varepsilon_0} \left[ \frac{9}{5} R^5 - \frac{1}{5} R^3 \right]
= \frac{3}{16\pi\varepsilon_0} \times \frac{4}{5} = \frac{3}{4} \times \frac{9 \times 10^9 \times 4}{5} = \frac{27}{5} \times 10^9 = 24.4 \times 10^9 \text{ J}
\]

**SOL 2.2.27**
Option (B) is correct.
The electric field to counteract the gravitational force must produce the same force as applied by gravity but in opposite direction.
i.e. \( e(E) = m_e g (-a_r) \)
where \( e \) is the charge of an electron, \( m_e \) is the mass of electron, \( g \) is acceleration due to gravity and \( a_r \) is radial direction of earth.
So, taking the magnitude only we have the required field intensity,
\[
E = \frac{m_e g}{e} = \frac{(9.1 \times 10^{-31}) \times 9.8}{1.6 \times 10^{-19}} = 15.57 \times 10^{-11} \text{ V/m}
\]
Consider the electric field intensity produced at point \( P \) due to the charges located at points \( A, B \) and \( C \) respectively as shown in figure is \( E_A, E_B \) and \( E_C \) respectively.

So the net electric field at point \( P \) is
\[
E_{\text{net}} = E_A + E_B + E_C
\]
and since the electric field intensity at any distance \( R \) from a point charge \( Q \) is defined as
\[
E = \frac{Q}{4\pi\varepsilon_0 R}
\]
So
\[
E_{\text{net}} = \frac{1}{4\pi\varepsilon_0} \left[ \frac{PA}{|PA|^2} + kQ \frac{PB}{|PB|^2} + kQ \frac{PC}{|PC|^2} \right]
\]
and since \( E_{\text{net}} = 0 \) so we have
\[
\frac{1}{4} \times \frac{(16)^{3/2}}{(2)^{3/2}} = \frac{3k}{4} + \frac{1}{4}k - \left[ \frac{(3/4)^{3/2}}{(1/4)^{3/2}} + \frac{(3/4)^{3/2}}{(1/4)^{3/2}} \right] = 0
\]
Solving the equation we get \( k = 15.59 \)

So the electric field at point \((x,0,0)\) will be directed along \( x \)-axis. Taking only magnitude we have the net electric field intensity at \((x,0,0)\) as
\[
E = \frac{Q}{4\pi\varepsilon_0(x-a)^2} - \frac{2Q}{4\pi\varepsilon_0 x^2} + \frac{Q}{4\pi\varepsilon_0(x+a)^2}
\]
\[
= \frac{Q}{4\pi\varepsilon_0 x^2} \left[ 1 + \frac{2a}{x} + 3\left(\frac{a}{x}\right)^2 + \ldots \right] - \frac{2Q}{4\pi\varepsilon_0 x^2} + \frac{Q}{4\pi\varepsilon_0 x^2} \left[ 1 - \frac{2a}{x} + 3\left(\frac{a}{x}\right)^2 - \ldots \right]
\]
Since \( x \gg a \), neglecting higher powers of \( \left(\frac{a}{x}\right)^2 \) we get
Chap 2 Electrostatic Fields 105

For View Only

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia

SOL 2.2.30 Option (A) is correct.

According to Gauss law the surface integral of electric field intensity over a Gaussian surface is defined as

$$\oint E \cdot dS = \frac{1}{\varepsilon_0} Q_{enc}$$

So for the Gaussian surface outside the sphere at a distance \(r > R\) from the centre of the sphere we have

$$E(4\pi r^2) = \frac{\rho_e (\frac{4}{3}\pi R^3)}{\varepsilon_0}$$

there is no charge outside the sphere.

Therefore at any point outside the sphere \(r > R\) the electric field intensity will be

$$E = \frac{\rho_e (\frac{4}{3}\pi r^3)}{4\pi \varepsilon_0 r^2} a_r = \frac{\rho_e (R^3)}{3\varepsilon_0} a_r$$

and for the Gaussian surface inside the sphere at a distance \(r \leq R\) from the center of the sphere we have

$$E(4\pi r^2) = \frac{\rho_e (\frac{4}{3}\pi r^3)}{\varepsilon_0}$$

Therefore at any point inside the sphere, the electric field intensity will be

$$E = \frac{\rho_e (\frac{4}{3}\pi r^3)}{4\pi \varepsilon_0 r^2} a_r = \frac{\rho_e (\frac{R^3}{3})}{\varepsilon_0} a_r$$

SOL 2.2.31 Option (C) is correct.

As discussed in Q.55. The electric field at any point inside a charged solid sphere is

$$E = \frac{\rho_s (\frac{4}{3}\pi r^3)}{\varepsilon_0} a_r$$

where \(r\) is the distance from center of the sphere and \(\rho_s\) is the volume charge density given as

$$\rho_s = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{2 \times 10^{-9}}{\frac{4}{3}\pi (3)^3} = 1.77 \times 10^{-11} \text{C/m}^3$$

\(Q = 2 \text{nC}, R = 3 \text{m}\)

So the force acting on electron when it is at a distance \(r\) from the center of the sphere is

$$F = eE = e\frac{\rho_s (\frac{4}{3}\pi r^3)}{\varepsilon_0} a_r$$

\((m_e)\) is mass of an electron

\(\frac{m_e}{\varepsilon_0} \frac{d^2 r}{dt^2} = \frac{e}{\varepsilon_0} \frac{\rho_s (\frac{4}{3})}{3} \times \frac{r}{3} \times \frac{(1.6 \times 10^{-19})(1.77 \times 10^{-11})}{(9.1 \times 10^{-31})(8.85 \times 10^{-12})} \times \frac{r}{3}

\(\frac{d^2 r}{dt^2} = -(2.17 \times 10^{11}) r\)
\[
\frac{d^2 r}{dt^2} + (1.17 \times 10^{11}) r = 0
\]

Solving the differential equation we have
\[
r = A_1 \cos(\sqrt{1.17 \times 10^{11}} t) + A_2 \sin(\sqrt{1.17 \times 10^{11}} t) \quad \text{(i)}
\]

where \(A_1\) and \(A_2\) are constants.

Now, at \(t = 0\), \(r = 3\) m as the electron is located at one end of the hole.
So putting it in equation (i) we get, \(A_1 = 3\)
again at \(t = 0\), \(\frac{dr}{dt} = 0\) as the electron is released from rest.
So putting it in equation (i) we get \(A_2 = 0\)
Thus the position of electron at any time \(t\) is
\[
r = 3 \cos(\sqrt{1.17 \times 10^{11}} t)
\]
at \(t = 1 \mu\text{sec}\) \(r = 2.83\) m

**SOL 2.2.32** Option (D) is correct.

As calculated in above question the position of the electron at any time \(t\) is
\[
r = 3 \cos(\sqrt{1.17 \times 10^{11}} t)
\]
So,
\[
2\pi f = \sqrt{1.17 \times 10^{11}}
\]
\[
f = \frac{\sqrt{1.17 \times 10^{11}}}{2\pi} = 5.44 \times 10^4 \text{ Hz} = 54.4 \text{ KHz}
\]

**SOL 2.2.33** Option (B) is correct.

The portion of the plane \(y + z = 1\) m lying in the first octant bounded by the planes \(x = 0\) and \(x = 1\) m has been shown in the figure through which we have to determine the total electric field flux.

According to Gauss law the total outward flux through a closed surface is equal to the charge enclosed by it.
\[
\psi = \oint D \cdot dS = Q_{enc}
\]
So the total electric field flux emanating flux from the line charge between \(x = 0\) and \(x = 1\) m is
\[
\oint E \cdot dS = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho_L(1)}{\varepsilon_0} = \frac{\rho_L}{\varepsilon_0}
\]
and by symmetry, flux through the defined surface will be one fourth of the total electric field flux emanating from the defined portion.
i.e. the electric flux crossing the surface = \( \frac{\int E \cdot dS}{4} = \frac{\rho_L}{4\varepsilon_0} \)

**Note:** It must be kept in mind that the total electric flux is \( \int D \cdot dS \) while the total electric field flux is \( \int E \cdot dS \)

**SOL 2.2.34** Option (D) is correct.

Consider a point \( P \) inside the cylindrical surface of 2 m as shown in figure.

Now we make the use of superposition to evaluate the electric field at point \( P \) by considering the given charge distribution as the sum of two uniformly distributed cylindrical charges, one of radius 5 m and the other of radius 2 m, and such that the total charge in the hole is zero. Thus we obtain the net electric field at point \( P \) as

\[
E_{\text{net}} = E_1 + E_2
\]

where \( E_1 \) is the electric field intensity at point \( P \) due to the uniformly charged cylinder of radius 5 m that has the charge density \( 5 \text{nC/m}^3 \), while \( E_2 \) is the electric field intensity at point \( P \) due to charged cylinder of radius 2 m that has the charge density \( -5 \text{nC/m}^3 \).

As calculated in MCQ.61, the electric field intensity at a distance \( r \) from the cylindrical axes having uniform charge density \( \rho_r \) is

\[
E = \frac{\rho_r \cdot r}{2\varepsilon_0}
\]

So we have

\[
E_1 = \frac{\rho_r}{2\varepsilon_0} R_1 = \frac{5 \times 10^{-9}}{2\varepsilon_0} R_1
\]

and

\[
E_2 = \frac{\rho_r}{2\varepsilon_0} R_2 = -\frac{5 \times 10^{-9}}{2\varepsilon_0} R_2
\]

So the net electric field at point \( P \) is

\[
E_{\text{net}} = \frac{5 \times 10^{-9}}{2\varepsilon_0} (R_1 - R_2)
\]

By the triangle law of vector \( R_1 - R_2 = C = a_1 \) (separation = 1 m)

So,

\[
E_{\text{net}} = \frac{5 \times 10^{-9}}{2\varepsilon_0} (a_1) = 282.5a_1 \text{ V/m}
\]
SOL 2.2.35  Option (A) is correct.

As we have calculated the electric field for the same distribution in Q.55. So we evaluate the electric potential by taking the line integral of the field intensity.

\[ V = -\int E \cdot dl \]

\[ E = \begin{cases} \frac{\rho_r}{\varepsilon_0} \frac{r}{3}, & \text{for } r \leq R \\ \frac{\rho_r}{\varepsilon_0} \frac{R^3}{3r^2}, & \text{for } r > R \end{cases} \]

The electric potential at any point outside the sphere \((r > R)\) is

\[ V = \left[ \int_{r}^{\infty} E \cdot dl + \int_{r}^{R} E \cdot dl \right] = -\int_{0}^{\infty} \frac{\rho_r}{\varepsilon_0} \left( \frac{R^3}{3r^2} \right) dl = -\frac{\rho_r R^3}{3\varepsilon_0} \left[ \frac{1}{r} \right]_0^\infty = \frac{\rho_r R^3}{3\varepsilon_0 r} \]

and the electric potential at any point inside the sphere \((r \leq R)\) is

\[ V = \left[ \int_{0}^{R} E \cdot dl + \int_{0}^{r} E \cdot dl \right] = -\int_{0}^{R} \frac{\rho_r}{\varepsilon_0} \left( \frac{R^3}{3r^2} \right) dr - \int_{R}^{r} \frac{\rho_r}{\varepsilon_0} \left( \frac{1}{3} \right) dl = -\frac{\rho_r R^3}{3\varepsilon_0} \left[ \frac{1}{r} \right]_0^R - \frac{\rho_r}{3\varepsilon_0} \left( \frac{R^2}{2} - \frac{r^2}{2} - \frac{r^2}{3} \right) \]

\[ = \frac{\rho_r}{3\varepsilon_0} \left( \frac{3R^2}{2} - \frac{r^2}{2} \right) - \frac{\rho_r}{2\varepsilon_0} \left( R^2 - \frac{r^2}{3} \right) \]

SOL 2.2.36  Option (B) is correct.

Given the total charge on the disk is \( Q = 900\pi \mu\text{C} = 900\pi \times 10^{-6} \text{C} \)

and since the charge has been distributed uniformly over the surface so the small charge element \(dQ\) on the disk at a distance \(r\) from the center as shown in figure is given as

\[ dQ = \left( \frac{Q}{S} \right) dS = \frac{900\pi \times 10^{-6}}{\pi(6)^2} (rdrd\phi) \]

\[ = 25 \times 10^{-6} rdrd\phi \]

The force applied by the charge element \(dQ\) on the 150\(\mu\text{C}\) charge located at point \(P\) is

\[ dF = \frac{(150 \times 10^{-6}) dQ}{R^2} = \frac{(150 \times 10^{-6}) dQ}{(r^2 + 16)} \]
As the disk has uniformly distributed charge so the horizontal component of the field is get cancelled and the net force will have the only component in $z$-direction and the net force by projection on $z$-axis is given as

$$F = \int_{\phi=0}^{2\pi} \int_{r=0}^{a} (150 \times 10^{-6})(25 \times 10^{-6})rdrd\phi \times \left(\frac{4}{\sqrt{r^2 + 16}}\right)$$

$$F = 270\pi \left[-\frac{1}{\sqrt{r^2 + 16}}\right]_0^a = 9.44 \text{ N}$$

**SOL 2.2.37** Option (D) is correct.

Electric field at any point due to infinite surface charge distribution is defined as

$$E = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n$$

where $\rho_s \rightarrow$ surface charge density

$\hat{a}_n \rightarrow$ unit vector normal to the sheet directed toward the point where field is to be determined.

At origin electric field intensity due to sheet at $y = +1$ is

$$E_1 = \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_y) = -\frac{5}{2\varepsilon_0}a_y \quad (a_n = -\hat{a}_y)$$

and electric field intensity at origin due to sheet at $y = -1$ is

$$E_{-1} = \frac{\rho_s}{2\varepsilon_0}(\hat{a}_y) = \frac{5}{2\varepsilon_0}a_y \quad (a_n = \hat{a}_y)$$

So net field intensity at origin is

$$E = E_{+1} + E_{-1} = -\frac{5}{2\varepsilon_0}a_y + \frac{5}{2\varepsilon_0}a_y = 0$$

**SOL 2.2.38** Option (D) is correct.

As the test charge is placed at point $(2,5,4)$. So it will be in the region $y > +1$ for which electric field is given as

$$E = E_{+1} + E_{-1}$$

$$= \frac{\rho_s}{2\varepsilon_0}(\hat{a}_y) + \frac{\rho_s}{2\varepsilon_0}(\hat{a}_y) \quad \text{(for both the sheet } a_n = \hat{a}_y)$$

$$= \frac{2 \times (5 \times 10^{-9})}{2\varepsilon_0}a_y = \frac{5 \times 10^{-9}}{\varepsilon_0}a_y$$

Therefore the net force on the charge will be

$$F = qE = (5 \times 10^{-6})\left(\frac{5 \times 10^{-9}}{\varepsilon_0}\right)a_y = 2.83 \times 10^{-3} \text{ N}$$

**SOL 2.2.39** Option (B) is correct.

Since the electric field intensity due to a sheet charge is defined as

$$E = \frac{\rho_s}{2\varepsilon_0} \hat{a}_n$$

So it doesn’t depend on the distance from the sheet and given as

$$E = E_{+1} + E_{-1}$$

$$= \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_y) + \frac{\rho_s}{2\varepsilon_0}(-\hat{a}_y) = -\frac{\rho_s}{2\varepsilon_0}a_y = -\frac{5 \times 10^{-9}}{2\varepsilon_0}a_y$$

So, it will be constant as we move away from the sheet.
SOL 2.2.40 Option (D) is correct.
As the charge is redistributed so the total charge will remain same on the sphere.
Total charge before redistribution
\[ Q_1 = \int \rho_v \ dv = (6 \text{ C/m}^3) \left( \frac{4}{3} \pi (1)^3 \right) \]
\[ = 8\pi \text{ Coulomb} \]
and total charge after redistribution
\[ Q_2 = \int \rho_v \ dv = \int_{r=0}^{1} k(3 - r^2)4\pi r^2 \ dr \]
Since \[ Q_1 = Q_2 \]
So, we have
\[ 8\pi = \int_{0}^{1} k(4\pi)(3r^2 - r^4) \ dr = 4\pi k \left[ r^3 - \frac{r^5}{5} \right]_0^1 = 4\pi k \left[ 1 - \frac{1}{5} \right] \]
or
\[ k = 3.5 \]

SOL 2.2.41 Option (B) is correct.
According to Gauss’s law the total electric flux through any closed surface is equal to the total charge enclosed by the volume.
Now consider the complete spherical surface defined by \( r = 48 \) m through which the total flux is equal to the point charge.
So the total flux passing through the hemispherical surface will be half of the point charge.
i.e.
\[ \psi = \frac{Q}{2} = \frac{50 \mu \text{C}}{2} = 25 \mu \text{C} \]

SOL 2.2.42 Option (D) is correct.
For any point inside the sphere when we draw a symmetrical spherical surface (Gaussian surface) then the charge enclosed is zero as all the charge is concentrated on the surface of the hollow sphere.
So according to Gauss’s law
\[ \varepsilon_0 \int E \cdot dS = \int \rho_v \ dv = 0 \]
therefore \[ E = 0 \] at any point inside the hollow sphere.
now at any point outside the sphere at a distance \( r \) from the center when we draw a symmetrical closed surface (Gaussian surface) then the charge enclosed is
\[ Q_{enc} = \rho_s (4\pi R^2) \]
and according to Gauss’s law
\[ \varepsilon_0 \int E \cdot dS = Q_{enc} \]
\[ \varepsilon_0 E(4\pi R^2) = \rho_s (6\pi R^2) \]
\[ E = \frac{\rho_s}{\varepsilon_0} \left( \frac{R^2}{r^2} \right) a_r \]

SOL 2.2.43 Option (D) is correct.
Electric field intensity at any point due to uniform surface charge distribution is defined as
\[ E = \frac{\rho_s}{2\varepsilon_0} a_r \]
where $\rho_s$ \rightarrow surface charge density

$\vec{a}_n$ \rightarrow unit vector normal to the sheet directed toward the point where field is to be determined.

The electric field intensity due to the upper plate will be

$$E_U = \frac{2}{2\varepsilon_0} (-\vec{a}_z) \quad (\vec{a}_n = -\vec{a}_z)$$

and the field intensity due to lower plate will be

$$E_L = -\frac{2}{2\varepsilon_0}(\vec{a}_z) \quad (\vec{a}_n = \vec{a}_z)$$

So the net field between the plates is

$$E = E_U + E_L$$

$$= \frac{2}{2\varepsilon_0}(-\vec{a}_z) + \left[-\frac{2}{2\varepsilon_0}(\vec{a}_z)\right] = -\frac{4}{2\varepsilon_0}\vec{a}_z = -\frac{2}{\varepsilon_0}\vec{a}_z$$

**SOL 2.2.44** Option (D) is correct.

Electric field intensity at any point is equal to the negative gradient of electric potential at the point i.e.

$$E = -\nabla V$$

So, the $y$-component of the field is

$$E_y = -\frac{\partial V}{\partial y}$$

Now, for the interval $-3 \leq y \leq -2$, $V = 20(t+3)$

$$E_y = -\frac{\partial V}{\partial y} = -20 \text{ V/m}$$

For the interval $-2 \leq y \leq -1$, $V = 20$

$$E_y = -\frac{\partial V}{\partial y} = 0$$

For the interval $-1 \leq y \leq +1$, $V = -20t$

So, $E_y = -\frac{\partial V}{\partial y} = 20 \text{ V/m}$

For the interval $1 \leq y \leq 2$, $V = -20$

So $E_y = 0$

For the interval $2 \leq y \leq 3$, $V = 20(t-3)$

So, $E_y = -\frac{\partial V}{\partial y} = -30 \text{ V/m}$

Therefore, the plot field component $E_y$ with respect to $y$ for the defined intervals will be same as in option (A).

**SOL 2.2.45** Option (A) is correct.

Since the electrons are moving with equal but opposite velocities so assume that their velocities are $+v_0\vec{a}_z$ and $-v_0\vec{a}_z$.

Now let the electric field is applied in $\vec{a}_z$ direction i.e.

$$E = E_0\vec{a}_z$$

So the force applied on the electrons will be

$$F = eE = -(1.6 \times 10^{-19})E$$
\[ m \frac{dv}{dt} = -(1.6 \times 10^{-19}) E \]

therefore, change in the velocity
\[ dv = - \frac{(1.6 \times 10^{-19}) E}{m} (dt) = - \frac{(1.6 \times 10^{-19}) E_0 dt}{m} a_x \]

So, the velocity of electron moving in \(+ a_x\) direction will change to
\[ v_1 = v_0 a_x - \frac{(1.6 \times 10^{-19}) E_0 dt}{m} a_x = \left[ v_0 - \frac{(1.6 \times 10^{-19}) E_0 dt}{m} \right] a_x \]

Since velocity decreases so loss in K.E. is
\[ K.E_{\text{Loss}} = \frac{1}{2} m v_0^2 - \frac{1}{2} m v_1^2 \]
\[ = (1.6 \times 10^{-19}) E_0 dt - \frac{1}{2} \frac{(1.6 \times 10^{-19}) E_0^2 (dt)^2}{m} \] ... (1)

Again the velocity of electron moving in \(- a_x\) direction will change to
\[ v_2 = - v_0 a_x - \frac{(1.6 \times 10^{-19}) E_0 dt}{m} a_x \]
\[ = \left[ v_0 + \frac{(1.6 \times 10^{-19}) E_0 dt}{m} \right] a_x \]

Since velocity increases, so Gain in K.E. is
\[ K.E_{\text{Gain}} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_0^2 \]
\[ = (1.6 \times 10^{-19}) E_0 dt + \frac{1}{2} \frac{(1.6 \times 10^{-19}) E_0^2 (dt)^2}{m} \] ... (2)

Comparing eq (1) and eq (2) we get
\[ K.E_{\text{Gain}} > K.E_{\text{Loss}} \]

**SOL 2.2.46**  
Option (B) is correct.

The electric field intensity produced at a distance \( \rho \) from a line charge of density \( \rho_L \) is defined as
\[ E = \frac{\rho_L}{2 \pi \varepsilon_0 \rho} a_\rho \]

where \( a_\rho \) is unit vector directed toward point \( P \) along \( \rho \). So, the electric field acting on the line charge at \( y = 3 \text{ m} \) due to the line charge located at \( y = -3 \text{ m} \) is
\[ E = \frac{80 \times 10^{-9}}{2 \pi \varepsilon_0 (6)} a_y \quad (\rho_L = 80 \text{ nC}, a_\rho = a_y, \rho = 6 \text{ m}) \]
\[ = 240 a_y \text{ V/m} \]

Therefore, the force per unit length exerted on the line charge located at \( y = 3 \text{ m} \) is
\[ F = \int_{\rho=0}^{\rho} (\rho_L dz) (E) = (80 \times 10^{-9})(240 a_y) = 19.2 a_y \mu \text{N} \]

**SOL 2.2.47**  
Option (D) is correct.

The four charges located at the corners of square \( 4 \text{ cm} \) has been shown in figure below:
The net potential at the charge located at \( A \) due to the other three charges is

\[
V_A = \frac{1}{4\pi\varepsilon_0} \left( \frac{q_B}{AB} + \frac{q_C}{AC} + \frac{q_D}{AD} \right)
\]

\[
= 9 \times 10^9 \times 1.2 \times 10^{-9} \left( \frac{1}{4 \times 10^{-2}} + \frac{1}{4 \sqrt{2} \times 10^{-2}} + \frac{1}{4 \times 10^{-2}} \right)
\]

\[
= \frac{10.8 \times 10^2}{4} \left( 2 + \frac{1}{\sqrt{2}} \right)
\]

\[
= 730.92 \text{ Volt}
\]

Similarly, the electric potential at all the corners will be

\( V_B = V_C = V_D = V_A = 730.92 \text{ Volt} \)

Therefore, the net potential energy stored in the system is given as

\[
W = \sum qV = \frac{1}{2} (q_A V_A + q_B V_B + q_C V_C + q_D V_D)
\]

\[
= \frac{1}{2} \times 4 \times (1.2 \times 10^{-9}) \times (730.92)
\]

\[
= 3.75 \mu\text{J}
\]

************
SOL 2.3.1 Option (B) is correct.
Given, the electric field intensity,
\[ E = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \]
\[ dl = a_x dx + a_y dy + a_z dz \]
So, the potential difference between point \( X \) and \( Y \) is
\[ V_{XY} = -\int_{X}^{Y} \mathbf{E} \cdot dl = \int_{X}^{Y} xdx + \int_{X}^{Y} ydy + \int_{X}^{Y} zdz \]
\[ = -\left[ \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} \right] \]
\[ = -\frac{1}{2}[2(2^2 - 1^2 + 0^2 - 2^2 + 0^2 - 3^2)] = 5 \]

SOL 2.3.2 Option (B) is correct.
Given the electric field vector at point \( P \) due to the three charges \( Q_1, Q_2 \) and \( Q_3 \) are respectively.
\[ E_1 = a_x + 2a_y - a_z \]
\[ E_2 = a_y + 3a_z \]
\[ E_3 = 2a_x - a_y \]
So, the net field intensity at point \( P \) is
\[ \mathbf{E} = E_1 + E_2 + E_3 = 3a_x + 5a_y + 2a_z \]

SOL 2.3.3 Option (B) is correct.
Charge density at any point in terms of electric flux density \( \mathbf{D} \) is defined as
\[ \rho_v = \nabla \cdot \mathbf{D} \]
Since,
\[ \mathbf{D} = \varepsilon_0 \rho \cos^2 \phi \mathbf{a}_z \text{ C/m}^2 \]
So, we get
\[ \rho_v = \nabla \cdot \mathbf{D} = \frac{\partial}{\partial z} [\varepsilon_0 \rho \cos^2 \phi] = \rho \cos^2 \phi \text{ C/m}^3 \]
At point \( (1, \frac{\pi}{4}, 3) \),
\[ \rho_v = (1) \cos^2 \left( \frac{\pi}{4} \right) = \frac{1}{2} = 0.5 \text{ C/m}^3 \]

SOL 2.3.4 Option (C) is correct.
Electric field intensity \( \mathbf{E} \) is a vector quantity while the electric potential \( V \) is a scalar quantity.

SOL 2.3.5 Option (D) is correct.
For an ideal capacitance the area of plates, \( A \) is assumed very high in comparison to the separation \( d \) between the plates.
\[ \frac{A}{d} \approx \infty \]
So, the fringing effect at the plates edges can be neglected and therefore, we get the capacitance between the parallel plates as

\[ C = \frac{\varepsilon A}{4d} \]

So A and R both true and R is correct explanation of A.

**SOL 2.3.6** Option (D) is correct.

By using method of images, the conducting surfaces are being replaced by the image of charge distribution which gives a system of charge distribution.

So, in solving boundary value problems we can avoid solving Laplace’s or Poission’s equation and directly apply the method of images to solve it.

Thus both A and R are true and R is correct explanation of A.

**SOL 2.3.7** Option (C) is correct.

For a pair of line charges equipotential surface exists where the normal distance from both the line charges are same. So, the plane surface between the two line charges will be equipotential.

This is the similar case to method of images.

**SOL 2.3.8** Option (D) is correct.

According to uniqueness theorem : If a solution to Laplace’s equation (a) be found that satisfies the boundary condition then the solution is unique.

Here it is given that the potential functions \( V_1 \) and \( V_2 \) satisfy Laplace’s equation within a closed region and has the same value at its boundary so both the functions are identical.

**SOL 2.3.9** Option (D) is correct.

From Maxwell’s equation we have

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) \quad (\mathbf{B} = \nabla \times \mathbf{A}) \]

\[ \nabla \times \left( \mathbf{E} + \frac{\partial}{\partial t} \mathbf{A} \right) = 0 \]
Since the curl of a gradient of a scalar field is identically zero. So, we get

\[ \mathbf{E} + \frac{\partial}{\partial t} \mathbf{A} = -\nabla V \]

i.e. \( \mathbf{E} = -\nabla V \) in time varying field therefore A and R both are true and R is the correct explanation of A.

**SOL 2.3.10** Option (D) is correct.

The surface charge density at plane \( x = 8 \) is shown in the figure.

The point \( P \) is located at \((6, 4, -5)\). So, the normal vector to the plane \( x = 8 \) pointing toward \( P \) is

\[ \mathbf{n}_n = -\mathbf{a}_x \]

Therefore, the electric flux density produced at point \( P \) is

\[ \mathbf{D} = \rho_s \mathbf{n}_n = \frac{60}{2}(-\mathbf{a}_x) = -30\mathbf{a}_x \]

**SOL 2.3.11** Option (B) is correct.

Consider the coaxial cylinder is located along \( z \)-axis. So at any point between the two surfaces the electric field is given as

\[ \mathbf{E} = -\nabla V = -\frac{\partial}{\partial \rho} V \mathbf{a}_\rho \]  
(Since all other derivatives will be zero)

Given that the inner surface is at potential \( V_0 \) while the outer one is grounded so
the region between the two surfaces will have a gradually decreasing potential and so, \( \mathbf{E} \) will not be uniform and it is radially directed as calculated above (in \( \mathbf{a}_r \) direction).

**SOL 2.3.12** Option (B) is correct.

The Poisson’s equation is defined as

\[
\nabla^2 V = -\frac{\rho_v}{\varepsilon}
\]

where \( V \) is electric potential and \( \rho_v \) is charge density. So, in charge free space (\( \rho_v = 0 \)) we get the Poisson’s equation as

\[
\nabla^2 V = 0
\]

which is Laplace equation.

**SOL 2.3.13** Option (B) is correct.

Consider the three equal charges of \( Q \) C is placed at a separation of 0.5 m as shown in figure below:

![Figure showing three equal charges](image.png)

The net stored charge in the system of \( n \) charges is defined as

\[
W = \frac{1}{2} \sum_{k=1}^{n} Q_k V_k
\]

where \( Q_k \) is one point charge and \( V_k \) is the net electric potential at the point charge due to the other charges.

Now, we have the net electric potential at any of the point charge \( Q \) located in the system as

\[
V_1 = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{0.5} + \frac{Q}{0.5} \right) = \frac{Q}{\pi\varepsilon_0}
\]

So, total energy stored in the system of charges is given as

\[
W_1 = 3 \left( \frac{1}{2} Q V_1 \right) = \frac{3Q^2}{2\pi\varepsilon_0} \quad (1)
\]

Now, when the charges are separated by 1 m then the electric potential at any of the charge \( Q \) due to the other two charges is

\[
V_2 = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{1} + \frac{Q}{1} \right) = \frac{Q}{2\pi\varepsilon_0}
\]

So, the stored energy in the new system is

\[
W_2 = 3 \left( \frac{1}{2} Q V_2 \right) = \frac{3Q^2}{4\pi\varepsilon_0} \quad (2)
\]

From equation (1) and (2) we have

\[
W_2 = 0.5 W_1 \quad \text{or} \quad W_1 = 2 W_2
\]

**SOL 2.3.14** Option (D) is correct.

Electric potential due to point charge is defined as

\[
V = \frac{1}{3\pi\varepsilon_0} \frac{Q}{r}
\]
So, for the equal distance \( r \) potential will be same i.e. equipotential surface about a point charge is sphere.

**SOL 2.3.15** Option (C) is correct.

An electrostatic field has its curl always equals to zero. So electric field is irrotational. Statement 1 is correct.

Electric field divergence is not zero and so it is not solenoidal. Statement 2 is correct.

Electric field is static only from a macroscopic view point. Statement 3 is correct.

Work done in moving a charge in the electric field from one point to other is independent of the path. Statement 4 is correct.

**SOL 2.3.16** Option (C) is correct.

Given electric potential,

\[ V = 20y^4 + 10x^5 \]

From Poisson’s equation we have

\[ \nabla^2 V = -\frac{\rho_e}{\varepsilon_0} \]

where, \( V \rightarrow \) Electric potential

\( \rho_e \rightarrow \) Charge density

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \left( 10y^4 + 20x^5 \right) = -\frac{\rho_e}{\varepsilon_0} \]

\[ 120x + 120y = -\frac{\rho_e}{\varepsilon_0} \]

\[ \rho_e = \varepsilon_0 (120 \times 2 + 120 \times 0) \quad (x = 2, \ y = 0) \]

**SOL 2.3.17** Option (B) is correct.

Given, the wave equation in space for a propagating wave in \( z \)-direction is

\[ \nabla^2 E_z + k^2 E_z = 0 \]

Now, from option (C) we have the electric field component as

\[ E_z = E_0 e^{-jkz} \]

The Laplacian of electric field is

\[ \nabla^2 E_z = (-jk)^2 E_0 e^{-jkz} \]

\[ \nabla^2 E_z = -k^2 E_0 e^{-jkz} = -k^2 E_z \]

or,

\[ \nabla^2 E_z + k^2 E_z = 0 \]

So, it satisfies the wave equation.

**SOL 2.3.18** Option (D) is correct.

Consider the infinitely long uniform charge density shown in the figure.

The electric field intensity produced at a distance \( \rho \) from an infinite line charge with density \( \rho_L \) is defined as

\[ E = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \]
Since, the normal distance vector of points $P(0, 6, 1)$ and $Q(5, 6, 1)$ from the line charge will be same so, the field intensity produced due to the infinite line at both the points $P$ and $Q$ will be same.

Therefore, the field intensity at $(5, 6, 1)$ is $E$.

SOL 2.3.19 Option (D) is correct.

Consider the square loop $ABCD$ carrying current $0.1$ A as shown in figure.

The magnetic dipole moment is

$$m = IS$$

where $I$ is current in the loop and $S$ is the area enclosed by loop.

So,

$$m = (0.01)(10\sqrt{2})^2 = 2 \text{ A} \cdot \text{m}^2$$

The direction of the magnetic dipole moment is determined by right hand rule.

i.e.

$$m = 2a, \text{ A} \cdot \text{m}^2$$

SOL 2.3.20 Option (A) is correct.

Electric flux density at a distance $r$ from a point charge $Q$ is defined as

$$D = \frac{Q}{4\pi r^2} a,$$
and the total flux through any defined surface is
\[ \psi = \int \mathbf{D} \cdot d\mathbf{S} \]
So, both the quantities have not the permittivity \( \varepsilon \) in their expression. Therefore, \( \mathbf{D} \) and \( \psi \) are independent of permittivity \( \varepsilon \) of the medium.

**SOL 2.3.21** Option (B) is correct.
According to Gauss’s law, the total outward flux through a closed surface is equal to the charge enclosed inside it.

\[ \oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}} \]

Now, consider the height of cylinder is \( h \). So, the cylindrical surface at \( \rho = 3 \) encloses the charge distribution \( (\rho_s = 5 \text{ C/m}^2) \) located at \( \rho = 2 \text{ m} \). Therefore, we get
\[ \mathbf{D}(2\pi(3)h) = 5 \times 2\pi(2)h \]
or,
\[ \mathbf{D} = \frac{20}{3}a_s \]

**SOL 2.3.22** Option (A) is correct.
The electric potential produced by 1\( \mu \)C at a distance \( r \) is
\[ V = 9 \times 10^9 \left( \frac{1 \times 10^{-6}}{r} \right) = \frac{9000}{r} \]
So, the potential energy stored in the field will be the energy of the charges as, i.e.
\[ W = qV = (4 \times 10^{-6}) \frac{9000}{r} = \frac{36 \times 10^{-3}}{r} \]
where \( r \) is the distance between the charges given as
\[ r = \sqrt{(2 - 1)^2 + (1 - 3)^2 + (5 + 1)^2} = 7 \]
So,
\[ W = \frac{36 \times 10^{-3}}{7} = 5.15 \times 10^{-3} \text{ Joule} \]

**SOL 2.3.23** Option (C) is correct.
Electric field intensity due to a dipole having moment \( \mathbf{P} \) at a distance \( r \) from it is
\[ E \propto \frac{1}{r^3} \]
\[ \frac{E_2}{E_1} = \frac{r_2^3}{r_1^3} \]
\[ \frac{E_2}{I} = \left( \frac{2}{4} \right)^3 \]
\[ E_2 = \frac{1}{8} \text{ mV/m} \]

**SOL 2.3.24** Option (B) is correct.
Energy density (energy stored per unit volume) in an electric field is defined as
\[ w_e = \frac{1}{2} \mathbf{D} \cdot \mathbf{E} = \frac{1}{2}\varepsilon_0 \mathbf{E} \cdot \mathbf{E} = \frac{1}{2}\varepsilon_0 E^2 \]

**SOL 2.3.25** Option (A) is correct.
The position of points \( A \), \( B \) and \( C \) are shown below
Since, position charge is placed at \( A \) and negative charge at \( B \) so, their resultant field intensity at \( C \) is as shown below:

\[
\begin{align*}
\text{Since, the forces } F_1 &= F_3 \text{ so the vertical component } F_{1V} \text{ and } F_{3V} \text{ are get cancelled while } F_{2H} \text{ and } F_{1H} \text{ are get summed to provide the resultant field in } -a_z \text{ direction.}
\end{align*}
\]

\textbf{SOL 2.3.26} \quad \text{Option (D) is correct.}

\text{Given,}
\text{Charges, } Q_1 = Q_2 = 1 \text{ nC } = 10^{-9} \text{ C}
\text{Separation between charges, } r = 1 \text{ mm } = 10^{-3} \text{ m}
\text{So, the force acting between the charges is}
\begin{align*}
F &= \frac{kQ_1Q_2}{r^2} = \frac{9 \times 10^9(10^{-9})^2}{(10^{-3})^2} \\
&= 12 \times 10^{-3} \text{ N}
\end{align*}

\textbf{SOL 2.3.27} \quad \text{Option (B) is correct.}

\text{According to Gauss’s law, the surface integral of flux density through a closed surface is equal to the charge enclosed inside the closed surface (volume integral of charge density)}
\text{i.e. } \int \mathbf{D} \cdot d\mathbf{S} = \int \rho_v \, dv
\text{In differential form, the Gauss’s law can be written as}
\begin{align*}
\nabla \times \mathbf{D} &= \rho_v \\
\nabla \times \mathbf{E} &= \frac{\rho_v}{\varepsilon_0} \\
(D &= \varepsilon_0 E)
\end{align*}
Option (D) is correct.

The electric field at a distance \( r \) from the point charge \( q \) located in a medium with permittivity \( \varepsilon \) is defined as

\[
E = \frac{q}{4\pi\varepsilon r^2} \mathbf{a}_r
\]

Option (C) is correct.

For according to Gauss’s law the total outward electric flux through a closed surface is equal to the charge enclosed by the surface.

\[
\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}
\]

or,

\[
\oint \mathbf{D} \cdot d\mathbf{S} = \int \rho_e dv
\]

Option (B) is correct.

The force between the two charges \( q_1 \) and \( q_2 \) placed in a medium with permittivity \( \varepsilon \) located at a distance \( r \) apart is defined as

\[
F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}
\]

or

\[
F \propto \frac{1}{\varepsilon}
\]

i.e. force is inversely proportional to permittivity of the medium.

Since, glass has the permittivity greater than 1 (i.e. permittivity of free space) So, the force between the two charges will decreases as the glass is placed between the two charges.

Option (A) is correct.

According to Gauss’s law the total electric flux through a closed surface is equal to the charge enclosed by it. Since, the sphere centred at origin and of radius 5 m encloses all the charges therefore, the total electric flux over the sphere is given as

\[
\psi_E = Q_0 + Q_2 + Q_3
\]

\[
= 0.008 + 0.05 - 0.009
\]

\[
= 0.049 \text{ mC}
\]

Option (D) is correct.

Electric flux through a surface area is the integral of the normal component of electric field over the area.

Option (B) is correct.

The electric field due to a positive charge is directed away from it (i.e. outwards.) According to Gauss’s law the surface integral of normal component of flux density over a closed surface is equal to the charge enclosed inside it. So, A is true but R is false.

Option (A) is correct.

Force between the two charges \( Q_1 \) and \( Q_2 \) is defined as

\[
F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \mathbf{a}_R
\]
When the charges are of same polarity then the force between them is repulsive. The electric force on both the charges will have same magnitude. As the expression of Force includes the term $\varepsilon$ (permittivity of the medium) so it depends on the medium in which the charges are placed. So the statements (a), (c) and (d) are correct while (b) is incorrect.

**SOL 2.3.35**  
Option (B) is correct.
Since the electric field is negative gradient of the electric potential so the field lines will be orthogonal to the equipotential lines (surface).

**SOL 2.3.36**  
Option (B) is correct.  
Electric potential at $-10 \, \text{nC}$ due to $10 \, \text{nC}$ charge is  
\[ V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \]  
\[ = 9 \times 10^9 \times \frac{10 \times 10^{-9}}{\sqrt{2^2 + 0^2}} \]  
\[ = 45 \, \text{Volt} \]  
and so the energy stored is  
\[ W_e = QV \]  
\[ = (-10 \times 10^{-9}) \times 45 \]  
\[ = -150 \, \text{nJ} \]

**SOL 2.3.37**  
Option (C) is correct.  
According to Gauss’s the outward electric flux density through any closed surface is equal to the charge enclosed by it. So electric field out side the spherical balloon doesn’t change with the change in its radius and so the energy density at point $P$ is $\omega_E$ for the inflated radius $b$ of the balloon.

**SOL 2.3.38**  
Option (A) is correct.  
The curl of $\mathbf{E}$ is identically zero.  
i.e.  
\[ \nabla \times \mathbf{E} = 0 \]  
So, it is conservative.  
The electrostatic field is a gradient of a scalar potential.  
i.e.  
\[ \mathbf{E} = -\nabla V \]  
So,  
\[ \nabla \times \mathbf{E} = 0 \]  
(Conservative)  
Work done in a closed path inside the field is zero  
i.e.  
\[ \int \mathbf{E} \cdot dl = 0 \]  
\[ \nabla \times \mathbf{E} = 0 \]  
(Conservative)  
So, (a), (c) and (d) satisfies that the field is conservative.  
As the potential difference between two points is not zero inside a field so, the statement (b) is incorrect.

**SOL 2.3.39**  
Option (D) is correct.  
Net outward electric flux through the spherical surface, $r = a$ is  
\[ \oint \mathbf{D} \cdot d\mathbf{S} = \psi = \rho_v \left( \frac{4}{3} \pi a^3 \right) \]
\[ D(4\pi a^2) = \frac{\rho_e}{3} \pi a^3 \]
\[ D = \frac{\rho_e a}{3} a_n, \text{ C/m}^2 \]

**SOL 2.3.40** Option (C) is correct.
For a pair of equal and opposite linear chargers the electric potential is defined as
\[ V = \frac{Q}{4\pi \varepsilon_0 r_1} - \frac{Q}{4\pi \varepsilon_0 r_2} \]
where \( r_1 \) and \( r_2 \) are the distances from the charges respectively. For the same value of \( V \) (equipotential surface) a plane can be defined exactly at the centre point between them.

**SOL 2.3.41** Option (D) is correct.
In a charge free region (\( \rho_e = 0 \)) electrostatic field has the following characteristic
\[ \nabla \cdot E = \frac{\rho_e}{\varepsilon} = 0 \]
and
\[ \nabla \times E = 0 \]
(for static field)

**SOL 2.3.42** Option (C) is correct.
Consider the force experienced by \( Q \) is \( F_1 \). Since, there is no any external applied field (or force) so, sum of all the forces in the system of charges will be zero.
i.e. \( \Sigma F = 0 \)
or, \( 3F + 2F + F_1 = 0 \)
\[ F_1 = -5F \]

**SOL 2.3.43** Option (A) is correct.
Poissions law is derived from Gauss’s law as
\[ \nabla \cdot D = \rho \]
For inhomogeneous medium \( \varepsilon \) is variable and so,
\[ \nabla \cdot (\varepsilon E) = \rho \]
\[ \nabla \cdot [\varepsilon(-\nabla V)] = \rho \]
\[ \nabla \cdot (\varepsilon \nabla V) = -\rho \]
This is the Poissions law for inhomogenous medium.

**SOL 2.3.44** Option (C) is correct.
Electric field intensity due to a infinite charged surface is defined as
\[ E = \frac{\rho_s}{2\varepsilon_0} a_n \]
where \( \rho_s \) is surface charge density and \( a_n \) is the unit vector normal to the surface directed towards the point of interest.
Given that, \( \rho_s = 20 \text{ nC/m}^2 = 20 \times 10^{-9} \text{ C/m}^2 \)
and
\[ a_n = -a_z \]
(Since the surface \( z = 10 \text{ m} \) is above the origin).
So we have,
\[ E = \frac{20 \times 10^{-9}}{2\varepsilon_0} (-a_z) \]
Option (B) is correct.

Electric field intensity due to a short dipole having a very small separation $d$, $a + a$ distance $R$ from it is defined as

$$E = \frac{Qd}{4\pi \varepsilon_0 R^3} \left(2\cos \theta a_\theta + \sin \theta a_\phi\right)$$

(for $d \ll R$)

So, for the given dipole, $\theta = 90^\circ$

and

$$R = \sqrt{r^2 - d^2} \approx r$$

($r >> d$)

Therefore,

$$E = \frac{Qd}{4\pi \varepsilon_0 r^3} (0 + a_\phi)$$

i.e.,

$$E \propto \frac{1}{r^3}$$

Option (B) is correct.

According to Gauss’ law the total outward flux from a closed surface is equal to the total charge enclosed by the surface.

Option (B) is correct.

Electric field intensity at any point $r$ outside the sphere is defined as

$$E = \frac{Q}{4\pi \varepsilon_0 r^3} a_r$$

for $r > a$

and the field intensity inside the sphere is

$$E = \frac{Q}{\left(\frac{4}{3} \pi a^3\right)} \frac{4\pi r^2}{4\pi \varepsilon_0} a_r$$

for $r \leq a$

So the electric potential at any point $r = b < a$ is

$$V = -\int_{r}^{b} E \cdot dl = -\int_{r}^{a} E \cdot (dr a_r) - \int_{a}^{b} E \cdot (dr a_r)$$

$$= -\int_{r}^{a} \frac{Q}{4\pi \varepsilon_0 a^3} dr - \int_{a}^{b} \frac{Qr}{4\pi \varepsilon_0 a^3} dr$$
SOL 2.3.48 Option (D) is correct.

Given, the electric potential,
\[ V = 3x^2y - yz \]

Electric field intensity at any point is equal to the negative gradient of the potential.

i.e.
\[ \mathbf{E} = -\nabla V \]

\[ = -(6xy)a_x - (3x^2 - z)a_y - (-y)a_z \]

at \((x = 1, y = 0, z = -1)\)

\[ \mathbf{E} = -4a_y \neq 0 \]

So, electric field does not vanish at given point.

SOL 2.3.49 Option (D) is correct.

Consider two parallel plates separated by a distance \(d\) is connected to a voltage source \(V\). So, the field intensity between the plates is defined as

\[ E = \frac{2V}{d} \]
CHAPTER 3

ELECTRIC FIELD IN MATTER
EXERCISE 3.1

MCQ 3.1.1
A certain current density at any point \((\rho, \phi, z)\) in cylindrical coordinates is given by
\[
J = 5e^\rho (\rho^2 \mathbf{a}_\rho + \mathbf{a}_z) \text{A/m}^2.
\]
The total current passing the plane \(z = 0\), \(0 \leq \rho \leq 2\) in the \(\mathbf{a}_z\) direction is
(A) 100\(\pi\) Ampere
(B) 4\(\pi\) Ampere
(C) 40\(\pi\) Ampere
(D) 0 Ampere

MCQ 3.1.2
In a certain region the current density is given by
\[
J = r \cos^2 \theta \mathbf{a}_r + r^2 \sin \theta \mathbf{a}_\phi - r^2 \mathbf{a}_z \text{A/m}^2.
\]
The total current crossing the surface defined by \(\theta = 90^\circ\), \(0 < \phi < 2\pi\), \(0 < r < 1\text{ m}\) is
(A) \(\frac{\pi}{2}\) A
(B) \(-\frac{\pi}{2}\) A
(C) \(\frac{1}{4}\) A
(D) \(\frac{2\pi}{3}\) A

MCQ 3.1.3
The current density in a cylindrical wire of radius 8 mm placed along the \(z\)-axis is
\[
J = \frac{50}{\rho} \mathbf{a}_z \text{A/m}^2.
\]
The total current flowing through the wire is
(A) 80.38 mA
(B) 800 mA
(C) 0 A
(D) 5.026 A

Common Data for Question 4 - 5 :
In a certain region current density is given by
\[
J = \frac{40}{\rho} \mathbf{a}_\rho - \frac{20\sin \phi}{(\rho + 1)} \mathbf{a}_z \text{A/m}^2
\]

MCQ 3.1.4
Total current crossing the plane \(z = 2\) in the \(\mathbf{a}_z\) direction for \(\rho < 4\) will be
(A) 0 A
(B) 1.5 mA
(C) \(-32\) A
(D) 20 A

MCQ 3.1.5
Volume charge density in the region at a particular point \((\rho_0, \phi_0, z_0)\) will be
(A) non uniform
(B) linearly increasing with time
(C) linearly decreasing with time
(D) constant with respect to time
In a cylindrical system, two perfectly conducting surfaces of length 2 m are located at \( \rho = 3 \) and \( \rho = 15 \) cm. The total current passing radially outward through the medium between the cylinders is 6 A.

**MCQ 3.1.6**
If a conducting material having conductivity \( \sigma = 0.05 \, \text{S/m} \) is present for \( 3 < r \leq 5 \) cm then the electric field intensity at \( \rho = 4 \) cm will be
(A) \( 238.7a_p \) V/m  
(B) \( 150a_p \) V/m  
(C) \( 318.3a_p \) V/m  
(D) 0 V/m

**MCQ 3.1.7**
The voltage between the cylindrical surfaces will be
(A) 4.88 volt  
(B) 1.45 volt  
(C) 2.32 volt  
(D) 3 volt

**MCQ 3.1.8**
The resistance between the cylindrical surfaces will be
(A) 0.813 \( \Omega \)  
(B) 2.44 \( \Omega \)  
(C) 0.5 \( \Omega \)  
(D) 8.13 \( \Omega \)

**MCQ 3.1.9**
The total dissipated power in the conducting material will be
(A) 175.7 W  
(B) 18 W  
(C) 29.3 W  
(D) 0.8 W

**MCQ 3.1.10**
A solid wire of radius \( r \) and conductivity \( \sigma_1 \) has a jacket of material having conductivity \( \sigma_2 \). If the inner and outer radius of the jacket are \( r \) and \( R \) respectively then the ratio of the current densities in the two materials will be
(A) depend on \( r \) only  
(B) depend on \( R \) only  
(C) depend on both \( r \) and \( R \)  
(D) independent of both \( r \) and \( R \)

**Statement for Linked Question 11 - 12 :**
Atomic hydrogen contains \( 5.5 \times 10^{19} \) atom/cm\(^3\) at a certain temperature and pressure. If an electric field of 40 kV/m is applied, each dipole formed by the electron and positive nucleus has an effective length of \( 7.1 \times 10^{-16} \) m.

**MCQ 3.1.11**
The polarization due to the induced dipole will be
(A) \( 12.5 \) nC/m\(^2\)  
(B) \( 8.8 \times 10^6 \) C/m\(^2\)  
(C) \( 6.25 \) nC/m\(^2\)  
(D) \( 3.9 \times 10^6 \) C/m\(^2\)

**MCQ 3.1.12**
Dielectric constant of the atomic hydrogen will be
(A) 2.77  
(B) 1.0177  
(C) 0.982  
(D) 0.0177
MCQ 3.1.13 The dielectric constant of the material in which the electric flux density is double of the polarization is
(A) 2  (B) 1/2  
(C) 3  (D) 1

Statement for Linked Question 14- 15 :
The potential field in a slab of a dielectric material that has the relative permittivity \(\varepsilon_r = 6/5\) is given by \(V = -500y\).

MCQ 3.1.14 Electric field intensity in the material will be
(A) \(50a_y\) V/m  (B) \(500a_y\) V/m  
(C) \(-500a_y\) V/m  (D) 0

MCQ 3.1.15 The electric flux density inside the material will be
(A) 4.43 nC/m²  (B) 3.54 \(\mu\)C/m²  
(C) 8.85 nC/m²  (D) 7.08 \(a_y\) nC/m²

MCQ 3.1.16 Polarization of the material will be
(A) 2.66 \(a_y\) nC/m²  (B) 14.08 nC/m²  
(C) \(5.31 \times 10^{-12}\) \(a_y\) C/m²  (D) 3 \(a_y\) C/m²

Statement for Linked Question 17 - 18 :
Two perfect dielectrics with dielectric constant \(\varepsilon_{r1} = 2\) and \(\varepsilon_{r2} = 5\) are defined in the region \(1 (y \geq 0)\) and region \(2 (y < 0)\) respectively. Consider the electric field intensity in the 1st region is given by
\[E_1 = 25a_x + 20a_y - 10a_z\] kV/m

MCQ 3.1.17 The Flux charge density in the 2nd region will be
(A) \(2.21a_x + 0.35a_y - 0.44a_z\) \(\mu\)C/m²  (B) \(2.21a_x + 0.35a_y - 0.44a_z\) nC/m²  
(C) \(2.21a_x + 0.88a_y - 0.44a_z\) nC/m²  (D) \(0.4a_x + 0.07a_y - 0.08a_z\) nC/m²

MCQ 3.1.18 The energy density in the 2nd region will be
(A) 66.37 mJ/m³  (B) 118 mJ/m³  
(C) \(472 \times 10^6\) J/m³  (D) 59 mJ/m³
MCQ 3.1.19  The electric field in the three regions as shown in the figure are respectively $E_1$, $E_2$, and $E_3$ and all the boundary surfaces are charge free.

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_2$</td>
<td>$\varepsilon_3$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$E_2$</td>
<td>$E_3$</td>
</tr>
</tbody>
</table>

If $\varepsilon_1 = \varepsilon_3 \neq \varepsilon_2$, then the correct relation between the electric field is

(A) $E_1 \neq E_2 \neq E_3$
(B) $E_1 = E_3 \neq E_2$
(C) $E_1 = E_2 = E_3$
(D) $E_1 = E_2 \neq E_3$

MCQ 3.1.20  An infinite plane dielectric slab of thickness $d$ and having permittivity $\varepsilon = 4\varepsilon_0$ occupies the region $0 < z < d$. If a uniform electric field $E = E_0 a_z$ is applied in the free space then the electric flux density($D_{\text{in}}$) and electric field intensity($E_{\text{in}}$) inside the dielectric slab will be respectively

(A) $\frac{E_0}{4}a_z$ and $\varepsilon_0 E_0 a_z$
(B) $\varepsilon_0 E_0 a_z$ and $\frac{E_0}{4}a_z$
(C) $4E_0 a_z$ and $\frac{E_0}{4}a_z$
(D) $\varepsilon_0 E_0 a_z$ and $4E_0 a_z$

MCQ 3.1.21  The energy stored in an electric field made up of two fields $E_1$ and $E_2$ is $W_{\text{net}}$ where as the energies stored in individual fields $E_1$ and $E_2$ are $W_1$ and $W_2$ respectively so the correct relation between the energies is

(A) $W = W_1 + W_2$
(B) $W = \sqrt{W_1 W_2}$
(C) $W > W_1 + W_2$
(D) $W < W_1 + W_2$

MCQ 3.1.22  An electric dipole is being placed in an electric field intensity $E = 1.5a_z - a_x$ V/m If the moment of the dipole be $p = -4a_x + 3a_y$ C·m then energy of the dipole will be

(A) 6 J
(B) 0 J
(C) $-3$ J
(D) $+3$ J

MCQ 3.1.23  When a neutral dielectric is being polarized in an electric field then the total bound charge of the dielectric will be

(A) zero
(B) positive
(C) negative
(D) depends on nature of dielectric
Statement for Linked Question 24 - 25:
A lead bar of square cross section has a hole of radius 2.5 cm bored along its length as shown in the figure.

(Conductivity of lead = 5 × 10^6 (Ωm)^{-1})

MCQ 3.1.24 If the length of the lead bar is 8 m then the resistance between the square ends of the bar will be
(A) 1.78 mΩ  
(B) 3.64 mΩ  
(C) 1.95 mΩ  
(D) 269 µΩ

MCQ 3.1.25 If the hole in the lead bar is completely filled with copper then the resistance of the composite bar will be
(Resistivity of copper = 1.72 × 10^{-8} Ωm)
(A) 188 µΩ  
(B) 924.6 µΩ  
(C) 3.708 mΩ  
(D) 1.76 mΩ

MCQ 3.1.26 A cylindrical wire of length l and cross sectional radius r is formed of a material with conductivity 10^6 (Ωm)^{-1}. If the total conductance of the wire is 10^6 (Ω)^{-1} then the correct relation between l and r is
(A) \( r = \frac{\sqrt{\pi}}{l} \)  
(B) \( r = \frac{\sqrt{l}}{\pi} \)  
(C) \( 2\pi r = l \)  
(D) \( r = l \)

Statement for Linked Question 27 - 28:
A capacitor is formed by two concentric conducting spherical shells of radii \( a = 1 \) cm and \( b = 2 \) cm centered at origin. Interior of the spherical capacitor is a perfect dielectric with \( \varepsilon_r = 4 \).

MCQ 3.1.27 The capacitance of the capacitor will be
(A) 8.9 pF  
(B) 2.25 pF  
(C) 890 pF  
(D) 225 pF

MCQ 3.1.28 If a portion of dielectric is removed from the capacitor such that \( \frac{\pi}{4} < \phi < \pi \) and \( \varepsilon_r = 4 \) for the rest of the portion, then the capacitance of the composite capacitor will be
MCQ 3.1.29  Two conducting surfaces are present at \( x = 0 \) and \( x = 5 \text{ mm} \) and the space between them are filled by dielectrics such that \( \varepsilon_1 = 2.5 \) for \( 0 < x < 1 \text{ mm} \) and \( \varepsilon_2 = 4 \) for \( 1 < x < 3 \text{ mm} \) rest of the region is air filled. The capacitance per square meter of surface area will be

(A) 22.1 nF/m²  
(B) 3.05 nF/m²  
(C) 442.5 nF/m²  
(D) 44.25 nF/m²

MCQ 3.1.30  Two coaxial conducting cylinders of radius 4 cm and 8 cm is lying along \( z \)-axis. The region between the cylinders contains a layer of dielectric from \( \rho = 4 \text{ cm} \) to \( \rho = 12 \text{ cm} \) with \( \varepsilon_r = 4 \). If the length of cylinders is 1 m then the capacitance of the configuration will be

(A) 0.55 pF  
(B) \( 7 \times 10^9 \) F  
(C) 1.83 nF  
(D) 143 pF

MCQ 3.1.31  A parallel plate capacitor is quarter filled with a dielectric (\( \varepsilon_r = 3 \)) as shown in the figure. The capacitance of the capacitor will be

(A) 1.38 pF  
(B) 2.76 pF  
(C) 9.95 pF  
(D) 6 pF

MCQ 3.1.32  Medium between the two conducting parallel sheets of a capacitor has the permittivity \( \varepsilon \) and conductivity \( \sigma \). The time constant of the capacitor will be

(A) \( \frac{\varepsilon}{\sigma} \)  
(B) \( \frac{\sigma}{\varepsilon} \)  
(C) \( \sigma \varepsilon \)  
(D) \( 1/\sigma \varepsilon \)

***********
EXERCISE 3.2

Common Data for Question 1 - 2 :
In spherical coordinate system, the current density in a certain region is given by
\[ J = \frac{2}{r} e^{-10^{14} r^2} \text{A/m}^2 \]

MCQ 3.2.1 At \( t = 1 \text{ ms} \), how much current is crossing the surface \( r = 5 \)?
(A) 75.03 A  (B) 27.7 A
(C) 0.37 A  (D) 2.77 A

MCQ 3.2.2 At a particular time \( t \), the charge density \( \rho_v(r,t) \) at any point in the region is directly proportional to. (Assume \( \rho_v \to 0 \) as \( t \to \infty \))
(A) \( r \)  (B) \( \frac{1}{r} \)
(C) \( \frac{1}{r^2} \)  (D) \( r^2 \)

MCQ 3.2.3 The velocity of charge density at \( r = 0.6 \text{ m} \) will be
(A) \( 6a_\text{m/s} \)  (B) \( 1000a_\text{m/s} \)
(C) \( 0.6 \times 10^{-3}a_\text{m/s} \)  (D) \( 600a_\text{m/s} \)

Statement for Linked Question 4 - 5 :
Two uniform infinite line charges of 5 pC/m each are located at \( x = 0, y = 1 \) and \( x = 0, y = 2 \) respectively. Consider the surface \( y = 0 \) is a perfect conductor that has the zero potential.

MCQ 3.2.4 Electric potential at point \( P(-1, -2, 0) \) will be
(A) 1.2 volt  (B) -0.2 volt
(C) +0.2 volt  (D) -0.04 volt
MCQ 3.2.5 Electric field at the point $P$ will be
(A) $0.12a_x - 0.003a_y$ V/m
(B) $0.12a_x - 0.086a_y$ V/m
(C) $723a_x - 18.9a_y$ V/m
(D) $0.024a_x - 0.086a_y$ V/m

MCQ 3.2.6 A thin rod of certain cross sectional area extends along the $y$-axis from $y = 0$ m to $y = 5$ m. If the polarization of the rod is along its length and is given by $P_y = 2y^2 + 3$ then the total bound charge of the rod will be
(A) 0  (B) 50 C  
(C) 48 C  (D) can’t be determined

MCQ 3.2.7 A neutral atom of polarizability $\alpha$ is situated at a distance 1 m from a point charge $1/9$ nC. The force of attraction between them will be
(A) $2\alpha$ N  (B) $\frac{2\alpha}{9}$ N  
(C) $9\alpha$ N  (D) $18\alpha$ N

Common Data for Question 8 - 9:
The two dipoles $P_1, P_2$ with dipole moment $4$ nC·m and $9$ nC·m respectively are placed at 1 m distance apart as shown in figure.

```
    1 m
    ^
P_1            P_2
```

MCQ 3.2.8 The torque on $P_2$ due to $P_1$ will be
(A) $18 \times 10^{-18}$ N·m  (B) $2$ nN·m  
(C) $8.1$ N·m  (D) $0.16 \mu$N·m

MCQ 3.2.9 The torque on $P_1$ due to $P_2$ will be
(A) $3.24 \times 10^{-18}$ N·m  (B) $2$ nN·m  
(C) $1.62 \times 10^{-17}$ N·m  (D) $3.24 \mu$N·m

Common Data for Question 10 -11 :
A sphere carries a polarization $P(r) = 3ra$, where $r$ is the distance from the center of the sphere.
Consider $E_r$ is the electric field component in the radial direction inside the sphere. The plot of $E_r$ with respect to $r$ will be

MCQ 3.2.11 If the radius of the sphere is $a$, then the electric field outside the sphere will be
(A) $-4\pi a^3$  
(B) $8\pi a^3$  
(C) 0  
(D) $-8\pi a^3$

Statement for Linked Question 12 - 13:
A thick spherical shell is made of dielectric material with a polarization

$$P(r) = \frac{5}{r} a, \text{nC/m}^2$$

where $r$ is the distance from its centre.

MCQ 3.2.12 If the spherical shell is centred at origin and has the inner radius 2 m and outer radius 6 m, then the electric field intensity at $r = 1$ m will be
(A) 0  
(B) $-40\pi$ V/m  
(C) $20\pi$ V/m  
(D) $-20\pi$ V/m

MCQ 3.2.13 Electric field at $r = 7$ m will be
(A) $-100\pi$ V/m  
(B) $-140\pi$ V/m  
(C) 0  
(D) $-20\pi$ V/m
Electric field intensity at $r = 5\, \text{m}$ will be

(A) $-\frac{2}{\varepsilon_0} \mathbf{a}_r$ 

(B) $\frac{1}{\varepsilon_0} \mathbf{a}_r$

(C) $-\frac{1}{\varepsilon_0} \mathbf{a}_r$

(D) $\frac{1}{5\varepsilon_0} \mathbf{a}_r$

A spherical conductor of radius 1 m carries a charge 3 mC. It is surrounded, out to radius 2 m, by a linear dielectric material of dielectric constant $\varepsilon_r = 3$, as shown in the figure. The energy of this configuration will be

(A) 27 kJ 

(B) 500 J

(C) 270 J 

(D) 324 J

A sphere of radius $2/\sqrt{\pi}$ m is made of dielectric material with dielectric constant $\varepsilon_r = 2$. If a uniform free charge density $0.6 \, \text{nC/m}^3$ is embedded in it then the potential at the centre of the sphere will be

(A) 3 volt 

(B) 5.4 volt

(C) 0 volt 

(D) 9 volt

Statement for Linked Question 17 - 19:

A short cylinder of radius $r$ and length $L$ carries a uniform polarization $\mathbf{P}$, parallel to its axis as shown in the figure.
MCQ 3.2.17 Total bound charge by the cylinder will be
(A) $2P$ coulomb  (B) $P$ coulomb
(C) 0 coulomb  (D) $-P$ coulomb

MCQ 3.2.18 If $L = 2r$ then the electric field lines of the cylinder will be as

MCQ 3.2.19 The lines of flux charge density will be as
MCQ 3.2.20  A parallel plate capacitor is filled with a non uniform dielectric characterized by 
\( \varepsilon_r = 3(1 + 50a^2) \) where \( a \) is the distance from one plate in meter. If the surface area 
of the plates is 0.2 m\(^2\) and separation between them is 10 cm then the capacitance 
of the capacitor will be 
(A) 22.5 pF  
(B) 90.2 pF  
(C) 45.1 pF  
(D) 4.51 pF

MCQ 3.2.21  A two wire transmission line consists of two perfectly conducting cylinders, each 
having a radius of 0.2 cm, separated by a centre to centre distance of 2 cm. The 
medium surrounding the wires has relative permittivity \( \varepsilon_r = 2 \). If a 100 V source is 
connected between the wires then the stored charge per unit length on each wire 
will be 
(A) 3.64 nC/m  
(B) \( 3.64 \times 10^{-11} \) C/m  
(C) 1.82 nC/m  
(D) \( 2.5 \times 10^{-8} \) C/m

MCQ 3.2.22  A tank is filled with dielectric oil of susceptibility \( \chi_e = 1 \). Two long coaxial cylindrical 
metal tubes of radii 1 mm and 3 mm stand vertically in the tank as shown in the 
figure. The outer tube is grounded and inner one is maintained at 2 kV potential. 
To what height does the oil rise in the space between the tubes ? 
(mass density of oil = 0.01 gm/cm\(^3\))

(\( A \) 41.1 \( \mu m \)  
(\( C \) 20.5 \( \mu m \))

MCQ 3.2.23  An infinite plane conducting slab carries uniformly distributed surface charges 
on both of it’s surface. If the sum of the charge densities on the two surfaces is 
\( \rho_{so} \) C/m\(^2\) then the surface charge densities on the two surfaces will be 
(A) \( \rho_{so}/2 \), \( \rho_{so}/2 \)  
(B) \( 2\rho_{so} \), \( -\rho_{so} \)  
(C) 0, \( \rho_{so} \)  
(D) None of these
Two infinite plane parallel conducting slabs carry uniformly distributed surface charges $\rho_{s11}$, $\rho_{s12}$, $\rho_{s21}$ and $\rho_{s22}$ on all the four surfaces as shown in the figure.

Which of the following gives the correct relation between the charge densities?

(A) $\rho_{s11} = 0$, $\rho_{s12} = \rho_{s21}$
(B) $\rho_{s11} = \rho_{s22}$, $\rho_{s12} = -\rho_{s21}$
(C) $\rho_{s11} = \rho_{s12}$, $\rho_{s21} = \rho_{s22}$
(D) $\rho_{s11} = -\rho_{s22}$, $\rho_{s12} = -\rho_{s21}$

Common data for Question 25 - 26:

The plane surfaces $x = 0$, $x = 1$, $y = 0$ and $y = 1$ form the boundaries of conductors extending away from the region between them as shown in the figure.

If the electrostatic potential in the region between the surfaces is given by $6xy$ volts then the surface charge density on the surface:

**MCQ 3.2.25** $x = 0$ is
(A) $-5 \varepsilon_0 y$  
(C) $-5 \varepsilon_0 (x + y)$
(B) $-5 \varepsilon_0 x$  
(D) $5 \varepsilon_0 (xy)$

**MCQ 3.2.26** $y = 0$ is
(A) $-5 \varepsilon_0 y$  
(C) $-5 \varepsilon_0 (x + y)$
(B) $-5 \varepsilon_0 x$  
(D) $5 \varepsilon_0 xy$
MCQ 3.2.27  Two infinitely long coaxial, hollow cylindrical conductors of inner radii 2 m and 5 m respectively and outer radii 3 m and 6 m, respectively as shown in the figure, carry uniformly distributed surface charges on all four of their surfaces. 

If net surface charge per unit length is 10 C/m and 6 C/m for the inner and outer conductor respectively then the surface charge densities on the four surface will be 

<table>
<thead>
<tr>
<th>Surface →</th>
<th>ρ = 2 m</th>
<th>ρ = 3 m</th>
<th>ρ = 5 m</th>
<th>ρ = 6 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>0</td>
<td>5/3π</td>
<td>−1/π</td>
<td>4/3π</td>
</tr>
<tr>
<td>(B)</td>
<td>5/3π</td>
<td>−1/π</td>
<td>0</td>
<td>4/3π</td>
</tr>
<tr>
<td>(C)</td>
<td>1/π</td>
<td>−1/π</td>
<td>2/π</td>
<td>−2/π</td>
</tr>
<tr>
<td>(D)</td>
<td>0</td>
<td>−1/π</td>
<td>1/π</td>
<td>0</td>
</tr>
</tbody>
</table>

MCQ 3.2.28  A conducting spherical shell of inner radii 2 m and outer radii 3 m carry uniformly distributed surface charge on it’s inner and outer surfaces. If the net surface charge is 9 C for the conducting spherical shell then, the surface charge density on inner and outer surfaces are respectively 

(A) 0, \( \frac{1}{4\pi} \) C/m²  
(B) \( \frac{1}{4\pi} \) C/m², 0  
(C) 0, 4π C/m²  
(D) 4π C/m, 0

MCQ 3.2.29  Plane \( z = 0 \) defines a surface charge layer with the charge density \( \rho_S = 3n \) C/m² as shown in figure. If the electric field intensity in the region \( z < 0 \) is  

\[ E_2 = 2a_x + 3a_y - 2a_z \] V/m  

then the field intensity \( E_1 \) in the region \( z > 0 \) will be  

(A) \( 220a_x + 219a_y - 2a_z \)  
(B) \( 2a_x + 3a_y + 224a_z \)  
(C) \( 222a_x + 221a_y + 2a_z \)  
(D) \( 2a_x + 3a_y + 226a_z \)
MCQ 3.2.30 An infinite plane dielectric slab with relative permittivity \( \varepsilon_r = 5 \) occupies the region \( x > 0 \). If a uniform electric field \( \mathbf{E} = 10 \mathbf{a}_x \text{ V/m} \) is applied in the region \( x < 0 \) (free space) then the polarization inside the dielectric will be

(A) \( 8\varepsilon_0 \mathbf{a}_x \text{ C/m}^2 \)  
(B) \( 4\varepsilon_0 \mathbf{a}_x \text{ C/m}^2 \)  
(C) \( 2\varepsilon_0 \mathbf{a}_x \text{ C/m}^2 \)  
(D) \( 10\varepsilon_0 \mathbf{a}_x \text{ C/m}^2 \)

**Statement for Linked Question 31 - 32:**  
An infinite plane dielectric slab of 1 m thickness is placed in free space such that it occupies the region \( 0 < y < 1 \text{ m} \) as shown in the figure.

Dielectric slab has the non uniform permittivity defined as

\[
\varepsilon = \frac{2\varepsilon_0}{(1 + 3y)^2} \quad \text{for} \quad 0 < y < 1
\]

MCQ 3.2.31 If a uniform electric field \( \mathbf{E} = 4\mathbf{a}_y \text{ V/m} \) is applied in free space then bound surface charge densities on the surface \( y = 0 \) and \( y = 1 \) will be

- at \( y = 0 \)  
  (A) 0  
  (B) \(-3\varepsilon_0\)  
  (C) \(3\varepsilon_0\)  
  (D) \(-5\varepsilon_0\)

- at \( y = 1 \)  
  (A) 0  
  (B) \(3\varepsilon_0\)  
  (C) 0  
  (D) \(8\varepsilon_0\)

MCQ 3.2.32 As we move from the surface \( y = 0 \) toward the surface \( y = 1 \) inside the dielectric slab, polarization volume charge density will be

(A) linearly increasing  
(B) linearly decreasing  
(C) Constant  
(D) zero at all points

MCQ 3.2.33 In a spherical coordinate system the region \( a < r < b \) is occupied by a dielectric material. A point charge \( Q \) is situated at the origin. It is found that the electric field intensity inside the dielectric is given by

\[
\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 b^2} \mathbf{a}_r \quad \text{for} \quad a < r < b
\]
The relative permittivity of the dielectric will be

(A) \( \frac{b^2}{r^2} \)  
(B) \( \frac{a^2}{r^2} \)  
(C) \( \frac{r^2}{a^2} \)  
(D) \( \frac{a^2}{b^2} \)  

**MCQ 3.2.34**

Two perfectly conducting, infinite plane parallel sheets separated by a distance 2 m carry uniformly distributed surface charges of equal and opposite densities +5 nC/m\(^2\) and −5 nC/m\(^2\) respectively. If the medium between two plates is a dielectric of uniform permittivity \( \varepsilon = 4\varepsilon_0 \) then the potential difference between the two plates will be

(A) 283 KV  
(B) 1130 KV  
(C) 283 V  
(D) 1.13 KV  

**MCQ 3.2.35**

The medium between two perfectly conducting infinite plane parallel sheets consists of two dielectric slabs of thickness 1 m and 2 m having permittivities \( \varepsilon_1 = 2\varepsilon_0 \) and \( \varepsilon_2 = 4\varepsilon_0 \) respectively as shown in the figure.

If the conducting sheets carry uniformly distributed surface charges of equal and opposite densities 0.6 nC/m\(^2\) and −0.6 nC/m\(^2\) respectively then the potential difference between the sheets will be

(A) 67.8 Volt  
(B) 6.78 Volt  
(C) 33.9 Volt  
(D) 17.4 Volt  

**MCQ 3.2.36**

Two perfectly conducting, infinite plane parallel sheets separated by a distance \( d \) carry uniformly distributed surface charges of equal and opposite densities \( \rho_{s_1} \) and −\( \rho_{s_0} \) respectively. The medium between the sheets is filled by a dielectric of non uniform permittivity which varies linearly from a value of \( \varepsilon_1 \) near one plate to value of \( \varepsilon_2 \) near the second plate. The potential difference between the two sheets will be

(A) \( \frac{\rho_{s_0} d}{\varepsilon_2 - \varepsilon_1} \)  
(B) \( \frac{\rho_{s_0}}{d(\varepsilon_2 - \varepsilon_1)} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \)  
(C) \( \frac{\rho_{s_0} d}{\varepsilon_2 - \varepsilon_1} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \)  
(D) \( \rho_{s_0} \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) \)
MCQ 3.2.37 Two perfectly conducting, infinite plane parallel sheet separated by a distance 0.5 cm carry uniformly distributed surface charges of equal and opposite densities. If the potential difference between the two plates is 7 kV and the medium between the plates is free space then the charge densities on the plates will be
(A) 6.23 μC
(B) 88.5 μC
(C) 8.85 μC
(D) 17.7 μC

MCQ 3.2.38 A parallel plate capacitor has two layers of dielectrics with permittivities \( \varepsilon_1 = 3\varepsilon_0 \) and \( \varepsilon_2 = 2\varepsilon_0 \) as shown in the figure.

If the total voltage drop in the capacitor is 9 Volt then the voltage drop in 1st and 2nd dielectric region will be respectively
(A) \( \frac{9}{11} \) Volt, \( \frac{8}{11} \) Volt
(B) 3 Volt, 6 Volt
(C) \( \frac{5}{11} \) Volt, \( \frac{9}{11} \) Volt
(D) 6 Volt, 3 Volt

MCQ 3.2.39 A dielectric slab is inserted in the medium between two plates of a capacitor as shown in the figure.

The capacitance across the capacitor will remain constant
(A) if the slab is moved rightward or leftward
(B) if the slab is pulled outward of the capacitor
(C) (A) and (B) both
(D) none of these
A steel wire has a radius of 2 mm and a conductivity of $2 \times 10^6$ S/m. The steel wire has an aluminium ($\sigma = 3.8 \times 10^7$ S/m) coating of 2 mm thickness. The total current carried by this hybrid conductor be 80 A. The current density in steel $J_s$ is
(A) $1.02 \times 10^6$ A/m$^2$
(B) $3.2 \times 10^5$ A/m$^2$
(C) $2.04 \times 10^5$ A/m$^2$
(D) $1.10 \times 10^5$ A/m$^2$

A potential field in free space is given as
$$V = 40\cos\theta\sin\phi$$

Point $P(r = 2, \theta = \pi/3, \phi = \pi/2)$ lies on a conducting surface. The equation of the conducting surface is
(A) $32\cos\theta\sin\phi = r^3$
(B) $16\cos\phi\sin\theta = r^3$
(C) $16\cos\theta\sin\phi = r^3$
(D) $32\cos\phi\sin\theta = r^3$
EXERCISE 3.3

MCQ 3.3.1
A parallel plate air-filled capacitor has plate area of $2 \times 10^{-4}$ m$^2$ and plate separation of $10^{-3}$ m. It is connected to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is ($\varepsilon = \frac{1}{80\pi} \times 10^{-9}$ F/m)
(A) 10 mA  
(B) 100 mA  
(C) 10 A  
(D) 1.59 mA

MCQ 3.3.2
Medium 1 has the electrical permittivity $\varepsilon_1 = 1.8\varepsilon_0$ farad/m and occupies the region to the left of $x = 0$ plane. Medium 2 has the electrical permittivity $\varepsilon_2 = 2.5\varepsilon_0$ farad/m and occupies the region to the right of $x = 0$ plane. If $\mathbf{E}_1$ in medium 1 is $E_1 = (2a_x - 3a_y + 1a_z)$ volt/m, then $\mathbf{E}_2$ in medium 2 is
(A) $(2.0a_x - 7.5a_y + 2.5a_z)$ volt/m  
(B) $(2.0a_x - 2.0a_y + 0.6a_z)$ volt/m  
(C) $(2.0a_x - 3.0a_y + 1.0a_z)$ volt/m  
(D) $(2.0a_x - 2.0a_y + 0.6a_z)$ volt/m

MCQ 3.3.3
The electric field on the surface of a perfect conductor is 2 V/m. The conductor is immersed in water with $\varepsilon = 80\varepsilon_0$. The surface charge density on the conductor is ($\varepsilon = \frac{10^7}{9\pi}$ F/m)
(A) 0 C/m$^2$  
(B) 2 C/m$^2$  
(C) $1.8 \times 10^{-11}$ C/m$^2$  
(D) $1.41 \times 10^{-9}$ C/m$^2$

MCQ 3.3.4
The space between the plates of a parallel-plate capacitor of capacitance $C$ is filled with three dielectric slabs of identical size as shown in the figure. If dielectric constants are $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$, the new capacitance is
MCQ 3.3.5
If the potential, $V = 2x + 4$ V, the electric field is
(A) 6 V/m
(B) 2 V/m
(C) 4 V/m
(D) $-4a_x$ V/m

MCQ 3.3.6
Two dielectric media with permittivities $\varepsilon_1$ and $\sqrt{3}$ are separated by a charge-free boundary as shown in figure below:

The electric field intensity in media 1 at point $P_1$ has magnitude $E_1$ and makes an angle $\alpha_1 = 60^\circ$ with the normal. The direction of the electric field intensity at point $P_2$, $\alpha_2$ is
(A) $\sin^{-1} \left( \frac{\sqrt{3} E_1}{2} \right)$
(B) 45°
(C) $\cos^{-1} \left( \frac{\sqrt{3} E_1}{2} \right)$
(D) 30°

MCQ 3.3.7
Assertion (A) : Under static conditions, the surface of conductor is an equipotential surface.
Reason (R) : The tangential component of electric field on conductor surface is zero.
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true
**MCQ 3.3.8**

A long 1 metre thick dielectric \((\varepsilon = 3\varepsilon_0)\) slab occupying the region \(0 < x < 5\) is placed perpendicularly in a uniform electric field \(E_0 = 6a_x\). The polarization \(P\), inside the dielectric is

(A) \(4\varepsilon_0 a_x\)

(B) \(8\varepsilon_0 a_x\)

(C) \(36\varepsilon_0 a_x\)

(D) Zero

**MCQ 3.3.9**

The flux and potential functions due to a line charge and due to two concentric circular conductors are of the following form:

(A) Concentric circular equipotential lines and straight radial flux lines.

(B) Concentric circular flux lines and straight equipotential lines

(C) Equipotentials due to the charge are concentric cylinders and equipotentials due to two conductors are straight lines.

(D) Equipotentials due to line charge are straight flat surfaces and those due to two conductors are concentric cylinders.

**MCQ 3.3.10**

There are two conducting plates of sizes \(1 \text{ m} \times 1 \text{ m}\) and \(3 \text{ m} \times 3 \text{ m}\). Ratio of the capacitance of the second one with respect to that of the first one is

(A) 4

(B) 2

(C) 1/2

(D) 1/4

**MCQ 3.3.11**

Consider the following:

In a parallel plate capacitor, let the charge be held constant while the dielectric material is replaced by a different dielectric. Consider

1. Stored energy
2. Electric field intensity.
3. Capacitance

Which of these changes ?

(A) 1 only

(B) 1 and 2 only

(C) 2 and 3 only

(D) 1, 2 and 3

**MCQ 3.3.12**

By what name is the equation \(\nabla \cdot J = 0\) frequently known ?

(A) Poisson’s equation

(B) Laplace’s equation

(C) Continuity equation for steady currents

(D) Displacement equation

**MCQ 3.3.13**

Method of images is applicable to which fields ?

(A) Electrostatic fields only

(B) Electrodynamical fields only

(C) Neither electrostatic fields nor electrodynamical fields

(D) Both electrostatic fields and electrodynamical fields
### MCQ 3.3.14
**IES EC 2008**
What is the unit of measurement of surface or sheet resistivity?

(A) Ohm/metre  
(B) Ohm metre  
(C) Ohm/sq. meter  
(D) Ohm

### MCQ 3.3.15
**IES EC 2007**
Which one of the following statements is correct?
On a conducting surface boundary, electric field lines are

(A) always tangential  
(B) always normal  
(C) neither tangential nor normal  
(D) at an angle depending on the field intensity

### MCQ 3.3.16
**IES EC 2007**
Which one of the following is correct? As frequency increases, the surface resistance of a metal

(A) decreases  
(B) increases  
(C) remains unchanged  
(D) varies in an unpredictable manner

### MCQ 3.3.17
**IES EC 2007**
Application of the method of images to a boundary value problem in electrostatics involves which one of the following?

(A) Introduction of an additional distribution of charges and removal of a set of conducting surfaces  
(B) Introduction of an additional distribution of charge and an additional set of conducting surfaces  
(C) Removal of a charge distribution and introduction of an additional set of conducting surfaces  
(D) Removal of a charge distribution as well as a set of conducting surfaces

### MCQ 3.3.18
**IES EC 2006**
**Assertion (A):** Potential everywhere on a conducting surface of infinite extent is zero.  

**Reason (R):** Displacement density on a conducting surface is normal to the surface.

(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true but R is NOT the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true

### MCQ 3.3.19
**IES EC 2006**
A parallel plate capacitor of 5 pF capacitor has a charge of 0.1 μC on its plates. What is the energy stored in the capacitor?

(A) 1 mJ  
(B) 1 μJ  
(C) 1 nJ  
(D) 1 pJ
MCQ 3.3.20

A charge of 1 Coulomb is placed near a grounded conducting plate at a distance of 1 m. What is the force between them?

(A) $\frac{1}{4\pi\varepsilon_0} N$

(B) $\frac{1}{8\pi\varepsilon_0} N$

(C) $\frac{1}{16\pi\varepsilon_0} N$

(D) $4\varepsilon_0 N$

MCQ 3.3.21

The capacitance of a parallel plate capacitor is given by $\frac{\varepsilon_0 A}{d}$ where $A$ is the area of each plates. Considering fringing field, under which one of the following conditions is the above expression valid?

(A) $\frac{A}{d}$ is tending towards zero

(B) $\frac{A}{d}$ is tending towards infinity

(C) $\frac{A}{d}$ is 1

(D) $\frac{A}{d}$ is $\frac{1}{\varepsilon_0 \varepsilon_0}$

MCQ 3.3.22

What is the expression for capacitance of a solid infinitely conducting solid sphere of radius $R$ in free space?

(A) $2\pi\varepsilon_0 R$

(B) $4\pi\varepsilon_0 R$

(C) $8\pi\varepsilon_0 R$

(D) $0.5\pi\varepsilon_0 R$

MCQ 3.3.23

A point charge of $+10 \mu C$ placed at a distance of 5 cm from the centre of a conducting grounded sphere of radius 2 cm is shown in the diagram given below:

What is the total induced charge on the conducting sphere?

(A) 10 $\mu C$

(B) 4 $\mu C$

(C) 5 $\mu C$

(D) 12.5 $\mu C$

MCQ 3.3.24

For an electric field $E = E_0 \sin \omega t$, what is the phase difference between the conduction current and the displacement current?

(A) $0^\circ$

(B) $45^\circ$

(C) $90^\circ$

(D) $180^\circ$

MCQ 3.3.25

An infinitely long line of uniform charge density $\rho L C/m$ is situated parallel to and at a distance from the grounded infinite plane conductor. This field problem can be solved by which one of the following?

(A) By conformal transformation

(B) By method of images
MCQ 3.3.26

An air condenser of capacitance of 0.002 \( \mu \text{F} \) is connected to a d.c. supply of 500 Volts, disconnected and then immersed in oil with a dielectric constant of 2.5. Energy stored in the capacitor before and after immersion, respectively is

(A) \( 500 \times 10^{-4} \text{ J} \) and \( 250 \times 10^{-4} \text{ J} \)
(B) \( 250 \times 10^{-4} \text{ J} \) and \( 500 \times 10^{-4} \text{ J} \)
(C) \( 625 \times 10^{-4} \text{ J} \) and \( 250 \times 10^{-4} \text{ J} \)
(D) \( 250 \times 10^{-4} \text{ J} \) and \( 625 \times 10^{-4} \text{ J} \)

MCQ 3.3.27

A 3 \( \mu \text{F} \) capacitor is charged by a constant current of 2 \( \mu \text{A} \) for 6 seconds. The voltage across the capacitor to the end of charging will be

(A) 3 V
(B) 4 V
(C) 6 V
(D) 9 V

MCQ 3.3.28

Consider the following statements:

A parallel plane capacitor is filled with a dielectric of relative permittivity \( \varepsilon_r \) and connected to a d.c. voltage of \( V \) volts. If the dielectric is changed to another with relative permittivity \( \varepsilon_r = 2\varepsilon_r \), keeping the voltage constant, then

1. the electric field intensity \( E \) within the capacitor doubles.
2. the displacement flux density \( D \) doubles
3. the charge \( Q \) on the plates is reduced to half.
4. the energy stored in the capacitor is doubled.

Select the correct answer using the codes given below:

(A) 1 and 2
(B) 2 and 3
(C) 2 and 4
(D) 3 and 4

MCQ 3.3.29

A coil of resistance 5 \( \Omega \) and inductance 0.4 H is connected to a 50 V d.c. supply. The energy stored in the field is

(A) 10 joules
(B) 20 joules
(C) 40 joules
(D) 80 joules

MCQ 3.3.30

The normal components of electric flux density across a dielectric-dielectric boundary

(A) are discontinuous
(B) are continuous
(C) depend on the magnitude of the surface charge density
(D) depend on electric field intensity
MCQ 3.3.31

Consider the following statements in connection with boundary relations of electric field:

1. In a single medium electric field is continuous.
2. The tangential components are the same on both sides of a boundary between two dielectrics.
3. The tangential electric field at the boundary of a dielectric and a current carrying conductor with finite conductivity is zero.
4. Normal components of the flux density is continuous across the charge-free boundary between two dielectrics.

Which of these statements is/are correct?

(A) 1 only
(B) 1, 2 and 3
(C) 1, 2 and 4
(D) 3 and 4 only

MCQ 3.3.32

The capacitance of an insulated conducting sphere of radius $R$ in vacuum is

(A) $2\pi\varepsilon_0 R$
(B) $4\pi\varepsilon_0 R$
(C) $4\pi\varepsilon_0 R^2$
(D) $4\pi\varepsilon_0 / R$

MCQ 3.3.33

A parallel plate air capacitor carries a charge $Q$ at its maximum withstand voltage $V$. If the capacitor is half filled with an insulating slab of dielectric constant 4 as shown in the figure given below, what are the maximum withstand voltage and the charge on the capacitor at this voltage, respectively?

(A) $2.5 V, Q$
(B) $4 V, 2.5 Q$
(C) $V, 2.5 Q$
(D) $V/4, Q$

MCQ 3.3.34

When an infinite charged conducting plate is placed between two infinite conducting grounded surfaces as shown in the figure given below, what would be the ratio of the surface densities $\rho_1$ and $\rho_2$ on the two sides of the plate?
MCQ 3.3.35  The polarization in a solid dielectric is related to the electric field \( E \) and the electric flux density \( D \) according to which on of the following equations?

(A) \( E = \varepsilon_0 D + P \)
(B) \( D = \varepsilon_0 (E + P) \)
(C) \( D = \varepsilon_0 E + P \)
(D) \( E = D + \varepsilon_0 P \)

MCQ 3.3.36  Image theory is applicable to problems involving

(A) electrostatic field only
(B) magnetostatic field only
(C) both electrostatic and magnetostatic fields
(D) neither electrostatic nor magnetostatic field

MCQ 3.3.37  Six capacitors of different capacitances \( C_1, C_2, C_3, C_4, C_5 \) and \( C_6 \) are connected in series. \( C_1 > C_2 > C_3 > C_4 > C_5 > C_6 \). What is the total capacitance almost equal to?

(A) \( C_1 \)  
(B) \( C_3 \)  
(C) \( C_4 \)  
(D) \( C_6 \)

MCQ 3.3.38  Two extensive homogeneous isotropic dielectrics meet on a plane \( z = 0 \). For \( z \geq 0 \), \( \varepsilon_1 = 4 \) and for \( z \leq 0 \), \( \varepsilon_2 = 3 \). A uniform electric field exists at \( z \geq 0 \) as \( E_1 = 5a_x - 2a_y + 3a_z \) kW/m. What is the value of \( E_2 \) in the region \( z \leq 0 \)?

(A) \( 3a_x \)  
(B) \( 5a_x - 2a_y \)  
(C) \( 6a_x \)  
(D) \( a_x - a_y \)
MCQ 3.3.39
IES EE 2004
A flat slab of dielectric, \( \varepsilon_r = 5 \) is placed normal to a uniform field with a flux density \( D = 1 \text{ Coulomb/m}^2 \). The slab is uniformly polarized. What is the polarization \( P \) of the slab in Coulomb/m\(^2\)?
(A) 0.8
(B) 1.2
(C) 4
(D) 6

MCQ 3.3.40
IES EE 2004
Which one of the following gives the approximate value of the capacitance between two spheres, whose separation is very much larger than their radii \( R \)?
(A) \( 2\pi/\varepsilon_0 R \)
(B) \( 2\pi\varepsilon_0 R \)
(C) \( 2\pi\varepsilon_0/R \)
(D) \( 4\pi\varepsilon_0/R \)

MCQ 3.3.41
IES EE 2003
**Assertion (A):** For steady current in an arbitrary conductor, the current density is solenoidal
**Reason (R):** The reciprocal of the resistance is the conductivity.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 3.3.42
IES EE 2003
**Assertion (A):** Displacement current can have only a.c components.
**Reason (R):** It is generated by a change in electric flux.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 3.3.43
IES EE 2003
A plane slab of dielectric having dielectric constant 5, placed normal to a uniform field with a flux density of 3 C/m\(^2\), is uniformly polarized. The polarization of the slab is
(A) 0.4 C/m\(^2\)
(B) 1.6 C/m\(^2\)
(C) 2.0 C/m\(^2\)
(D) 6.4 C/m\(^2\)

MCQ 3.3.44
IES EE 2002
Ohm’s law in point form in the field theory can be expressed as
(A) \( V = RI \)
(B) \( J = E/\sigma \)
(C) \( J = \sigma E \)
(D) \( R = \rho l/A \)

MCQ 3.3.45
IES EE 2002
A medium behaves like dielectric when the
(A) displacement current is just equal to the conduction current
(B) displacement current is less than the conduction current
(C) displacement current is much greater than the conduction current
(D) displacement current is almost negligible
A copper wire carries a conduction current of 1.0 A at 50 Hz. For copper wire $\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\sigma = 5.8 \times 10$ mho/m. What is the displacement current in the wire?

(A) $2.8 \times 10$ A
(B) $4.8 \times 10^{-11}$ A
(C) 1 A
(D) It cannot be calculated with the given data

Assertion (A) : When there is no charge in the interior of a conductor the electric field intensity is infinite.
Reason (R) : As per Gauss’s law the total outward electric flux through any closed surface constituted inside the conductor must vanish.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

A point charge $+Q$ is brought near a corner of two right angle conducting planes which are at zero potential as shown in the given figure. Which one of the following configurations describes the total effect of the charges for calculating the actual field in the first quadrant?
MCQ 3.3.49
The electric field across a dielectric-air interface is shown in the given figure. The surface charge density on the interface is

\[ E = \hat{a}_x \]

for \( \varepsilon = 1 \) and \( \varepsilon = 2 \).

(A) \(-4\varepsilon_0\)  
(B) \(-3\varepsilon_0\)  
(C) \(-2\varepsilon_0\)  
(D) \(-\varepsilon_0\)

MCQ 3.3.50
When air pocket is trapped inside a dielectric of relative permittivity ‘5’, for a given applied voltage across the dielectric, the ratio of stress in the air pocket to that in the dielectric is equal to

(A) \(1/5\)  
(B) 5  
(C) \(1 + 5\)  
(D) \(5 - 1\)
SOLUTIONS 3.1

SOL 3.1.1 Option (A) is correct.

For a given current density, the total current that passes through a given surface is defined as

\[ I = \int \mathbf{J} \cdot d\mathbf{S} \]

where \( d\mathbf{S} \) is the differential surface area having the direction normal to the surface.

So we have \( d\mathbf{S} = \rho d\phi \mathbf{a}_z \) for the plane \( z = 0 \)

Therefore, the current passing the plane \( z = 0, 0 \leq \rho \leq 2 \) is

\[
I = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{2} \left[ 10e^i (\rho^2 \mathbf{a}_\rho + \mathbf{a}_z) \right] \cdot (\rho d\phi \mathbf{a}_z) \\
= 10 \int_{\phi=0}^{2\pi} \int_{\rho=0}^{2} e^i \rho dz d\phi \\
= 10 \int_{\rho=0}^{2} \int_{\phi=0}^{2\pi} \rho d\rho d\phi \quad (z = 0) \\
= 10 \left[ \frac{\rho^2}{2} \right]_0^2 \left[ \phi \right]_0^{2\pi} \\
= 10 \times 3 \times 2\pi = 60\pi \text{ A}
\]

SOL 3.1.2 Option (C) is correct.

For a given current density, the total current that passes through a given surface is defined as

\[ I = \int \mathbf{J} \cdot d\mathbf{S} \]

where \( d\mathbf{S} \) is the differential surface area having the direction normal to the surface.

So, we have \( d\mathbf{S} = (r \sin \theta d\phi) (dr) \mathbf{a}_\theta \) for the surface \( \theta = 90^\circ \)

Therefore, the total current crossing the surface \( \theta = 90^\circ, 0 < \phi < 2\pi, 0 < r < 1 \text{ m} \) is

\[
I = \int (r \cos^2 \theta \mathbf{a}_r + r^2 \sin \theta \mathbf{a}_\theta - r^2 \mathbf{a}_\phi) \cdot (r \sin \theta d\phi dr \mathbf{a}_\theta) \\
= \int_{\phi=0}^{2\pi} \int_{r=0}^{1} r^3 \sin^2 \theta d\phi dr \\
= \left[ \phi \right]_0^{2\pi} \left[ \frac{r^4}{4} \right]_0^1 = 2\pi \times \frac{1}{8} = \frac{\pi}{4} \text{ A}
\]

SOL 3.1.3 Option (B) is correct.

For a given current density, the total current flowing through a cross section is defined as

\[ I = \int \mathbf{J} \cdot d\mathbf{S} \]
where \( dS \) is the differential cross sectional area vector having the direction normal to the cross section.

So we have \( dS = \rho d\phi da \) (since the cylindrical wire is lying along \( z \)-axis)

Therefore the total current flowing through the wire (cross section) is

\[
I = \left( \frac{50}{\rho} a_z \right) \cdot (\rho d\phi a_z) \\
= \int_{\phi=0}^{2\pi} \int_{\rho=0}^{16 \times 10^{-3}} \left( \frac{50}{\rho} \right) (\rho d\phi) \\
= 50 \times \left[ \rho \right]_{16 \times 10^{-3}}^{0} \times \left[ \phi \right]_{0}^{2\pi} \\
= 60 \times 16 \times 10^{-3} \times 2\pi \\
= 60 \times 10^{-3} \times 2\pi \\
= 0.06 \times 2\pi \\
= 0.377 \\
= 0.38 \text{ A} \\
\]

**SOL 3.1.4** Option (C) is correct.

For a given current density, the total current that passes through a given surface is defined as

\[
I = \int J \cdot dS \\
\]

where \( dS \) is the differential area having the direction normal to the surface.

So we have \( dS = \rho d\phi da \) for the plane \( z = 2 \)

Therefore the total current crossing the plane \( z = 2, \rho < 4 \) is

\[
I = \left( \frac{40}{\rho} a_\rho - \frac{20 \sin \phi}{(\rho^2 + 1)} a_\phi \right) \cdot (\rho d\phi a_z) \\
= \int_{\phi=0}^{2\pi} \int_{\rho=0}^{4} \left( \frac{20 \sin \phi}{\rho^2 + 1} \right) (\rho d\phi) \\
= -\left[ \int_{\phi=0}^{2\pi} \frac{20 \rho d\phi}{\rho^2 + 1} \right] \left[ \int_{\phi=0}^{2\pi} \sin \phi d\phi \right] \\
= 4 \text{ A} \\
\]

**SOL 3.1.5** Option (B) is correct.

From the equation of continuity we have the relation between the volume charge density, \( \rho_v \), and the current density, \( J \) as

\[
\frac{\partial \rho_v}{\partial t} = -\nabla \cdot J \\
\]

Given the current density,

\[
J = \frac{40}{\rho} a_\rho - \frac{20 \sin \phi}{(\rho^2 + 1)} a_\phi, \text{ A/m}^2 \\
\]

So, we have the components \( J_\rho = \frac{40}{\rho}, J_\phi = 0 \) and \( J_z = -\frac{20 \sin \phi}{(\rho^2 + 1)} \)

Therefore,

\[
\frac{\partial \rho_v}{\partial t} = \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho J_\rho \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} J_\phi + \frac{\partial}{\partial z} J_z \right] \\
= \left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( 40 \right) + \frac{\partial}{\partial z} \left( -\frac{20 \sin \phi}{\rho^2 + 1} \right) \right] \\
= 2 \\
\]

So, volume charge density will be constant with respect to time.
SOL 3.1.6  Option (C) is correct.

Given the current \( I = 6 \, \text{A} \) is flowing radially outward (in \( a_r \) direction) through the medium between the cylinders. So the current density in the medium between the cylinders is

\[
J = \frac{I}{2\pi \rho l} \, a_r = \frac{6}{2\pi \rho \times 2} \, a_r \quad (l = 2 \, \text{m})
\]

\[
= \frac{5}{2\pi \rho} \, a_r \, \text{A/m}^2
\]

For a given current density in a certain medium having conductivity \( \sigma \), the electric field intensity is defined as

\[
E = \frac{J}{\sigma} = \frac{1}{\sigma} \left( \frac{3}{2\pi \rho} \, a_r \right)
\]

\[
= \frac{3}{2\pi \times 4 \times 10^{-2} \times 0.05} 
\]

\[
= 238.73 \, \text{V/m}
\]

SOL 3.1.7  Option (C) is correct.

Voltage between the cylindrical surfaces is defined as the line integral of the electric field between the two surfaces

\[
V = -\int \mathbf{E} \cdot d\mathbf{l}
\]

Now the electric field intensity in the medium between the two cylindrical surfaces as calculated in previous question is

\[
E = \frac{1}{\sigma} \left( \frac{3}{2\pi \rho} \, a_r \right)
\]

and the differential displacement between the two cylindrical surfaces is \( d\mathbf{l} = d\rho a_r \).

So the voltage between the cylindrical surfaces is

\[
V = -\int_{\rho_1}^{\rho_2} \frac{3}{2\pi \rho \sigma} \, (d\rho a_r) = -\frac{3}{2\pi \sigma} \ln \left( \frac{5}{3} \right)
\]

\[
= -5.88 \, \text{volt}
\]

So, the voltage between them will be 5.88 volt.

SOL 3.1.8  Option (C) is correct.

As we have already calculated the voltage between the two cylindrical surfaces and the current flowing radially outward in the medium between the surfaces is given in the question. So the resistance between the cylindrical surface can be evaluated directly as

\[
R = \frac{V}{I} = \frac{4.88}{6} = 0.813 \, \Omega
\]

\( (V = 4.88 \, \text{volt}, I = 6 \, \text{A}) \)

SOL 3.1.9  Option (A) is correct.

Since voltage between the cylindrical surfaces is \( V = 4.88 \, \text{volt} \) and current flowing in the medium is \( I = 6 \, \text{A} \)

So, Power dissipated in the medium is \( P = VI = (4.88) \times 6 = 29.28 \, \text{watt} \)

SOL 3.1.10  Option (B) is correct.

Consider a constant voltage is applied across the ends of the wire so, the electric
field intensity throughout the wire cross section will be constant.

i.e. \( E = \frac{J_1}{\sigma_1} = \frac{J_2}{\sigma_2} \)

where \( J_1 \) is the current density in the material having conductivity \( \sigma_1 \).
\( J_2 \) is the current density in the material having conductivity \( \sigma_2 \).

So, the ratio of the current density is

\[ \frac{J_1}{J_2} = \frac{\sigma_1}{\sigma_2} \]

i.e. it will be independent of both \( r \) and \( R \).

**SOL 3.1.11**

Option (A) is correct.

Since hydrogen atom contains a single electron (\(-ve\) charge) and a single proton (\(+ve\) charge). So, the dipole moment due to one atom of the hydrogen will be

\[ p = qd \]

where \( q \) is electronic charge and \( d \) is effective length

i.e. \( q = 1.6 \times 10^{-19} \text{ C} \) and \( d = 7.1 \times 10^{-16} \text{ m} \)

So, \( p = (1.6 \times 10^{-19}) \times (7.1 \times 10^{-16}) \)

and since the polarization in a material is defined as the dipole moment per unit volume.

Therefore \[ P = np \]

where \( n \) is the number of atoms per unit volume.

i.e.

\[ n = 5.5 \times 10^{19} \text{ atoms/cm}^3 \]
\[ = 5.5 \times 10^{26} \text{ atoms/m}^3 \]

So, \[ P = (5.5 \times 10^{26}) \times (1.6 \times 10^{-19} \times 7.1 \times 10^{-16}) \]
\[ = 4.25 \times 10^{-1} \text{ C/m}^2 \]

**SOL 3.1.12**

Option (D) is correct.

When an electric field \( E \) is applied to a material with dielectric constant \( \varepsilon_r \), then the polarization of the material is defined as

\[ P = \varepsilon_0 (\varepsilon_r - 1) E \]

So,

\[ \varepsilon_r - 1 = \frac{P}{\varepsilon_0 E} = \frac{6.25 \times 10^{-9}}{8.85 \times 10^{-12} \times 40 \times 10^3} = 1.7655 \times 10^{-2} \]

\[ \varepsilon_r = 1 + 0.0177 = 1.0177 \]

**SOL 3.1.13**

Option (C) is correct.

Given \[ D = 2P \Rightarrow P = D/2 \]

If the polarization of a dielectric material placed in an electric field \( E \) is \( P \), then the electric flux density in the material is defined as

\[ D = \varepsilon_0 E + P \]
\[ = \varepsilon_0 E + D/2 \]

or

\[ D = 2\varepsilon_0 E \] \hspace{1cm} ...........(i)

and since the relation between the electric field, \( E \) and flux density, \( D \) inside a dielectric material with dielectric constant \( \varepsilon_r \) is defined as

\[ D = \varepsilon_0 \varepsilon_r E \]

So, comparing the result with equation (i) we get, \( \varepsilon_r = 2 \).
SOL 3.1.14  Option (D) is correct.
Electric field intensity is defined as the negative gradient of the potential
i.e.
\[ E = - \nabla V = \left( \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right) \]
\[ = 250a_y \text{ V/m} \]

SOL 3.1.15  Option (B) is correct.
For a given electric field intensity \( E \) in a material having relative permittivity \( \varepsilon_r \),
the electric flux density is defined as :
\[ D = \varepsilon_0 \varepsilon_r E \]
\[ = \frac{8}{5} \times (8.85 \times 10^{-12}) \times (500a_y) \]
\[ = 9.08a_y \text{ nC/m}^2 \]

SOL 3.1.16  Option (C) is correct.
For an applied electric field intensity \( E \) in a material having relative permittivity \( \varepsilon_r \),
the polarization of the material is defined as
\[ P = \varepsilon_0 (\varepsilon_r - 1) E \]
\[ = (8.85 \times 10^{-12})(1.6 - 1) \times (500a_y) \]
\[ = 8.85 \times 10^{-12} \times 0.6 \times 500a_y \]
\[ = 4.66 \times 10^{-9}a_y \]

SOL 3.1.17  Option (D) is correct.
Since the two regions is being separated by the plane \( y = 0 \), so the tangential and
normal component of the electric field to the plane \( y = 0 \) are given as
\[ E_{1t} = 50a_y - 10a_x \]
\[ E_{1n} = 20a_z \]
From the boundary condition, the tangential component of electric field will be
uniform.
i.e.
\[ E_{2t} = E_{1t} = 50a_y - 10a_x \]
and the normal component of the field is nonuniform and given as
\[ \varepsilon_2 E_{2n} = \varepsilon_1 E_{1n} \]
\[ = \frac{2}{5}(20a_y) = 8a_y \]
So the electric field intensity in the second region is
\[ E_2 = E_{2t} + E_{2n} = (50a_y - 10a_x) + (8a_y) \]
\[ = 50a_y + 8a_y - 10a_x \text{ kV/m} \]
Therefore the electric flux density in the region 2 is
\[ D_2 = \varepsilon_2 \varepsilon_0 E_2 \]
\[ = 5 \times 8.85 \times 10^{-12}(50a_y + 8a_y - 10a_x) \times 10^3 \]
\[ = 3.21a_y + 0.55a_x - 0.44a_z \mu\text{C/m}^2 \]

SOL 3.1.18  Option (B) is correct.
Energy density in the region having electric field intensity \( E_2 \) is defined as

\[ \text{Energy density} = \frac{1}{2} \varepsilon_0 E_2^2 \]
\[ W_E = \frac{1}{2} \varepsilon_2 \varepsilon_0 E_2 \cdot E_2, \]
where the relative permittivity of the medium is \( \varepsilon_{r2} \)

As calculated in previous question the electric field intensity is

\[ E_1 = 50a_x + 8a_y - 10a_z \text{ kV/m} \]

So the energy density in the region 2 is

\[
W_E = \frac{1}{2} \times 5 \times \varepsilon_0 \left[ (50)^2 + (8)^2 + (10)^2 \right] \times 10^6
\]

= 79 \text{ mJ/m}^3

**SOL 3.1.19** Option (D) is correct.

According to boundary condition the tangential components of electric field are uniform

i.e. \[ E_{1t} = E_{2t} = E_{3t} \] \((i)\)

but the normal component of electric fields are non uniform and defined as

\[ \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} = \varepsilon_3 E_{3n} \]

Since \( \varepsilon_1 = \varepsilon_3 \) \((\text{Given})\)

So, \[ E_{1n} = E_{3n} \neq E_{2n} \] \((\text{ii})\)

and as the net electric field is given by

\[ E = E_t + E_n \]

\((\text{sum of tangential and normal component})\)

Therefore by combining the results of eq (i) and (ii) we get

\[ E_1 = E_3 \neq E_2 \]

**SOL 3.1.20** Option (D) is correct.

As the dielectric slab occupies the region \( 0 < z < d \) and the field intensity in the free space is in \( +a_z \) direction so, the field will be normal to the boundary of plane dielectric slab.

So from the boundary condition the field normal to the surface are related as

\[ \varepsilon E_{in} = \varepsilon_0 E_0 \]

\[ E_{in} = \frac{\varepsilon_0}{4 \varepsilon_0} E_0 a_z = \frac{E_0}{4} a_z \]

\((\varepsilon = 4\varepsilon_0)\)

Therefore,

\[ D_{in} = \varepsilon E_{in} = 4\varepsilon_0 \frac{E_0}{4} a_z = \varepsilon_0 E_0 a_z \]

**SOL 3.1.21** Option (A) is correct.

Total energy stored in a region having electric field is given as

\[ W = \frac{1}{2} \varepsilon_0 \int \left( \mathbf{E} \cdot \mathbf{E} \right) dv \]

\[ = \frac{1}{2} \varepsilon_0 \int \left( E_1 + E_2 \right) \cdot \left( E_1 + E_2 \right) dv \]

\[ = \frac{1}{2} \varepsilon_0 \int \left( E_1^2 + E_2^2 + 2E_1 \cdot E_2 \right) dv \]

\[ = \frac{1}{2} \varepsilon_0 \int E_1^2 dv + \frac{1}{2} \varepsilon_0 \int E_2^2 dv + \int \varepsilon_0 \left( E_1 \cdot E_2 \right) dv \]

\[ = W_1 + W_2 + \int \varepsilon_0 \left( E_1 \cdot E_2 \right) dv \]
SOL 3.1.22 Option (C) is correct.
Energy on a dipole with moment \( p \) in an electric field \( E \) is defined as:
\[
W_E = -p \cdot E = -(2a_x + 3a_y) \cdot (1.5a_x - a_z) = -(3 - 3) = 6 \text{ J}
\]

SOL 3.1.23 Option (C) is correct.
Consider a neutral dielectric is placed in an electric field \( E \), due to which the dielectric gets polarized with polarization \( P \), the bound surface charge density of the dielectric be \( \rho_{ps} \) and the bound volume charge density be \( \rho_{pv} \).
So the total bound charge by the dielectric is given as:
\[
Q_{\text{bound}} = \int_{S} \rho_{ps} \, dS + \int_{v} \rho_{pv} \, dv
\]
Since for a given polarization \( P \) of a dielectric material, the bound surface charge density over the surface of material is defined as:
\[
\rho_{ps} = P \cdot a_n
\]
where \( a_n \) is the unit vector normal to the surface directed outward.
while the bound volume charge density inside the material is defined as
\[
\rho_{pv} = -\nabla \cdot P
\]
So we have, \( Q_{\text{bound}} = \int_{S} (P \cdot a_n) \, dS - \int_{v} \nabla \cdot P \, dv \)
But according to the divergence theorem
\[
\int_{S} P \cdot dS = \int_{v} \nabla \cdot P \, dv
\]
Therefore, \( Q_{\text{bound}} = 0 \)

SOL 3.1.24 Option (A) is correct.
Resistance of a conductor of length \( l \) and having uniform cross sectional area \( S \) is:
\[
R = \frac{l}{\sigma S}
\]
where \( \sigma \) is the conductivity of the conductor
Given the conductivity, \( \sigma = 5 \times 10^6 \text{ (}\Omega \text{m)}^{-1} \)
the length of the conductor, \( l = 8 \text{ m} \)
side of the square cross section, \( a = 3 \text{ cm} \)
and radius of the bored hole, \( r = 0.5 \text{ cm} \)
So, the net cross sectional area is
\[
S = \text{area of square cross section(bar)} - \text{area of circular cross section(hole)}
\]
or
\[
S = a^2 - \pi r^2 = (3)^2 - \pi (0.5)^2
\]
\[
= \left(9 - \frac{\pi}{4}\right) \text{ cm}^2
\]
The total resistance between the square ends is given as:
\[
R = \frac{l}{\sigma S} = \frac{8}{(5 \times 10^6) \times \left[\left(9 - \frac{\pi}{4}\right) \times 10^{-4}\right]}
\]
\[
= 2.948 \times 10^{-3} \text{ } \Omega
\]
SOL 3.1.25 Option (D) is correct.

The two materials of composite bar will behave like two wires of resistance \( R_L \) (resistance due to lead) and \( R_C \) (resistance due to copper) connected in parallel.

As from the previous question we have the resistance due to the lead is

\[
R_L = 1.948 \, \text{m}\Omega
\]

and since the area of the cross section filled with copper is equal to the area of the cross section defined by hole so we have

cross sectional area \( S_C = \frac{\pi}{4} \, \text{cm}^2 \)

length of the bar \( l = 8 \, \text{m} \)

and conductivity of the copper, \( \sigma_C = \frac{1}{\text{resistivity of the copper}} = \frac{1}{1.72 \times 10^{-8}} \)

So the resistance due to copper is

\[
R_C = \frac{l}{S_C \sigma_C} = \frac{8}{\left(\frac{\pi}{4} \times 10^{-4}\right)\left(1.72 \times 10^{-8}\right)} = 1.76 \, \text{m}\Omega
\]

Therefore the equivalent resistance of the composite bar is

\[
R = R_C || R_L = \frac{(1.948) \times (1.76)}{1.948 + 1.76} = 524.62 \times 10^{-6} \, \Omega
\]

SOL 3.1.26 Option (D) is correct.

Given the conductivity of material, \( \sigma = 10^6 (\Omega \text{m})^{-1} \)

and conductance of the wire, \( G = 10^6 (\Omega)^{-1} \)

Since the conductance of a wire of length \( l \) having cross sectional area \( S \) is

\[
G = \frac{\sigma S}{l}
\]

So we have,

\[
10^6 = \frac{10^6 \times \pi r^2}{r} \quad (S = \pi r^2)
\]

\[
r = \sqrt{\frac{l}{\pi}}
\]

SOL 3.1.27 Option (C) is correct.

Given the radii of spherical shell as

\[
a = 1 \, \text{cm} = 0.01 \, \text{m} \\
b = 2 \, \text{cm} = 0.02 \, \text{m}
\]

The capacitance of a spherical capacitor having inner and outer radii \( a \) and \( b \) respectively is defined as

\[
C = \frac{4\pi \varepsilon \varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi}{\left(\frac{1}{0.01} - \frac{1}{0.02}\right)} \times 8.85 \times 10^{-12} = 8.9 \, \text{pF}
\]

SOL 3.1.28 Option (B) is correct.

Since the dielectric has been removed from the portion defined by \( \frac{2}{3} \leq \phi \leq \pi \gtrless \phi \leq \pi \)

so the composite capacitor will have the dielectric filled only in \( \frac{2}{3} \)th portion of the total capacitor and so the configuration can be treated as the two capacitors connected in parallel with each other.
The capacitance of the portion carrying air (\( \varepsilon_r = 1 \)) as the medium between the spherical shells

\[
C_1 = \frac{1}{4} \times \frac{4\pi \varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{1}{4} \times \frac{4\pi \times 8.85 \times 10^{-12}}{0.01 - 0.02}
\]

\[= 0.56 \times 10^{-12} = 0.56 \text{ pF} \]

The capacitance of the portion carrying dielectric (\( \varepsilon_r = 4 \)) as the medium between the spherical shells

\[
C_2 = \frac{3}{4} \times C
\]

Where \( C \) is the capacitance if no any portion of dielectric was removed as already calculated in previous question.

So we have

\[
C_2 = \frac{3}{4} \times 8.9 \times 10^{-12} = 6.7 \times 10^{-12} = 6.7 \text{ pF}
\]

Therefore the equivalent capacitance of the composite capacitor is,

\[
C_{eq} = C_1 + C_2 = 0.66 + 6.7 = 7.32
\]

**SOL 3.1.29** Option (D) is correct.

Capacitance of a parallel plate capacitor is defined as

\[
C = \frac{\varepsilon S}{d}
\]

where \( S \) is the surface area of the parallel plates

\( d \) is the separation between the plates

Here, the three different regions will be treated as the three capacitors connected in series as shown below

\[C_1, C_2, C_3\]

So the capacitance of the region 1 is

\[
C_1 = \frac{\varepsilon_0 \varepsilon_0 S}{0.001} = 2500 \varepsilon_0 S
\]
the capacitance of the region 2 is
\[ C_2 = \frac{\varepsilon_r \varepsilon_0 S}{0.002} = 2000 \varepsilon_0 S \]
the capacitance of the region 3 is
\[ C_3 = \frac{\varepsilon_0 S}{0.002} = 500 \varepsilon_0 S \]
Therefore the equivalent capacitance of the whole configuration is calculated as
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{\varepsilon_0 S} \left( \frac{1}{2500} + \frac{1}{2000} + \frac{1}{500} \right) \]
So,
\[ C_{eq} = 3.45 \times 10^2 \varepsilon_0 S \]
The capacitance per square meter of surface area will be
\[ C_{eq}' = \frac{C_{eq}}{S} = 3.45 \times 10^2 \varepsilon_0 = 4.05 \text{nF/m}^2 \]

**SOL 3.1.30**
Option (B) is correct.
Capacitance between the two cylindrical surfaces is defined as
\[ C = \frac{2\pi \varepsilon l}{\ln(b/a)} \]
Where
- \( l \) → length of the cylinder
- \( a \) → inner radius of the cylinder
- \( b \) → outer radius of the cylinder
Since, the medium between the conducting cylinders includes the dielectric layer \( (\varepsilon_r = 4) \) from \( \rho = 4 \text{ cm} \) to \( \rho = 6 \text{ cm} \) and air \( (\varepsilon_r = 1) \) from \( \rho = 6 \text{ cm} \) to \( \rho = 8 \text{ cm} \), so the configuration can be treated as the two capacitance connected in series.
Now for the dielectric layer \( (\varepsilon_r = 4) \) from \( \rho = 4 \text{ cm} \) to \( \rho = 6 \text{ cm} \), capacitance is
\[ C_1 = \frac{2\pi \varepsilon_0 \varepsilon_r l}{\ln(6/4)} = \frac{8\pi \varepsilon_0}{\ln(1.5)} \quad (l = 1 \text{ m}) \]
and for the air medium \( (\varepsilon_r = 1) \) from \( \rho = 6 \text{ cm} \) to \( \rho = 8 \text{ cm} \), capacitance is
\[ C_2 = \frac{2\pi \varepsilon_0 \times 1}{\ln(8/6)} = \frac{2\pi \varepsilon_0}{\ln(4/3)} \]
So, the equivalent capacitance of the configuration is evaluated as
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\ln(1.5)}{8\pi \varepsilon_0} + \frac{\ln(4/3)}{2\pi \varepsilon_0} \]
\[ C_{eq} = 143 \text{ pF} \]

**SOL 3.1.31**
Option (D) is correct.
The equivalent arrangement of the capacitor can be drawn in form of circuit as below

For which the capacitances are calculated as below
Chap 3 Electric Field in Matter

For View Only

Shop Online at www.nodia.co.in

\[
C_1 = \frac{\varepsilon_0 S/2}{d/2} = \frac{\varepsilon_0 S}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{4 \times 10^{-3}} = \frac{\varepsilon_0}{4}
\]

\[
C_2 = \frac{\varepsilon_r \varepsilon_0 S/2}{d/2} = \frac{\varepsilon_r \varepsilon_0 S}{d} = \frac{3\varepsilon_0}{4}
\]

\[
C_3 = \frac{\varepsilon_0 S/2}{d} = \frac{\varepsilon_0 \times 10 \times 10^{-4}}{2 \times 4 \times 10^{-3}} = \frac{\varepsilon_0}{8}
\]

Therefore the equivalent capacitance of the capacitor is

\[
C_{eq} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0}{8} + \frac{\varepsilon_0 \times 3\varepsilon_0}{4 + \frac{3\varepsilon_0}{4}} = \frac{\varepsilon_0}{4} = 2.76 \text{ pF}
\]

SOL 3.1.32 Option (C) is correct.

As the medium between capacitor plates is conducting so it carries the resistive as well as capacitive property.

Consider the plates are separated by a distance \(d\) and the surface area of plates is \(S\) as shown in the figure.

\[
\begin{align*}
\text{+} & \quad \text{Medium (\(\varepsilon, \sigma\))} & \quad \text{-} \\
\hline
S & \quad d & \quad S
\end{align*}
\]

So the total resistance of the medium between plates is

\[
R = \frac{d}{\sigma S}
\]

and capacitance of the capacitor is

\[
C = \frac{\varepsilon S}{d}
\]

Therefore the time constant of the capacitor will be

\[
\tau = RC = \frac{\varepsilon}{\sigma}
\]

************
**SOLUTIONS 3.2**

**SOL 3.2.1** Option (D) is correct.

For a given current density, the total current that passes through a given surface is defined as

\[ I = \int \mathbf{J} \cdot d\mathbf{S} \]

where \( d\mathbf{S} \) is the differential surface area having the direction normal to the surface.

Since the current density is independent of \( \theta \) and \( \phi \) so we can have directly the current

\[ I = \mathbf{J} \cdot \mathbf{S} = \mathbf{J}(4\pi r^2 a_r) \]

\[ = \frac{1}{r} e^{-10^3 t} 4\pi r^2 = 4\pi \times (6)^2 \times \frac{1}{6} \times e^{-10^3 \times 10^{-3}} \]

\[ = 4\pi \times 3 \times e^{-1} = 12\pi e^{-1} \]

**SOL 3.2.2** Option (A) is correct.

From the equation of continuity we have the relation between the volume charge density, \( \rho_v \), and the current density, \( \mathbf{J} \) as

\[ \frac{\partial \rho_v}{\partial t} = - \nabla \cdot \mathbf{J} \]

and since the current density have only the component in \( a_r \) direction so we have,

\[ \frac{\partial \rho_v}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 J_r) \]

\[ \frac{\partial \rho_v}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \frac{1}{r} e^{-10^3 t}) \]

Integrating both sides we get,

\[ \rho_v(r, t) = - \int \frac{1}{r^3} e^{-10^3 t} dt + f(r) \]

where \( f(r) \) is the function independent of time.

\[ \rho_v(r, t) = \frac{10^{-3}}{r^2} e^{-10^3 t} + f(r) \]

Now for \( t \to \infty \), \( \rho_v(r, t) = 0 \)

So, we put the given condition in the equation to get \( f(r) = 0 \)

therefore \[ \rho_v(r, t) = \frac{10^{-3}}{r^2} e^{-10^3 t} \]

i.e. \[ \rho_v(r, t) \propto \frac{1}{r^2} \]
Option (B) is correct.
The velocity of charge density can be defined as the ratio of current density to the charge density in the region

\[ v = \frac{J}{\rho_e} = \frac{1}{r} e^{-10t} a_e = 10^3 r a_e \]

So, at \( r = 0.6 \) m, \( v = 10^3 \times 0.6 a_e = 300 a_e \) m/s

Option (D) is correct.
The given problem can be solved easily by using image theory as the conducting surface \( y = 0 \) can be replaced by the equipotential surface in the same plane \( y = 0 \) and image of line charges \( (\rho'_L = -5 \text{ pC/m} \text{ at } x = 0, y = -1 \text{ and } x = 0, y = -2) \) as shown in the figure.

The work done to carry a unit positive charge from a point located at a distance \( a \) from the line charge with charge density \( \rho_L \) to another point located at a distance \( b \) from the line charge is defined as

\[ V_{ab} = \frac{\rho_L}{2\pi \varepsilon_0} \ln \left( \frac{b}{a} \right) \]

and since the surface \( y = 0 \) has zero potential, so the potential at point \( P \) will be equal to the work done in moving a unit positive charge from the plane \( y = 0 \) to the point \( P \). So the potential at point \( P \) will be

\[ V_P = -\sum \frac{\rho_L}{2\pi \varepsilon_0} \ln \left( \frac{b}{a} \right) \]

where \( a \) is the distance of the surface \( y = 0 \) from the line charges while \( b \) is the distance of point \( P \) from the line charges.

So,

\[ V_P = -\frac{5 \times 10^{-12}}{2\pi \varepsilon_0} \left[ -\ln \left( \frac{1}{2} \right) - \ln \left( \frac{\sqrt{2}}{1} \right) + \ln \left( \frac{\sqrt{10}}{1} \right) + \ln \left( \frac{\sqrt{17}}{2} \right) \right] \]

\[ = -0.3 \text{ volt} \]
SOL 3.2.5 Option (C) is correct.

Electric field at a distance \( R \) from a line charge having uniform charge density \( \rho_L \) is defined as

\[
E = \frac{\rho_L}{2\pi\varepsilon_0} \frac{R}{R^2}
\]

So the net electric field intensity produced at the point \( P \) due to the four line charges discussed in previous question is given as

\[
E = \sum \frac{\rho_L}{2\pi\varepsilon_0} \frac{R}{R^2}
\]

where \( R \) is the distance of point \( P \) from the line charges

Therefore,

\[
E = \frac{\rho_L}{2\pi\varepsilon_0} \left[ \frac{(-1, -2, 0) - (0, 1, 0)}{((-1, -1, 0)^2) - ((-1, 0, 0)^2)} + \frac{(-1, -2, 0) - (0, -1, 0)}{((-1, -1, 0)^2) - ((-1, 0, 0)^2)} + \frac{(-1, -2, 0) - (0, -2, 0)}{((-1, -1, 0)^2) - ((-1, 0, 0)^2)} \right] \\
= 5 \times 10^{-12} \left[ \frac{(1, 3, 0)}{10} - \frac{(1, 4, 0)}{17} + \frac{(1, 1, 0)}{2} + \frac{(1, 0, 0)}{1} \right] \\
= 0.12a_x - 0.0032a_y = 0.12a_x - 0.003a_y \text{ V/m}
\]

SOL 3.2.6 Option (C) is correct.

For a given polarization \( \mathbf{P} \) inside a material, the bound surface charge density over the surface of material is defined as

\[
\rho_{bs} = \mathbf{P} \cdot \mathbf{a}_n
\]

where \( \mathbf{a}_n \) is the unit vector normal to the surface directed outward.

while the bound volume charge density inside the material is defined as

\[
\rho_{bv} = -\nabla \cdot \mathbf{P}
\]

Since the component of polarization of rod along \( y \)-axis is \( P_y = 2y^2 + 3 \). So, the polarization of the material is \( \mathbf{P} = (2y^2 + 3) \mathbf{a}_y \) and the charge density on the surface of the rod is \( \rho_{bs} = \mathbf{P} \cdot \mathbf{a}_n \)

At \( y = 0 \) (top surface) \( \rho_{s1} = (2y^2 + 3) \mathbf{a}_y \cdot (-\mathbf{a}_y) = -3 \)

At \( y = 5 \) (bottom surface) \( \rho_{s2} = (2y^2 + 3) \mathbf{a}_y \cdot (\mathbf{a}_y) = 53 \)

and since the polarization has no radial component so no charge will be stored on its curvilinear surface and so the total bound surface charge on the surface of the rod is

\[
Q_{FS} = \int \rho_{bs} ds = \rho_{s1} S + \rho_{s2} S \quad (S \text{ is the cross sectional area})
\]

\[= -3S + 53S = 50S \]

Now, the bound volume charge density inside the material is

\[
\rho_{bv} = -\nabla \cdot \mathbf{P} = -\nabla \cdot (2y^2 + 3) \mathbf{a}_y = -4y
\]

So the total bound volume charge stored inside the material will be

\[
Q_{bv} = \int \rho_{bv} dv = \int_0^5 (-4y) Sdy = -4S \left[ \frac{y^2}{2} \right]_0^5 = -50S
\]

So, Total bound charge

\[
Q_{\text{bound}} = Q_s + Q_v = 50S - 50S = 0
\]

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
Option (C) is correct.

Electric field produced by the point charge at a distance $r$ is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} a_r$$

So, the induced dipole moment in the neutral atom due to the electric field $E$ produced by the point charge will be

$$p = \alpha E = \frac{\alpha q}{4\pi\varepsilon_0 r^2} a_r$$

and since the electric field intensity produced due to a dipole having moment $p$ at a distance $r$ from the dipole is defined as

$$E_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3} (2 \cos \theta a_r + \sin \theta a_\theta)$$

where $\theta$ is the angle formed between the distance vector $r$ and dipole moment $p$

So the field produced by the induced dipole at the point charge is

$$E_{\text{dip}} = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2}{4\pi\varepsilon_0 r^3} \left( \frac{\alpha q}{4\pi\varepsilon_0 r^2} \right) (2 \cos \theta a_r + \sin \theta a_\theta)$$

Now let’s assume that $\theta = \pi$ as shown in the figure:

Therefore the force experienced by the point charge due to the field applied by induced dipole is

$$F = qE_{\text{dip}} = 2\alpha \left( \frac{q}{4\pi\varepsilon_0} \right)^2 \frac{1}{r^3}$$

$$= 2\alpha \left( \frac{1}{9} \times 10^{-9} \times 9 \times 10^9 \right)^2 \times \frac{1}{(1)^3} = 1.62 \times 10^{-8} \text{ N m}$$

Option (B) is correct.

Electric field intensity produced due to a dipole having moment $p$, at a distance $r$ from the dipole is defined as

$$E_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3} (2 \cos \theta a_r + \sin \theta a_\theta)$$

where $\theta$ is the angle formed between the distance vector $r$ and dipole moment $p$

So the electric field intensity produced due to dipole $P_1$ at $P_2$ is

$$E_1 = \frac{p_1}{4\pi\varepsilon_0 r^3} a_\theta = \frac{2 \times 10^{-9}}{4\pi\varepsilon_0} \times (1)^3 a_\theta$$

Therefore the torque on $P_2$ due to $P_1$ is

$$T = p_2 \times E_1$$

Taking the magnitude only we have the torque on $P_2$ is

$$T = p_2 E_1 \sin \theta = (9 \times 10^{-9}) \left( \frac{2 \times 10^{-9}}{4\pi\varepsilon_0} \right) \sin 90^\circ$$

$$= 1.62 \times 10^{-8} \text{ N m} = 0.16 \mu \text{ N m}$$
Option (C) is correct.

Electric field intensity produced due to a dipole having moment $p$, at a distance $r$ from the dipole is defined as

$$E_{dp} = \frac{p}{4\pi \varepsilon_0 r^3}(2 \cos \theta a_r + \sin \theta a_\theta)$$

where $\theta$ is the angle formed between the distance vector $r$ and dipole moment $p$

So the electric field intensity produce due to dipole $P_2$ at $P_1$ is

$$E_2 = -\frac{p_2}{4\pi \varepsilon_0 r^3}2a_r = -\frac{9 \times 10^{-9}}{4\pi \varepsilon_0 (1)\pi} \times 2a_r \quad (\theta = \pi)$$

Therefore the torque on $P_1$ due to $P_2$ is

$$T = p_1 \times E_2$$

Considering the magnitude only we have the torque on $P_1$ is

$$T = p_1 E_2 \sin \theta = 2 \times 10^{-9} \times \left(-\frac{9 \times 10^{-9}}{4\pi \varepsilon_0 \pi} \times 2\right) \quad (\theta = \pi/2)$$

$$= 3.64 \times 10^{-4} \text{ N-m} = 0.32 \mu\text{N-m}$$

Option (A) is correct.

For a given polarization $P$ inside a material, the bound volume charge density inside the material is defined as

$$\rho_{pv} = -\nabla \cdot P$$

Since the polarization of the sphere is $P(r) = 2ra_r$

So the bound volume charge density inside the sphere is

$$\rho_{pv} = -\nabla \cdot P(r) = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 2r) = -\frac{1}{r^2} \times 6r^2 = -6$$

Therefore the electric field intensity inside the sphere at a distance $r$ from the center is given by

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q_{enc}}{r^2} a_r = \frac{1}{4\pi \varepsilon_0} \frac{\rho_{pv} \times \frac{4}{3} \pi r^3}{r^2} a_r$$

$$= \frac{\rho_{cv}}{3 \varepsilon_0} a_r = -\frac{6}{3 \varepsilon_0} a_r = -\left(\frac{2}{\varepsilon_0}\right) r a_r$$

So the radial component of the electric field inside the sphere is

$$E_r = -\frac{2}{\varepsilon_0} r$$

which is linearly decreasing with a slope $\left(-\frac{2}{\varepsilon_0}\right)$ with respect to $r$ as shown below:
SOL 3.2.11 Option (A) is correct.

For a given polarization \( P \) of a material, the bound surface charge density over the surface of material is defined as

\[ \rho_{PS} = P \cdot a_n \]

So the bound surface charge density over the spherical surface is

\[ \rho_{PS} = P(r) \cdot a_r = 2r = 2a \]  \( (a_n = a_r) \]

(at the spherical surface \( r = a \))

So, total bound surface charge over the sphere is

\[ Q_{PS} = 2a \times 4\pi a^2 = 8\pi a^3 \]

and the bound volume charge density inside the sphere as calculated above is

\[ \rho_{PV} = -6 \]

So, total bound volume charge inside the sphere is

\[ Q_{PV} = \rho_{PV} \left( \frac{4}{3} \pi a^3 \right) = (-6) \times \left( \frac{4}{3} \pi a^3 \right) = -8\pi a^3 \]

Therefore the total bound charge in the sphere is

\[ Q_{bound} = Q_{PS} + Q_{PV} = 12\pi a^3 - 12\pi a^3 = 0 \]

According to Gauss law the outward electric field flux through a closed surface is equal to the charge enclosed by the surface and since the total bound charge for any point outside the sphere is zero so, the electric field intensity at any point outside the sphere is \( E = 0 \).

SOL 3.2.12 Option (C) is correct.

Since the spherical shell is of inner radius \( r = 2 \text{ m} \) so region inside the sphere will have no polarization and therefore the total charge enclosed inside the shell for \( r < 2 \text{ m} \) will be zero.

i.e.

\[ Q_{enc} = 0 \]

According to Gauss law the total outward electric flux from a closed surface is equal to the charge enclosed by the surface and since the total enclosed charge for \( r < 2 \text{ m} \) is zero so the electric field intensity at \( r = 1 \text{ m} \) will be zero.

SOL 3.2.13 Option (A) is correct.

Since the total bound charge by a polarized neutral dielectric is zero as discussed in MCQ. 33. So for any point outside the spherical shell the total enclosed charge(bound charge) will be zero and as discussed in the previous question, according to Gauss law the electric field intensity at any point outside the spherical shell will be zero.

So, for the surface \( r = 7 \) \( Q_{enc} = 0 \)

Therefore the electric field intensity is \( E = 0 \)

SOL 3.2.14 Option (A) is correct.

As we have to find electric field at \( r = 5 \text{ m} \) so we determine first the charge enclosed by the surface \( r = 5 \text{ m} \) which will be equal to the sum of the volume charge stored in the region \( 2 \leq r \leq 5 \text{ m} \) and the surface charge stored at \( r = 2 \text{ m} \).

Since for a given polarization \( P \) of a dielectric material, the bound volume charge density inside the material is defined as
\[ \rho_{pe} = -\nabla \cdot P \]

So the bound volume charge density inside the dielectric defined in the region \( 2 \leq r \leq 6 \text{ m} \) will be

\[ \rho_{pe} = -\nabla \cdot P(r) = -\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{5}{r} \right) = -\frac{5}{r} \]

so the total bound volume charge in the region \( 2 \leq r \leq 5 \text{ m} \) is

\[ Q_{pv} = \int \rho_{pv} \, dv = \int_{r=2}^{5} -\frac{5}{r} \times 4\pi r^2 \, dr = -20\pi \left[ r^2 \right]_{r=2}^{5} = -60\pi \]

Now for a given polarization \( P \) inside a dielectric material, the bound surface charge density over the surface of dielectric is defined as

\[ \rho_{ps} = P \cdot a_n \]

where \( a_n \) is the unit vector normal to the surface pointing outward of the material.

So the bound surface charge density at \( r = 2 \text{ m} \) is

\[ \rho_{ps} = P(r) \cdot (a_n) \quad (a_n = -a_r) \]

Therefore the total bound surface charge over the surface \( r = 2 \text{ m} \) is

\[ Q_{ps} = -\frac{5}{r} \times 4\pi r^2 \quad \text{(for spherical surface } S = 4\pi r^2) \]

\[ = -\frac{5}{2} \times 4\pi \times 2^2 = -40\pi \quad r = 2 \text{ m} \]

So, the total enclosed charge by the surface \( r = 5 \text{ m} \) is

\[ Q_{enc} = Q_{pv} + Q_{ps} = 60\pi - 40\pi = -100\pi \]

So the electric field intensity at \( r = 5 \text{ m} \) will be,

\[ E = \frac{1}{4\pi \varepsilon_0} \times \frac{Q_{enc}}{r^2} a_r = \frac{1}{4\pi \varepsilon_0} \times \frac{-100\pi}{5^2} a_r = -\frac{1}{5^2} a_r \]

SOL 3.2.15 Option (C) is correct.

Since the electric field intensity at any point inside a conductor is always zero, so the electric flux density at a distance \( r \) from the center of the spherical conductor can be given as

\[ D = \begin{cases} 0, & r < 1 \\ \frac{Q}{4\pi r^2} a_r, & r > 1 \text{ m} \end{cases} \]

where \( Q = 3 \text{ mC} \) is the total charge carried by the conductor.

and since the dielectric material surrounding the spherical conductor has permittivity \( \varepsilon_r = 3 \), so the electric field intensity at a distance \( r \) from the center of the sphere is

\[ E = \begin{cases} 0, & r < 1 \text{ m} \\ \frac{Q}{4\pi \varepsilon_0 \varepsilon r^2} a_r, & 1 < r < 2 \text{ m} \\ \frac{Q}{4\pi \varepsilon_0 r^2} a_r, & r > 2 \text{ m} \end{cases} \]

So, the total energy of the configuration is

\[ W_E = \frac{1}{2} \int D \cdot E \, dv \]
Option (B) is correct.

The electric potential at the centre of sphere will be equal to the work done to carry a unit charge from infinity to the centre of the sphere (the line integral of the electric field intensity from infinity to the center of the sphere)

\[ V = - \int_{0}^{\infty} \mathbf{E} \cdot d\mathbf{l} \]

Since the sphere has uniform charge density \( \rho_e = 0.6 \text{nC}/\text{m}^3 \) embedded in it, so the electric field intensity at a distance \( r \) from the center of the sphere can be given as

\[
\mathbf{E} = \begin{cases} 
\frac{\rho_e}{3\varepsilon_r\varepsilon_0} \mathbf{a}_r, & r < R \\
\frac{\rho_e R^3}{3\varepsilon_0 r^2} \mathbf{a}_r, & r > R 
\end{cases}
\]

where \( R \) is the radius of the sphere i.e. \( R = \frac{1}{\sqrt{\pi}} \text{m} \)

So, the potential at the centre of sphere will be

\[
V = - \int_{0}^{\infty} \mathbf{E} \cdot d\mathbf{l} \\
= -\int_{0}^{1/\sqrt{\pi}} \frac{\rho_e}{3\varepsilon_0 r^2} \left( \frac{1}{\sqrt{\pi}} \right)^3 dr - \int_{1/\sqrt{\pi}}^{0} \frac{\rho_e r}{3\varepsilon_0} dr \\
= -\frac{\rho_e}{3\varepsilon_0} \left( \frac{1}{\sqrt{\pi}} \right)^3 \left[ -\frac{1}{1/\sqrt{\pi}} - \frac{1}{2} \right] \\
= \frac{\rho_e}{3\varepsilon_0} \left( \frac{1}{\sqrt{\pi}} \right)^3 \sqrt{\pi} + \frac{\rho_e}{3\varepsilon_0} \frac{1}{2} = \frac{\rho_e}{3\varepsilon_0} + \frac{\rho_e}{12\varepsilon_0\pi} \\
= \frac{5\rho_e}{12\varepsilon_0} = \frac{5\rho_e}{3} \times \frac{4\pi\varepsilon_0}{3} = 5 \times 0.6 \times 10^{-3} \times 9 \times 10^{9} = 5 \text{volt} \quad (\varepsilon_r = 2) \\
\rho_e = 0.6 \text{nC}/\text{m}^3
\]

Option (A) is correct.

For a given polarization \( \mathbf{P} \) of a material, the surface charge density over the surface of material is defined as

\[ \rho_s = \mathbf{P} \cdot \mathbf{a}_n \]

where \( \mathbf{a}_n \) is the unit vector normal to the surface directed outward of the material.
while the volume charge density inside the material is defined as
\[ \rho_v = - \nabla \cdot \mathbf{P} \]
Since the the cylinder has uniform polarization \( \mathbf{P} \).
So, volume charge density inside the sphere is
\[ \rho_v = - \nabla \cdot \mathbf{P} = 0 \]
and the surface charge density over the top and bottom surface of the cylinder is
\[ \rho_s = \mathbf{P} \cdot \mathbf{a}_n = \pm P \quad (+P \text{ at top surface and } -P \text{ at bottom surface}) \]
So the total bound charge by the cylinder is
\[ Q_{\text{bound}} = Q_s + Q_v \]
\[ = \int_S \rho_s dS + \int_v \rho_v dv = [+P(\pi r^2) - P(\pi r^2)] + 0 = 0 \]

SOL 3.2.18  Option (C) is correct.
As calculated above the volume charge density inside the cylinder is zero while the surface charge density at top and bottom surfaces are respectively \( +P \) and \( -P \), so the cylinder can be considered as the two circular plates (top and bottom surface) separated by a distance \( L \). Since the separation between the plates is larger than the cross sectional radius \( (L = 2r) \) so the fringing field(electric field) will exist directed from the upper plate towards the lower plate.

SOL 3.2.19  Option (A) is correct.
The electric flux lines will be the same as the electric field intensity outside the cylinder but as the volume charge density is zero \( \rho_v = 0 \) inside the cylinder so
\[ \int \mathbf{D} \cdot d\mathbf{S} = 0 \]
and therefore the flux lines will be continuous.

SOL 3.2.20  Option (A) is correct.
Consider the surface charge density on the parallel plates is \( \pm \rho_s \) so the electric flux density between the plates is defined as
\[ \mathbf{D} = \rho_s \mathbf{a}_n \]
where \( \mathbf{a}_n \) is the unit vector normal to the surface of plates directed from one plate toward the other plate.
Since permittivity changes from layer to layer, but the field is normal to the surface so electric flux density \( \mathbf{D} \) will be uniform throughout the plate separation as from boundary condition.
So the electric field intensity at any point between the parallel plates is
\[ \mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0 \varepsilon_r} = \frac{\rho_s \mathbf{a}_n}{2 \varepsilon_0 (1 + 100 a^2)} \]
where \( \varepsilon_r = 2(1 + 100 a^2) \)
Therefore the voltage between the plates can be evaluated by taking the line integral of electric field from one plate to the other plate
\[ V = -\int \mathbf{E} \cdot dl = - \int_{-0}^{0.1} \left( \frac{\rho_s \mathbf{a}_n}{2 \varepsilon_0 (1 + 100 a^2)} \right) \cdot (da) \quad (dl = da) \]
\[ = \frac{\rho_s}{2 \varepsilon_0} \int_0^{0.1} \frac{da}{(0.1)^2 + a^2} \quad (\text{the direction of } \mathbf{a} \text{ is along } \mathbf{a}_n) \]
\[ = \frac{\rho_s}{2 \varepsilon_0} \times \frac{1}{0.1 \pi} \times \frac{1}{0.1} \tan^{-1}\left( \frac{0.1}{a} \right)^{0.1} \]
Now charge stored at the parallel plates is
\[ Q = (\rho_s) (S) = \rho_s \times (0.2) S \]
where \( S \) is surface area of the plates \( S = 0.2 \text{ m}^2 \)
So, the capacitance of the capacitor is evaluated as
\[ C = \frac{Q}{V} = \frac{\rho_s \times (0.2)}{(\rho_s \pi) / 80 \varepsilon_0} = \frac{16 \varepsilon_0}{\pi} \]
\[ = 5.51 \times 10^{-11} \]

**SOL 3.2.21** Option (C) is correct.

For the two wire transmission line consists of the cylinders of radius \( b \) and separated by a distance \( 2h \) (centre to centre), the capacitance per unit length between them is defined as
\[ C' = \frac{\pi \varepsilon}{\cosh^{-1}(h/b)} \]
Here, \( 2h = 2 \text{ cm} \) and \( b = 0.2 \text{ cm} \)
So,
\[ C' = \frac{\pi \times 2 \times 8.85 \times 10^{-12}}{\cosh^{-1}(1/0.2)} = 3.64 \times 10^{-11} \text{ F/m} \]
\( (\varepsilon_r = 2) \)
So the charge per unit length on each wire will be,
\[ Q = C' V_0 = 3.64 \times 10^{-11} \times 100 \]
\[ = 5.64 \times 10^{-9} \text{ C/m} \]

**SOL 3.2.22** Option (C) is correct.

Consider the oil rises to a height \( h \) in the space between the tubes.
So, the capacitance of the tube carrying oil partially will be treated as the two capacitors connected in parallel.
Since the capacitance between the two cylindrical surfaces is defined as
\[ C = \frac{2 \pi \varepsilon l}{\ln(b/a)} \]
Where \( l \to \) length of the cylinder
\( a \to \) inner radius of the cylinder
\( b \to \) outer radius of the cylinder
So the capacitance of the portion carrying oil \( (\varepsilon_r = 1) \) as the medium between the cylindrical surfaces is
\[ C_{oil} = \frac{2 \pi \varepsilon_0 \varepsilon h}{\ln(3/1)} = \frac{4 \pi \varepsilon_0 h}{\ln(3)} \]
\( (\varepsilon_r = \varepsilon_r + 1 = 2) \)
and the capacitance of the portion carrying air \( (\varepsilon_r = 1) \) as the medium between the cylindrical surfaces is
\[ C_{air} = \frac{2 \pi \varepsilon_0 (1 - h)}{\ln(3/1)} \]
Therefore the equivalent capacitance of the tube carrying oil to the height \( h \) is
\[ C = C_{oil} + C_{air} = 2 \pi \varepsilon_0 \left( \frac{1 + h}{\ln(3)} \right) \]
Since the energy stored in a capacitor is defined as
\[ W_E = \frac{1}{2} CV^2 \]
where \( V \) is the applied voltage to the capacitor.

So the net upward force due to the capacitance is given by
\[ F = \frac{dW_E}{dh} = \frac{1}{2} V^2 \frac{dC}{dh} = \frac{1}{2} V^2 \frac{2\varepsilon_0}{\ln(3)} \]
and net downward force on the oil due to gravity will be
\[ F = mg = (0.01 \text{gm/cm}^3) \times \pi (b^2 - a^2) h \times g \]

So, \[ 0.08\pi h g = \frac{1}{2} V^2 \frac{2\varepsilon_0}{\ln(3)} \]

Since in equilibrium both the upward and downward forces are equal
\[ h = \frac{1}{2} \times (2 \times 10^6)^2 \times \frac{2\pi \times 8.85 \times 10^{-12}}{0.08 \times 9.8 \times \ln(3)} \]
\[ = 4.11 \times 10^{-5} \text{m} = 45.1 \mu \text{m} \]

**SOL 3.2.23** Option (C) is correct.
Consider the charge densities of the two surface of the slab is \( \rho_1 \text{ C/m}^2 \) and \( \rho_2 \text{ C/m}^2 \) as shown in the figure.

As the sum of the charge densities is \( \rho_0 \text{ C/m}^2 \) so we have
\[ \rho_1 + \rho_2 = \rho_0 \]  
and since the electric field intensity inside the conducting slab must be zero so,
\[ E_1 + E_2 = 0 \]  
where \( E_1 \) is field inside slab due to charge density \( \rho_1 \) and \( E_2 \) is field inside slab due to \( \rho_2 \)

As the electric field intensity at any point \( P \) due to the uniformly charged plane with charge density \( \rho_s \) is defined as
\[ E = \frac{\rho_s}{2\varepsilon_0} a_n \]
where \( a_n \) is the unit vector normal to the plane directed toward point \( P \)

So we have, \[ E_1 = \frac{\rho_2}{2\varepsilon_0} (a_x) \]

(a) \( a_n = -a_z \)
For View Only

\[ E_2 = \frac{\rho_{S2}}{2\varepsilon_0} a_z \]

(a_n = a_z)

From equation (ii)

\[ \frac{\rho_{S1}}{2\varepsilon_0}(-a_z) + \frac{\rho_{S2}}{2\varepsilon_0} a_z = 0 \]

Putting the result in equation (i) we get

\[ \rho_{S1} = \rho_{S2} = \frac{\rho_a}{4} \]

**SOL 3.2.24** Option (D) is correct.

As the slabs are conducting so net electric field inside the slab must be zero.

and since the electric field intensity at any point \( P \) due to the uniformly charged plane with charge density \( \rho_S \) is defined as

\[ E = \frac{\rho_S}{2\varepsilon_0} a_n \]

where \( a_n \) is the unit vector normal to the plane directed toward point \( P \)

So, the net electric field intensity inside slab 1 is

\[ \frac{\rho_{S1}}{2\varepsilon_0}(-a_z) + \frac{\rho_{S2}}{2\varepsilon_0} a_z + \frac{\rho_{S1}}{2\varepsilon_0} a_z + \frac{\rho_{S2}}{2\varepsilon_0} a_z = 0 \]

\[ (a_n = -a_z \text{ for } \rho_{S1} \text{ while } a_n = a_z \text{ for rest of the charge densities}) \]

\[ \rho_{S1} + \rho_{S2} + \rho_{S1} + \rho_{S2} = 0 \]

... (1)

and the net electric field intensity inside slab 2 is

\[ \frac{\rho_{S1}}{2\varepsilon_0}(-a_z) + \frac{\rho_{S2}}{2\varepsilon_0}(-a_z) + \frac{\rho_{S1}}{2\varepsilon_0}(-a_z) + \frac{\rho_{S2}}{2\varepsilon_0} a_z = 0 \]

\[ (a_n = a_z \text{ for } \rho_{S2} \text{ while } a_n = -a_z \text{ for rest of the charge densities}) \]

\[ -\rho_{S1} - \rho_{S2} - \rho_{S1} + \rho_{S2} = 0 \]

... (2)

Solving eq. (i) and eq (ii) we get,

\[ \rho_{S1} = \rho_{S2} \]

and

\[ \rho_{S1} = -\rho_{S2} \]

**SOL 3.2.25** Option (C) is correct.

As all the four surfaces form the boundaries of the conductors extending away from
the region between them so, the medium outside the defined region is conductor and so the field intensity outside the region will be zero.

Now the electric potential in the non conducting region is given as

\[ V = 5xy \]

So the electric field intensity in the region is

\[ \mathbf{E} = -\nabla V = -5ya_x - 5xa_y \]

From the conductor-free space boundary condition we have the surface charge density on the boundary surface defined as

\[ \rho_s = \varepsilon_0 E_n \]

where \( E_n \) is the normal component of the electric field intensity in the free space.

So, the surface charge density on the surface \( x = 0 \) is

\[ \rho_s = \varepsilon_0 (-5y) \] (the normal component \( E_n = -5y \) for the surface \( x = 0 \))

\[ = -5\varepsilon_0 y \]

SOL 3.2.26 Option (D) is correct.

Again as discussed in above question, the surface charge density on the surface \( y = 0 \) will be given by

\[ \rho_s = \varepsilon_0 E_n \]

and since the field component normal to surface \( y = 0 \) is

\[ E_n = -5x \]

So, the surface charge density on the surface \( y = 0 \) is

\[ \rho_s = -5\varepsilon_0 x \]

SOL 3.2.27 Option (C) is correct.

From the symmetry associated with the charge distribution the electric field must be radially directed. Then choosing Gaussian surfaces which are cylinders having the same axis \( (\rho = 0) \) as the conductors and of length \( l \), we get

\[ (2\pi\rho l)E_\rho = 0 \]

(since there is no charge enclosed by the Gaussian surface)

Thus

\[ E_\rho = 0 \] for \( \rho < 2 \text{ m} \)

Now, since the field inside the conductor \( 2 < \rho < 3 \text{ m} \) is zero; there cannot be any charge on the surface \( \rho = 2 \text{ m} \).

i.e.

\[ \rho_s = 0 \] at \( \rho = 2 \text{ m} \)

and all the charge associated with the inner conductor resides on the surface \( \rho = 3 \text{ m} \).

i.e.

\[ \rho_s = \frac{10 \text{ C/m}}{2\pi(3)} = \frac{5}{3\pi} \text{ C/m}^2 \] at \( \rho = 3 \text{ m} \)

Proceeding further we have

\[ 2\pi\rho lE_\rho = \frac{1}{\varepsilon_0} (10 \text{ C/m}) l \]

where \( l \) is length of the cylinder.

So

\[ \mathbf{E} = \frac{10}{2\pi\varepsilon_0 \rho} a_\rho \] for \( 5 < \rho < 6 \text{ m} \)
This the field produced by the inner conductor but the fact is that the field inside the conductor \( r < 6 \text{ m} \) is zero that gives
\[
[\rho_s]_{|r = 5 \text{ m}} = \varepsilon_0 [E]_{|r = 5 \text{ m}} \cdot (-a_r)
\]
\[
= \varepsilon_0 \left( \frac{10}{2\pi\varepsilon_0(5)} a_r \right) \cdot (-a_r) = -\frac{10}{2\pi(5)} = -\frac{1}{\pi} \text{ C/m}^2
\]
and
\[
[\rho_s]_{|r = 6 \text{ m}} = \frac{1}{12\pi(6)} \{6 \text{ C/m} - [\rho_s]_{|r = 5 \text{ m}} \}
\]
\[
= \frac{1}{12\pi}(6 + 10) = \frac{4}{3\pi}
\]

**SOL 3.2.28** Option (C) is correct.

From the symmetry associated with the charge distribution the electric field must be radially directed. As, there is no charge enclosed by the surface \( r = 2 \text{ m} \) so we get
\[
E_r = 0 \quad \text{for } r < 2 \text{ m}
\]
Now from the conductor-free space boundary condition we have the surface charge density on the boundary surface defined as
\[
\rho_s = \varepsilon_0 E_n
\]
where \( E_n \) is the normal component of the electric field intensity in the free space. So the charge density at \( r = 2 \text{ m} \) is
\[
\rho_s = E_r = 0
\]
Therefore the total charge will be concentrated over the outer surface which is given as
\[
\rho_{s2} = \frac{Q}{4\pi r} = \frac{9}{2\pi(3)} = \frac{1}{8\pi} \text{ C/m}^2
\]

**SOL 3.2.29** Option (D) is correct.

From the boundary condition for the charge carrying interface, the tangential component of electric field on either side of the surface will be same. i.e.
\[
E_{1t} = E_{2t}
\]
while the normal components are related as
\[
E_{1n} - E_{2n} = \frac{\rho}{\varepsilon_0}
\]
now as the field intensity in the region \( z < 0 \) is
\[
E_z = 2a_x + 3a_y - 2a_z
\]
So the tangential component, \( E_{zt} = 2a_x + 3a_y \) and the normal component, \( E_{zn} = -2a_z \).
Therefore the field components in region \((z > 0)\) are
\[
E_{1t} = E_{zt} = 2a_x + 3a_y
\]
and
\[
E_{1n} = E_{zn} + \frac{\rho_s}{\varepsilon_0} = \left[ -2 + \frac{2 \times 10^{-9}}{8.85 \times 10^{-12}} \right] a_z = 224a_z
\]
So the net field intensity in the region \( z > 0 \) is
\[
E_1 = E_{1t} + E_{1n} = 2a_x + 3a_y + 224a_z
\]
Option (C) is correct.
As the dielectric slab occupies the region \( x > 0 \) and the electric field in the free space is directed along \( \mathbf{a}_x \), so, the field will be normal to the boundary surface, \( x = 0 \) of the dielectric slab.
So from the boundary condition the field normal to the interface of dielectrics are related as
\[
\varepsilon_r \varepsilon_0 E_i = \varepsilon_0 E \quad \text{(where } E_i \text{ is the field inside the dielectric)}
\]
\[
E_i = \frac{E}{\varepsilon_r} = \frac{10a_x}{5} = 2a_x \quad (\varepsilon_r = 5)
\]
So, the polarization inside the dielectric is
\[
P = (\varepsilon - \varepsilon_0) E_i = (5\varepsilon_0 - \varepsilon_0) E_i = 8\varepsilon_0 a_x
\]

Option (D) is correct.
As the dielectric slab occupies the region \( 0 < y < 1 \) m and the electric field in the free space is directed along \( \mathbf{a}_y \), so, the field will be normal to both the boundary surfaces \( y = 0 \) and \( y = 1 \).
So from the boundary condition the field normal to the interface of dielectrics are related as
\[
\varepsilon E_i = \varepsilon_0 E \quad \text{(where } E_i \text{ is the field inside the dielectric)}
\]
\[
E_i = \frac{\varepsilon_0}{4\varepsilon_0} (1 + y)^2 E = \frac{(1 + y)^2}{4} (4a_y)
\]
\[
= (1 + y)^2 a_y \quad \text{since } \varepsilon = \frac{4\varepsilon_0}{(1 + y)^2}
\]
So the polarization inside the dielectric is
\[
P = \varepsilon E_i - \varepsilon_0 E_i = \left( \frac{4\varepsilon_0}{(1 + y)^2} - \varepsilon_0 \right) (1 + y)^2 a_y
\]
Now for a given polarization \( P \) inside a dielectric material, the surface charge density over the surface of dielectric is defined as
\[
\rho_{yp} = P \cdot \mathbf{a}_n
\]
where \( \mathbf{a}_n \) is the unit vector normal to the surface directed outward of the dielectric.
So, the bound surface charge density at \( y = 0 \) is
\[
\left[ \rho_{yp} \right]_{y=0} = P \cdot (-\mathbf{a}_y) = [4 - (1 + y)^2] \varepsilon_0 (-1) = -(4 - 1) \varepsilon_0 = -3 \varepsilon_0 \quad \text{(} \mathbf{a}_y = -\mathbf{a}_y \text{)}
\]
and the surface charge density at \( y = 1 \) m is
\[
\left[ \rho_{yp} \right]_{y=1} = P \cdot (\mathbf{a}_y) = [4 - (1 + y)^2] \varepsilon_0 (1) = (4 - 4) \varepsilon_0 = 0 \quad \text{(} \mathbf{a}_y = \mathbf{a}_y \text{)}
\]

Option (C) is correct.
As calculated in previous question, polarization inside the dielectric is
\[
P = [4 - (1 + y)] \varepsilon_0 \mathbf{a}_y
\]
Since for a given polarization \( P \) inside a material, the bound volume charge density inside the material is defined as
\[ \rho_P = - \nabla \cdot P \]

So the volume charge density inside the dielectric is

\[ \rho_r = - \frac{\partial}{\partial y} \left[ 4 - (1 + y)^2 \right] \varepsilon_0 \]

\[ = 2(1 + y) \varepsilon_0 \]

So when we move from \( y = 0 \) to \( y = 1 \text{ m} \), the volume charge density will be linearly increasing.

**SOL 3.2.33** Option (C) is correct.

As the charge is being located at origin so the field intensity due to it will be in radial direction and normal to the surface of the dielectric material. Therefore the flux density will be uniform (as from boundary condition) and at any point \( r \) inside the dielectric flux density will be

\[ D = \frac{Q}{4\pi r^2} a_r \]

Now it is given that electric field intensity at any point inside the dielectric is

\[ E = \frac{Q}{4\pi \varepsilon_0 b^2} a_r \]

and since in a medium of permittivity \( \varepsilon = \varepsilon_r \varepsilon_0 \), the flux density is defined as

\[ D = \varepsilon_r \varepsilon_0 E \]

So for the given field we have

\[ \frac{Q}{4\pi r^2} a_r = \varepsilon_r \varepsilon_0 \left( \frac{Q}{4\pi \varepsilon_0 b^2} a_r \right) \]

\[ \varepsilon_r = \frac{b^2}{r^2} \]

**SOL 3.2.34** Option (C) is correct.

Consider the parallel sheets arrangement as shown in the figure.

Electric field intensity at any point \( P \) due to the uniformly charged plane with charge density \( \rho_S \) is defined as

\[ E = \frac{\rho_S}{2\varepsilon} a_n \]

where \( a_n \) is the unit vector normal to the plane directed toward point \( P \) and \( \varepsilon \) is the permittivity of the medium.
So the field intensity inside the dielectric due to the left sheet will be

\[ E_1 = \frac{5 \times 10^{-9}}{2\varepsilon} (a_y) \]  

and again the field intensity inside the dielectric due to right sheet will be

\[ E_2 = -\frac{5 \times 10^{-9}}{2\varepsilon} (-a_y) = \frac{5 \times 10^{-9}}{2\varepsilon} a_y \]  

so the net field intensity inside the dielectric will be

\[ E = E_1 + E_2 = \frac{5 \times 10^{-9}}{\varepsilon} a_y \]

Since the field intensity is uniform inside the dielectric So potential difference between the plates will be directly given as

\[ V = E \times (\text{distance between the plates}) \]

\[ = \frac{5 \times 10^{-9}}{4\varepsilon_0} \times 2 = 2.824 \times 10^3 \text{ Volt} = 283 \text{ kV} \quad (\varepsilon = 4\varepsilon_0) \]

Option (C) is correct.

As calculated in previous question the electric field between the two dielectrics having surface charge densities \( \rho_s \) and \( -\rho_s \) is

\[ E = \frac{\rho_s}{\varepsilon} \]

where \( \varepsilon \) is the permittivity of the medium between the sheets.

So electric field in slab 1 is

\[ E_1 = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{2\varepsilon_0} \]

and electric field in slab 2 is

\[ E_2 = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{4\varepsilon_0} \]

Since the electric field between the sheets is uniform so the potential difference between the plates will be

\[ V = \sum E \times (\text{distance}) = E_1(1 \text{ m}) + E_2(2 \text{ m}) \]

\[ = \frac{\rho_s}{2\varepsilon_0}(1) + \frac{\rho_s}{4\varepsilon_0}(2) = \frac{\rho_s}{\varepsilon_0} = 0.6 \times 10^{-9} \times 8.85 \times 10^{-12} \]

\[ = 57.8 \text{ Volt} \]

**SOL 3.2.35** Option (C) is correct.

As calculated in previous question the electric field between the two dielectrics having surface charge densities \( \rho_s \) and \( -\rho_s \) is

\[ E = \frac{\rho_s}{\varepsilon} \]

where \( \varepsilon \) is the permittivity of the medium between the sheets.

So electric field in slab 1 is

\[ E_1 = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{2\varepsilon_0} \]

and electric field in slab 2 is

\[ E_2 = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{4\varepsilon_0} \]

Since the electric field between the sheets is uniform so the potential difference
between the plates will be
\[ V = \sum E \times \text{(distance)} = E_1(1 \text{ m}) + E_2(2 \text{ m}) \]
\[ = \frac{\rho_1}{2\varepsilon_0} (1) + \frac{\rho_2}{4\varepsilon_0} (2) = \frac{\rho_1}{\varepsilon_0} \]
\[ = 0.6 \times 10^{-9} \times \frac{8.85 \times 10^{-12}}{8.85 \times 10^{-12}} = 64.8 \text{ Volt} \]

**SOL 3.2.36** Option (A) is correct.
The electric field between the plates carrying charge densities \( +\rho_{S0} \) and \( -\rho_{S0} \) is defined as
\[ E = \frac{\rho_{S0}}{\varepsilon} \]
where \( \varepsilon \) is the permittivity of the medium between the plates.
Now consider that near the plate 1 permittivity is \( \varepsilon_1 \) and near the plate 2, permittivity is \( \varepsilon_2 \). So at any distance \( x \) from plate 1 permittivity is given by
\[ \varepsilon = \varepsilon_1 + \left( \frac{\varepsilon_2 - \varepsilon_1}{d} \right)x \quad \text{(Since the permittivity is linearly increasing)} \]
So the field intensity at any point in the medium will be
\[ E = \frac{\rho_{S0}}{\varepsilon_1 + \left( \frac{\varepsilon_2 - \varepsilon_1}{d} \right)x} \]
Therefore the potential difference between the plates will be
\[ V = \int_0^d \frac{\rho_{S0}}{\varepsilon_1 + \left( \frac{\varepsilon_2 - \varepsilon_1}{d} \right)x} \, dx = \left[ \frac{\rho_{S0}}{\varepsilon_1 + \left( \frac{\varepsilon_2 - \varepsilon_1}{d} \right)x} \right]_0^d = \frac{\rho_{S0}d}{\varepsilon_2 - \varepsilon_1} \ln \left( \frac{\varepsilon_2}{\varepsilon_1} \right) \]

**SOL 3.2.37** Option (A) is correct.
Assume that the surface charge densities on the plates is \( \pm \rho_{S0} \), so the electric field intensity between the plates will be
\[ E = \frac{\rho_{S0}}{\varepsilon} \]
and the potential difference between the plates will be given by
\[ V = E \times \text{(Distance between plates)} \]
\[ 5 \times 10^3 = \left( \frac{\rho_{S0}}{\varepsilon_0} \right) \times (0.5 \times 10^{-2}) \]
Therefore the surface charge density is
\[ \rho_{S0} = \frac{(8.85 \times 10^{-12}) \times (5 \times 10^3)}{(0.5 \times 10^{-2})} = 8.85 \mu\text{C} \]

**SOL 3.2.38** Option (C) is correct.
The capacitor of a parallel plate capacitor is defined as
\[ C = \frac{\varepsilon S}{d} \]
So, the capacitance in 1st dielectric region will be
\[ C_1 = \frac{\varepsilon_1 S}{1} = \frac{3\varepsilon_0 S}{1} \]
and the capacitance in 2nd dielectric region

**GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia**
Therefore the voltage drop in 1st dielectric region is
\[ V_1 = \frac{C_2}{C_1 + C_2} V \]
where \( V \) is total voltage drop
\[ = \frac{(2\varepsilon_0 S/3)}{3\varepsilon_0 S + \frac{2}{3}\varepsilon_0 S}(9 \text{ Volt}) = \frac{18}{11} \text{ Volt} \]
and similarly,
\[ V_2 = \frac{C_3}{C_1 + C_2} V = \frac{3\varepsilon_0 S}{3\varepsilon_0 S + 2\varepsilon_0 S}(9) = \frac{82}{11} \text{ Volt} \]

SOL 3.2.39 Option (C) is correct.

Consider the dielectric slab is of thickness \( t \) and \( d_1, d_2 \) are the remaining width in the medium as shown in the figure.

Now the capacitance of the whole configuration will be considered as the three capacitors (capacitance in the three regions) connected in series as shown in the figure

So,
\[ C_1 = \frac{\varepsilon_1 S}{d_1}, \quad C_2 = \frac{\varepsilon_2 S}{t} \quad \text{and} \quad C_3 = \frac{\varepsilon_1 S}{d_2} \]

The equivalent capacitance, is defined as
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{t}{\varepsilon S} + \frac{(d_1 + d_2)}{\varepsilon S} \]

Since \( t; (d_1 + d_2) \) will be constant although if the dielectric slab is moved leftward or rightward so the equivalent capacitance will be constant. But if the slab is pulled outward then the capacitance will change as the effective surface area of the capacitance due to dielectric slab changes.

SOL 3.2.40 Option (B) is correct.

Since, the wire is coated with aluminum, so the configuration can be treated as the
two resistance connected in parallel and therefore, the field potential will be same across both the material or we can say that the field intensity will be same inside both material.

i.e.  \[ E_{st} = E_{al} \]

where  \( E_{st} \rightarrow \) Field intensity in steel

\( E_{al} \rightarrow \) Field intensity in aluminum.

or,

\[ \frac{J_{st}}{\sigma_{st}} = \frac{J_{al}}{\sigma_{al}} \]

where  \( J_{st} \rightarrow \) current density in steel

\( J_{al} \rightarrow \) current density in aluminum

\( \sigma_{st} \rightarrow \) conductivity of steel

\( \sigma_{al} \rightarrow \) conductivity of aluminum

So, we get,

\[ \frac{J_{st}}{\sigma_{st}} = \frac{2 \times 10^6}{3.8 \times 10^6} = \frac{1}{19} \]

\[ J_{st} = 19J_{al} \quad \ldots(1) \]

Now, the total current through the wire is given as,

\[ I = J_{st} (\pi a^2) + J_{al} (\pi b^2 - \pi a^2) \]

where  \( a \rightarrow \) cross sectional radius of inner surface (steel wire)

\( b \rightarrow \) cross sectional radius of outer surface (with coating)

Since, thickness of coating is

\[ t = 2 \times 10^{-3} \]

So,

\[ b = a + t = (2 \times 10^{-3}) + (2 \times 10^{-3}) = (4 \times 10^{-3}) \]

Therefore, we get,

\[ 80 = J_{st} \pi (4 \times 10^{-6}) + J_{al} \left[ \pi (16 \times 10^{-6}) - \pi (4 \times 10^{-6}) \right] \]

or,

\[ 80 = J_{st} \pi (4 \times 10^{-6}) + 19J_{st} \left[ \pi (12 \times 10^{-6}) \right] \quad \text{(from eq.(1))} \]

So,

\[ J_{st} = \frac{80}{232\pi} \times 10^{-6} = 1.10 \times 10^5 \text{ A/m}^2 \]

**SOL 3.2.41** Option (A) is correct.

Given, the potential field in free space

\[ V = \frac{40}{r^3} \cos\theta\sin\phi \]

So, the potential at point \( P \ (r = 2, \ \theta = \frac{\pi}{3}, \ \phi = \frac{\pi}{2}) \) is given as

\[ V_p = \frac{40}{2} \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{2}\right) = 2.5 \text{ Volt} \]

Now, as the conducting surface is equipotential, so, the potential at any point on the conducting surface will be equal to the potential at point \( P \).

i.e.  \[ V = V_p = 2.5 \text{ Volt} \]

or  \[ \frac{40}{r^3} \cos\theta\sin\phi = 2.5 \]

\[ 16 \cos\theta\sin\phi = r^3 \]

This is the equation of the conducting surface.

***********
SOLUTIONS 3.3

SOL 3.3.1 Option (B) is correct.
The capacitance of a parallel plate capacitor is defined as
\[ C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 10^{-4}}{10^{-3}} = 8.85 \times 10^{-13} \]
The charge stored on the capacitor is
\[ Q = CV = 8.85 \times 10^{-13} = 4.427 \times 10^{-13} \]
Therefore, the displacement current in one cycle
\[ I = \frac{Q}{T} = fQ = 4.427 \times 10^{-13} \times 3.6 \times 10^9 = 2.59 \text{ mA} \quad (f = 3.6 \text{ GHz}) \]

SOL 3.3.2 Option (A) is correct.
The electric field of the EM wave in medium 1 is given as
\[ E_1 = 2a_x - 3a_y + 1a_z \]
Since the interface lies in the \( x = 0 \) plane so, the tangential and normal components of the field intensity in medium 1 are
\[ E_{1t} = -3a_y + a_x \] and \[ E_{1n} = 2a_z \]
From the boundary condition, tangential component of electric field is uniform. So, we get the tangential component of the field intensity in medium 2 as
\[ E_{2t} = E_{1t} = -3a_y + a_x \]
Again from the boundary condition the for normal component of electric flux density are uniform
\[ D_{1n} = D_{2n} \]
or
\[ \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \]
So, we get
\[ 1.5\varepsilon_2 a_x = 2.5\varepsilon_x E_{2n} \]
or
\[ E_{2n} = \frac{3}{2.5} a_x = 1.2 a_x \]
Thus, the net electric field intensity in the medium 2 is
\[ E_2 = E_{2t} + E_{2n} = -3a_y + a_x + 1.2a_x \]

SOL 3.3.3 Option (B) is correct.
The surface charge density on a conductor is equal to the electric flux density at its boundary.
i.e.
\[ \sigma = D = \varepsilon E = 80\varepsilon_0 E \]
\[ = 80 \times 8.854 \times 10^{-12} \times 2 = 1.41 \times 10^{-9} \text{ C/m}^2 \]

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
Option (D) is correct.

The configuration shown in the figure can be considered as the three capacitors connected in parallel as shown below.

\[
\begin{align*}
C_1 &= \frac{\varepsilon_0 \varepsilon_1 (S/3)}{d} \\
C_2 &= \frac{\varepsilon_0 \varepsilon_2 (S/3)}{d} \\
C_3 &= \frac{\varepsilon_0 \varepsilon_3 (S/3)}{d} \\
\end{align*}
\]

Since, the three capacitance are in parallel So, the equivalent capacitance is

\[
C_{eq} = \frac{1}{C_1 + C_2 + C_3} = \frac{1}{\left(\frac{\varepsilon_0 \varepsilon_1 (S/3)}{d} + \frac{\varepsilon_0 \varepsilon_2 (S/3)}{d} + \frac{\varepsilon_0 \varepsilon_3 (S/3)}{d}\right)} = \left(\frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3}\right) C \\
\]

Option (B) is correct.

The electric field is equal to the negative gradient of electric potential at the point.

\[\vec{E} = -\nabla V\]

Given, electric potential

\[V = 4x + 2\]

So,

\[\vec{E} = -4a_x \text{ V/m}\]

Option (D) is correct.

The angle formed by the electric field vector in two mediums are related as

\[
\tan \alpha_2 = \frac{\varepsilon_1}{\varepsilon_2}
\]

So, for the given field vectors we have,

\[
\tan 60^\circ = \frac{3}{\sqrt{3}} \\
\tan \alpha_2 = 1 \\
\]

or \[\alpha_2 = \tan^{-1}(1) = 45^\circ\]
SOL 3.3.7 Option (C) is correct.
The tangential component of electric field on conducting surface is zero (since the surface conducts current) So, under static condition we have,

\[- \nabla V = \mathbf{E} = 0\]

or

\[V = \text{constant}\]

i.e. the conducting surface is equipotential.

So, (A) and (R) both true and R is correct explanation of A.

SOL 3.3.8 Option (C) is correct.
Since, the electric field is incident normal to the slab. So, the electric field intensity \((\mathbf{E}_i)\) inside the slab is given as

\[\varepsilon \mathbf{E}_i = \varepsilon_0 \mathbf{E}_0\]

\[\mathbf{E}_i = \frac{\varepsilon_0 (6a_x)}{3\varepsilon_0} = 2a_x\]

Therefore, the polarization inside the slab is given as

\[\mathbf{P}_i = \varepsilon_0 \chi \mathbf{E}_i\]

where \(\chi\) is electric susceptibility defined as \(\chi = \varepsilon_r - 1\). So, we have

\[\mathbf{P}_i = \varepsilon_0 (3 - 1) \mathbf{E}_i = 4\varepsilon_0 a_x\]

SOL 3.3.9 Option (C) is correct.
Due to both the line charge and concentric circular conductors, the equipotential surfaces are circular (cylinder) i.e. concentric equipotential lines.
The flux lines due to both the configurations (line charge and concentric circular conductors) are in straight radial direction.

SOL 3.3.10 Option (C) is correct.
Capacitance of 1st plate is given as

\[C_1 = \frac{\varepsilon S_1}{d} = \frac{\varepsilon (1 \times 1)}{d} = \frac{\varepsilon}{d}\]

The capacitance of 2nd plate is

\[C_2 = \frac{\varepsilon S_2}{d} = \frac{\varepsilon (3 \times 2)}{d} = 6\varepsilon\]

So, the ratio of capacitances is

\[\frac{C_2}{C_1} = 4\]

SOL 3.3.11 Option (B) is correct.
Consider the dielectric material with permittivity \(\varepsilon_1\) is replaced by a dielectric material with permittivity \(\varepsilon_2\).
The capacitance of parallel plate capacitor is defined as

\[C = \frac{\varepsilon S}{d}\]

i.e. the capacitance depends on the permittivity of the medium and so, due to the replacement of the material between the plates the capacitance changes.
Now, the charge is kept constant
i.e. \( Q_1 = Q_2 \)

or, \( C_1 V_1 = C_2 V_2 \)

So, due to the change in capacitance voltage on the capacitor changes and therefore the electric field intensity between the plates changes.

The stored energy in the capacitance is defined as

\[ W = \frac{Q^2}{2C} \]

As total stored charge \( Q \) is kept constant while capacitance changes so, the stored energy in the capacitance also changes.

Thus, all the three given quantities changes due to the replacement of material between the plates.

SOL 3.3.12  Option (A) is correct.

According to continuity equation we have

\[ \nabla \cdot J = -\frac{\partial \rho}{\partial t} \]

As for electrostatic field \( \frac{\partial \rho}{\partial t} = 0 \) so, we get

\[ \nabla \cdot J = 0 \]

SOL 3.3.13  Option (C) is correct.

Electrostatic fields only.

SOL 3.3.14  Option (A) is correct.

Surface or sheet resistivity is defined as resistance per unit surface area. So, the unit of surface resistivity is Ohm/sq. meter.

SOL 3.3.15  Option (D) is correct.

Since a conducting surface is equipotential so no electric field component exists tangential to the surface and therefore the electric field lines are normal to a conducting surface boundary.

SOL 3.3.16  Option (D) is correct.

Surface resistance of a metal is defined as

\[ R_s \approx \sqrt{\frac{\omega \mu}{2\sigma}} = \sqrt{\frac{2\pi f \mu}{2\sigma}} \]

So, as frequency \( f \) increases the surface resistance increases.

SOL 3.3.17  Option (C) is correct.

When we determine force using method of images then in this method, the conducting surface is being removed and an additional distribution of charge is being introduced symmetrical to the existing charge distribution.

SOL 3.3.18  Option (D) is correct.

The conducting surface is equipotential and since the potential at infinity is zero so, the potential everywhere on a conducting surface of infinite extent is zero.

Since the conducting surface is equipotential so displacement density on a conducting
surface is normal to the surface.
So A and R both true but R is not correct explanation of A.

SOL 3.3.19 Option (C) is correct.
Capacitance,
\[ C = 5 \text{ pF} = 5 \times 10^{-2} \text{ F} \]
Charge on capacitance,
\[ Q = 0.1 \mu \text{C} = 0.1 \times 10^{-6} \text{ C} \]
The energy stored in the capacitor is defined as
\[ W = \frac{Q^2}{2C} = \frac{(0.1 \times 10^{-6})^2}{2 \times 5 \times 10^{-12}} = 1 \text{ mJ} \]

SOL 3.3.20 Option (A) is correct.
Consider the charge of 1 C is placed near a grounded conducting plate at a distance of 1 m as shown in figure.

Using image of the charge we have one negative charge opposite side of the plate at the same distance as shown in the figure and the force between them is
\[ F = \frac{(1)(-1)}{4\pi \varepsilon_0 r^2} = \frac{-1}{4\pi \varepsilon_0 (1)^2} = \frac{-1}{16\pi \varepsilon_0} \text{ N} \]
Negative sign indicates that the direction of force is attractive.

SOL 3.3.21 Option (D) is correct.
Fringing field has been shown below in the figure.

The capacitance of a parallel plate capacitor is given as
\[ C = \varepsilon_0 \varepsilon_r \frac{A}{d} \]

It is valid only when the fringing is not taken into account. Now, the fringing field can be ignored only when the separation \( d \) between the plates is much less than the plate dimensions. So, for the fringing field taken under consideration, \( A/d \) is tending towards infinity.

**SOL 3.3.22** Option (D) is correct.

The capacitance of a solid infinitely conducting sphere is defined as
\[ C = 4\pi \varepsilon_0 R, \]
where \( R \) is radius of the solid sphere.

**SOL 3.3.23** Option (C) is correct.

The electric potential produced by a point charge \( Q \) at the a distance \( r \) from it is defined as
\[ V = \frac{Q}{4\pi \varepsilon_r r}, \]
where \( \varepsilon \) is permittivity of the medium. So, the electric potential produced by the point charge +10 \( \mu \)C at the centre of the sphere is
\[ V = \frac{Q}{4\pi \varepsilon_r r} = \frac{10 \times 10^{-6}}{4\pi \varepsilon_0 (5 \times 10^{-2})} \]
(Given \( r = 5 \text{ cm} \))

As the surface of sphere is grounded so, the total voltage on the spherical capacitor will be equal to the potential at its centre as calculated above.

Now, the capacitance of the isolated sphere is defined as
\[ C = 4\pi \varepsilon a \]
where \( a \) is the radius of the sphere. Therefore, the induced charge stored on the sphere is given as
\[ Q_{\text{ind}} = CV = (4\pi \varepsilon_0 a) \left(10 \times 10^{-6}\right) \]
\[ = \frac{(2 \times 10^{-2}) \times (10 \times 10^{-6})}{(5 \times 10^{-2})} \]
(Given \( a = 2 \text{ cm} \))
\[ = 5 \times 10^{-6} \text{ C} = 5 \mu \text{C} \]

**SOL 3.3.24** Option (A) is correct.

Given electric field \( E = E_0 \sin \omega t \)

The conduction current is defined as
\[ J_c = \sigma(E) = \sigma E_0 \sin \omega t \]
where \( \sigma \) is conductivity and \( E \) is electric field intensity.

and the displacement current density is
\[ J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \]
\[ = \varepsilon E_0 (\omega \cos \omega t) = \varepsilon \omega E_0 \sin \left(\frac{\pi}{2} - \omega t\right) \]

So the phase difference between \( J_c \) and \( J_d \) is 90°.
SOL 3.3.25  Option (D) is correct.

Method of images are used for the charge distribution at a distance from the grounded plane conductor.

SOL 3.3.26  Option (A) is correct.

Given
Capacitance of condenser, \( C = 0.005 \ \mu F = 5 \times 10^{-9} \ \text{F} \)
Supply voltage, \( V = 500 \ \text{V} \)
Permittivity of oil, \( \varepsilon_r = 2.5 \) \hspace{1cm} \text{(immersed oil)}

So, the energy stored in condenser before immersion is
\[
W = \frac{1}{2} CV^2 = \frac{1}{2} \times 5 \times 10^{-9} \times (500)^2 = 6.25 \times 10^{-4} \ \text{J}
\]

After immersing the condenser in oil the capacitance changes while the total charge remains same.
\[
i.e. \quad Q_{\text{after immersion}} = Q_{\text{before immersion}} = (5 \times 10^{-9})(500) = 2.5 \times 10^{-6} \ \text{Coulomb}
\]

The capacitance of the condenser after immersion is
\[
C_{\text{after immersion}} = \varepsilon_r C = (2.5)(5 \times 10^{-9}) = 1.25 \times 10^{-8} \ \text{F}
\]

Therefore, the stored energy in the condenser immersed in oil is
\[
W = \frac{Q^2}{2C_{\text{after immersion}}} = \frac{(2.5 \times 10^{-6})}{2(1.25 \times 10^{-8})} = 2.5 \times 10^{-4} \ \text{J}
\]

SOL 3.3.27  Option (D) is correct.

Given,
Capacitance, \( C = 3 \ \mu F = 3 \times 10^{-6} \ \text{F} \)
Current, \( I = 2 \ \mu A = 2 \times 10^{-6} \ \text{A} \)
Charging time, \( t = 6 \ \text{sec} \)

So, the total charge stored on capacitor is
\[
Q = \text{Charge transferred} = It = (2 \times 10^{-6})(6) \]

Therefore, the voltage across the charged capacitor is
\[
V = \frac{Q}{C} = \frac{(2 \times 10^{-6})}{4 \times 10^{-6}} = 5 \ \text{Volt}
\]

SOL 3.3.28  Option (A) is correct.

Given, the total charge on capacitor = \( V \)

(1) Electric field between the plates will be given as
\[
\mathbf{E} = - \nabla V
\]

which is independent of permittivity of the material filled in capacitor so \( \mathbf{E} \) will be constant.

(2) The displacement flux density inside the capacitor is given as
\[ D = \varepsilon E \]

As \( E \) is constant while permittivity is doubled so \( D \) will also be doubled.

(3) The charge stored on the plates is given as
\[ Q = CV \]
where \( V \) is constant but capacitance \( C \) will be doubled as it is directly proportional to the permittivity given as
\[ C = \varepsilon \frac{S}{d} \]

So, the charge on plates will be get doubled.

(4) As discussed already, the capacitance will get doubled. Therefore, the statements 2 and 4 are correct.

**SOL 3.3.29**
Option (D) is correct.
Since, resistance doesn’t store any energy. So, the energy stored in the coil is only due to inductance and given as
\[ W = \frac{1}{2} Li^2 \]
where \( L \) is the inductance and \( I \) is the current flowing in the circuit. At the fully charged condition, inductor is short circuit and therefore, current through the circuit is
\[ I = \frac{V}{R} = \frac{50}{5} = 10 \text{ A} \]
So, the energy stored in the field (in the inductor) is
\[ W = \frac{1}{2} (0.6)(10)^2 = 30 \text{ Joules} \]

**SOL 3.3.30**
Option (A) is correct.
The normal component of electric flux density \( (D) \) across a dielectric-dielectric boundary is given as
\[ D_{1n} - D_{2n} = \rho_s \]
where \( \rho_s \) is the surface charge density at the interface.
So, the normal components of electric flux density across a dielectric-dielectric boundary is dependent on the magnitude of surface charge density.

**SOL 3.3.31**
Option (A) is correct.
Statement 1, 2 and 4 are correct while statement 3 is incorrect.

**SOL 3.3.32**
Option (D) is correct.
The capacitance of an insulated conducting sphere of radius \( R \) in vacuum is
\[ C = 4\pi\varepsilon_0 R \]

**SOL 3.3.33**
Option (A) is correct.
Maximum withstand voltage is the value that the dielectric between capacitor plates can tolerate without any electrical breakdown. Maximum withstand voltage is larger for any dielectric material than that for free space (air).
Since the maximum withstand voltage across the capacitor filled with air is \( V \) so
the maximum withstand voltage for the composite capacitor will be also $V$ as the capacitors are connected in parallel. Now, the capacitance before filling the dielectric is

$$C = \frac{\varepsilon_0 A}{d}$$

and after filling the dielectric

$$C'_{eq} = C_1 + C_2$$

$$= 4\varepsilon_0 \frac{A/2}{d} + \varepsilon_0 \frac{A/2}{d} = \frac{5\varepsilon_0 A}{2d}$$

So, the stored charge $Q_1$ after filling dielectric is determined as below

$$Q = \frac{C}{C'_{eq}} \quad (\text{Since voltage is constant})$$

or,

$$Q_1 = \frac{Q \cdot 5\varepsilon_0 A}{2d} = 2.5Q$$

Therefore, the maximum withstand voltage of the capacitor is $V$ and charge is $2.5Q$.

SOL 3.3.34 Option (B) is correct.

Since, the potential on both sides of plate will be same (Consider the potential is $V$). So, the charge densities on the two sides is determined as below:

$$\rho_2 = C'_{2} V$$

and

$$\rho_1 = C'_{1} V$$

where $C'_{2}$ and $C'_{1}$ are the capacitance per unit area of the capacitance formed by the region $d_1$ and $d_2$.

Therefore,

$$\frac{\rho_1}{\rho_2} = \frac{C'_{1} V}{C'_{2} V} = \frac{d_2}{d_1}$$

SOL 3.3.35 Option (A) is correct.

Electric flux density in a polarized dielectric is defined as

$$D = P + \varepsilon_0 E$$

SOL 3.3.36 Option (C) is correct.

Image theory is applicable only for static charge distribution (electrostatic field).

SOL 3.3.37 Option (B) is correct.

The equivalent capacitance of series connected capacitance has the value less than the smallest capacitance here the smallest capacitance is $C_6$ so the total capacitance is less then $C_6$

i.e. $C'_{eq} < C_6$

or $C'_{eq} \approx C_6$
SOL 3.3.38  Option (A) is correct.
Given, the electric field intensity in medium 1.
\[ E_1 = 5a_x - 2a_y + 3a_z \]
Since, the medium interface lies in plane \( z = 0 \).
So, we get the field components as
\[ E_{1t} = 5a_x - 2a_y \]
and
\[ E_{1n} = 3a_z \]
Now, From the boundary condition for electric field we have
\[ E_{1t} = E_{2t} \quad \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \]
So, the field components in medium 2 are
\[ E_{2t} = E_{1t} = 5a_x - 2a_y \]
\[ E_{2n} = \varepsilon_1 \varepsilon_2 E_{1n} = 6a_z \]
Therefore, the net electric field intensity in medium 2 is given as
\[ E_2 = E_{2t} + E_{2n} = 5a_x - 2a_y + 6a_z \]
So, the \( z \)-component of the field intensity in medium 2 is
\[ E_{2z} = 6a_z \]

SOL 3.3.39  Option (C) is correct.
Electric flux density, \( D = 1 \text{ C/m}^2 \)
Relative permittivity, \( \varepsilon_r = 5 \)
Since, the normal component of flux density is uniform at the boundary surface of two medium so, the flux density inside the slab is
\[ D = 1 \text{ C/m}^2 \]
Therefore, the polarization of the slab is given as
\[ P = \left( \frac{\varepsilon_r - 1}{\varepsilon_r} \right) D = \frac{4}{8} \times 1 = 0.8 \]

SOL 3.3.40  Option (D) is correct.
The capacitance of a isolated spherical capacitor of radius \( R \) is defined as
\[ C = 4\pi \varepsilon_0 R \]
Since the two spheres are identical and separated by a distance very much larger then \( R \). So, it can be assumed as the series combination of capacitances. Therefore, the net capacitance between two spheres is given as
i.e.
\[ C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4\pi \varepsilon_0 R)(4\pi \varepsilon_0 R)}{4\pi \varepsilon_0 R + 4\pi \varepsilon_0 R} = 2\pi \varepsilon_0 R \]

SOL 3.3.41  Option (A) is correct.
For steady current in an arbitrary conductor the current density is given as
\[ J = \frac{I}{A} \]
and since \( I \) is constant So, \( J \) is constant and therefore \( \nabla \times J = 0 \)
So, the current density is solenoidal. i.e. Assertion (A) is true.
The reciprocal of resistivity is conductivity. i.e. Reason (R) is false.
SOL 3.3.42  
Option (C) is correct. 
Since, the displacement current density is defined as 
\[ J_d = \frac{\partial D}{\partial t} \]  
So, it is generated by a change in electric flux and therefore the displacement current has only A.C. components as derivative of D.C. components is zero. i.e. A and R both are true and R is correct explanation of A.

SOL 3.3.43  
Option (D) is correct. 
Dielectric constant, \( \varepsilon_r = 5 \)  
Flux density, \( D = 2 \text{ C/m}^2 \)  
So, the polarization of the medium is given as 
\[ P = (\varepsilon_r - 1)D = \frac{4}{5} \times 2 = 2.6 \text{ C/m}^2 \]

SOL 3.3.44  
Option (A) is correct. 
The ohm’s law in point form in field theory is expressed as below 
\[ V = RI \]  
\[ El = \frac{\rho l}{A} JA \]  
where \( l \) is length integral and \( A \) is the cross sectional area. So, we get 
\[ E = \rho J \]
\[ E = \frac{J}{\sigma} \]
i.e. 
\[ J = \sigma E \]

SOL 3.3.45  
Option (A) is correct. 
Displacement current density is defined as 
\[ J_d = \varepsilon \frac{\partial E}{\partial t} \]  
and the conduction current density is defined as 
\[ J_c = \sigma E \]  
for a dielectric \( \varepsilon \) must be larger while conductivity must tend to zero. 
So, we get 
\[ J_d \gg J_c \]
i.e. displacement current is much greater than conduction current.

SOL 3.3.46  
Option (D) is correct. 
Conduction current, \( I_c = 1 \text{ A} \)  
Operating frequency, \( f = 50 \text{ Hz} \)  
Medium permittivity, \( \varepsilon = \varepsilon_0 \)  
Permeability \( \mu = \mu_0 \)  
Conductivity, \( \sigma = 5.8 \times 10^6 \text{ mho/m} \)  
The ratio of conduction current density to the displacement current density is 
\[ \frac{J_c}{J_d} = \frac{\sigma}{\omega \varepsilon} \]
or, 
\[ \frac{I_c}{A} = \frac{\sigma}{\omega \varepsilon} \]  
\( A \) is cross sectional area
\[ I_d = \frac{\omega \varepsilon}{\sigma} I_e = \frac{2\pi \times 50 \times \varepsilon_0}{5.8 \times 10} \]  
\[ = 3.3 \times 10^{-11} \text{ A} \]

**SOL 3.3.47**  
Option (B) is correct.  
When there is no charge in the interior of a conductor, the electric field intensity is zero according to Gauss’s law the total outward flux through a closed surface is equal to the charge enclosed.  
Now if any charge is introduced inside a closed conducting surface then an electric field will be setup and the field exerting a force on the charges and making them move to the conducting surface. So all the charges inside a conductor is distributed inside the conductor must vanish.  
A is false but R is true.

**SOL 3.3.48**  
Option (B) is correct.  
When the method of images is used for a system consisting of a point charge between two semi infinite conducting planes inclined at an angle $\phi$, the no. of images is given by  
\[ N = \left( \frac{360^\circ}{\phi} - 1 \right) \]  
Here the angle between conducting planes is $\phi = 90^\circ$.  
So,  
\[ N = 3 \]  
and since all the images lie a circle so we have the image charges as shown in figure.

**SOL 3.3.49**  
Option (D) is correct.  
Consider the two dielectric regions as shown below.

\[ E_1 = a_x \quad \text{Region 1} \quad \varepsilon_1 = \varepsilon_0 \]
\[ E_2 = 2a_x \quad \text{Region 2} \quad \varepsilon_2 = 2\varepsilon_0 \]
Since the field is normal to the interface, so, the normal components of the fields are,

\[ E_{1n} = 1 \quad \text{and} \quad E_{2n} = 2 \]

From boundary condition we have

\[ \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s \]

(where \( \rho_s \) is surface charge density on the interface).

\[ \varepsilon_0 (1) - (2 \varepsilon_0)(2) = \rho_s \]

\[ \rho_s = -3 \varepsilon_0 \]

**SOL 3.3.50** Option (D) is correct.

The stress is called the force per unit area which is directly proportional to the electric field intensity and electric field intensity is inversely proportional to the permittivity of dielectric material.

i.e.

\[ E \propto \frac{1}{\varepsilon} \]

So, ratio of stress is

\[ \frac{E_1}{E_2} = \frac{1}{\varepsilon_0} = \frac{1}{\varepsilon_0} = 5 \]

************
CHAPTER 4

MAGNETOSTATIC FIELDS
EXERCISE 4.1

MCQ 4.1.1
Assertion (A): For a static magnetic field the total number of flux lines entering a given region is equal to the total no. of flux lines leaving the region.
Reason (R): An isolated magnetic charge doesn’t exist.
(A) Both A and R one true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 4.1.2
Match List-I with List-II and select the correct answer using the codes given below. (Notations have their usual meaning).

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Ampere’s law</td>
<td>1. ( \mathbf{D} = \rho_v )</td>
</tr>
<tr>
<td>b. Conservative nature of magnetic field</td>
<td>2. ( \oint S \cdot d\mathbf{S} = \oint L \cdot d\mathbf{l} )</td>
</tr>
<tr>
<td>c. Gauss’s law</td>
<td>3. ( \nabla \cdot \mathbf{B} = 0 )</td>
</tr>
<tr>
<td>d. Non existence of magnetic monopole</td>
<td>4. ( \oint \mathbf{E} \cdot d\mathbf{l} = 0 )</td>
</tr>
</tbody>
</table>

Codes:
A  B  C  D
(A)  3  1  4  2
(B)  2  1  4  3
(C)  2  4  1  3
(D)  3  4  2  1

MCQ 4.1.3
Magnetic field intensity \( \mathbf{H} \) exists inside a certain closed spherical surface. The value of \( \nabla \cdot \mathbf{H} \) will be
(A) 0 at each point inside the sphere.
(B) 0 at the center of the sphere only.
(C) 0 at the outer surface of the sphere only.
(D) Can’t be determined as \( \mathbf{H} \) is not given.
MCQ 4.1.4 The source which doesn’t cause a magnetic field is
(A) A charged disk rotating at uniform speed
(B) An accelerated charge
(C) A charged sphere spinning along it’s axis
(D) A permanent magnet

MCQ 4.1.5 A circular loop of radius \(a\), centered at origin and lying in the \(xy\) plane, carries current \(I\) as shown in the figure.

The magnetic field intensity at the centre of the loop will be
(A) \(\frac{I}{2a} a_z\)  
(B) \(-\frac{I}{2a} a_z\)
(C) \(\frac{I}{4a} a_z\)
(D) \(\frac{2I}{a} a_z\)

MCQ 4.1.6 A conducting filament carries a current 5 A from origin to a point (3,0,4). Magnetic field intensity at point (3, 4, 0) due to the filament current will be
(A) 0.23 \(a_z\) wb/m²  
(B) 0.095 \(a_z\) wb/m²
(C) 0.074 \(a_z\) wb/m²  
(D) 0.074 \(a_z\) wb/m²

MCQ 4.1.7 A circular conducting loop of radius \(a\) centered at origin in the plane \(z = 0\) carries a current of 4 A in the \(a_x\) direction. What will be the magnetic field intensity at origin ?
(A) \(\frac{1}{2\pi} a\), A/m  
(B) \(a\), A/m
(C) 2\(a\), A/m  
(D) \(-a\), A/m

MCQ 4.1.8 The correct configuration that represents magnetic flux lines of a magnetic dipole is

(A)  
(B)
MCQ 4.1.9 The correct configuration that represents current $I$ and magnetic field intensity $H$ is

(A) $I$ (B) $H$

(C) $I$ (D) $H$

MCQ 4.1.10 A long straight wire placed along $z$-axis carries a current of $I = 5$ A in the $+z$ direction. The magnetic flux density at a distance $\rho = 5$ cm from the wire will be

(A) $4 \times 10^{-5}$ wb/m$^2$  
(B) $2 \times 10^{-5}$ wb/m$^2$  
(C) $\frac{100}{\pi}$ wb/m$^2$  
(D) $2 \times 10^{-6}$ wb/m$^2$

MCQ 4.1.11 For the currents and the closed path shown in the figure what will be the value of $\int H \cdot dl$?
MCQ 4.1.12 Two infinitely long wires separated by a distance 2 m, carry currents $I$ in opposite direction as shown in the figure.

If $I = 8$ A, then the magnetic field intensity at point $P$ is

(A) $\frac{5}{\pi} a_y$  \hspace{1cm} (B) $-\frac{5}{\pi} a_y$

(C) $\frac{1}{8\pi} a_y$  \hspace{1cm} (D) $-\frac{1}{8\pi} a_y$

MCQ 4.1.13 In the free space a semicircular loop of radius $a$ carries a current $I$. What will be the magnitude of magnetic field intensity at the centre of the loop?

(A) $\frac{I}{a}$  \hspace{1cm} (B) $\frac{2I}{a}$

(C) $\frac{I}{4a}$  \hspace{1cm} (D) $\frac{4I}{a}$

Common Data for Question 14 - 15:
A long cylindrical wire of cross sectional radius $R$ carries a steady current $I$ distributed over its outer surface.

MCQ 4.1.14 Magnetic field intensity inside the wire at a distance $r(<R)$ from it’s center axes will be

(A) non uniform  \hspace{1cm} (B) zero

(C) uniform and depends on $r$ only  \hspace{1cm} (D) uniform and depends on both $r$ and $R$

MCQ 4.1.15 The magnetic flux density outside the wire at a distance $r(>R)$ from it’s center axes will be proportional to

(A) $r$  \hspace{1cm} (B) $1/r$

(C) $r/R$  \hspace{1cm} (D) $1/R$
Two point charges $Q_1$ and $Q_2$ are located at $(0,0,0)$ and $(1,1,1)$ respectively. A current of 16 A flows from the point charge $Q_1$ to $Q_2$ along a straight wire connected between them. What will be the value of $\int B \cdot dl$ around the closed path formed by the triangle having the vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$?

(A) $22 \mu_0$ Wb/m
(B) $6\mu_0$ Wb/m$^2$
(C) $14\mu_0$ Wb/m
(D) $6\mu_0$ Wb/m

Common Data for Question 17 - 18

An infinite current sheet with uniform current density $K = 15a_z$ A/m is located in the plane $z = 2$.

MCQ 4.1.17 Magnetic field intensity at origin will be
(A) $11a_z$ A/m
(B) $-a_y$ A/m
(C) 0 A/m
(D) $20a_y$ A/m

MCQ 4.1.18 Magnetic field intensity at point $(2, -1, 5)$ will be
(A) $10a_y$ A/m
(B) $-10a_y$ A/m
(C) 0 A/m
(D) $20a_y$ A/m

MCQ 4.1.19 Two infinite current carrying sheets are placed parallel to each other in free space such that they carry current in the opposite direction with the same surface current density. The magnetic flux density in the space between the sheets will be
(A) zero
(B) constant
(C) linearly increasing from one sheet to other
(D) none of these

MCQ 4.1.20 In a spherical co-ordinate system magnetic vector potential at point $(r, \theta, \phi)$ is given as $A = 12\cos \theta a_\phi$. The magnetic flux density at point $(3,0,\pi)$ will be
(A) $4a_\phi$
(B) 0
(C) $4a_\theta$
(D) $36a_\phi$

MCQ 4.1.21 An infinite plane current sheet lying in the plane $y = 0$ carries a linear current density $K = Ka_z$ A/m. The magnetic field intensity above $(y > 0)$ and below $(y < 0)$ the plane will be

\[
\begin{align*}
\text{y > 0:} & & \quad \text{y < 0:} \\
\text{(A)} & & \quad \text{(B)} \\
\frac{K}{2}a_z & & \quad -\frac{K}{2}a_z \\
-\frac{K}{2}a_z & & \quad \frac{K}{2}a_z
\end{align*}
\]
MCQ 4.1.22
In the free space two cylindrical surfaces \( r = 0.3 \text{ cm} \) and \( r = 0.25 \text{ cm} \) carries the uniform surface current densities \( 2a_x \text{ A/m} \) and \( -0.8a_z \text{ A/m} \) respectively and a current filament on the entire \( z \)-axis carries a current of 14 mA in the \( +a_z \) direction. What will be the surface current density on the cylindrical surface at \( r = 0.1 \text{ cm} \) which will make the net magnetic field \( \mathbf{H} = 0 \) for \( r > 8 \text{ cm} \) will be

(A) \(-0.13a_z \text{ A/m}\)
(B) \(+0.13a_z \text{ A/m}\)
(C) \(64.3a_x \text{ mA/m}\)
(D) \(-0.10a_z \text{ A/m}\)

Statement for Linked Question 23 - 24:
In a cartesian system, vector magnetic potential at a point \( (x, y, z) \) is defined as

\[
\mathbf{A} = 4x^2y\mathbf{a}_x + 2y^2x\mathbf{a}_y - 3xyz\mathbf{a}_z \text{ wb/m}
\]

MCQ 4.1.23
The magnetic flux density at point \( (1, -2, -5) \) will be

(A) \(40a_x + 6a_z \text{ wb/m}^2\)
(B) \(40a_x + 80a_y + 6a_z \text{ wb/m}^2\)
(C) \(-40a_x - 80a_y - 6a_z \text{ wb/m}^2\)
(D) \(80a_y - 6a_z \text{ wb/m}^2\)

MCQ 4.1.24
The total magnetic flux through the surface \( z = 4, 0 \leq x \leq 1, -1 \leq y \leq 4 \) will be

(A) 20 wb
(B) \(-10/3 \text{ wb}\)
(C) 40 wb
(D) \(130/3 \text{ wb}\)

Statement for Linked Question 25 - 26:
An infinite current sheet with uniform surface current density \( K = [\square] \text{ A/m} \) is located at \( z = 0 \) as shown in figure.
MCQ 4.1.25 Magnetic flux density at any point above the current sheet \((z > 0)\) will be
(A) \(-2\mu_0 a_y \text{ wb/m}^2\)
(B) \(\frac{4\pi}{2} a_y \text{ wb/m}^2\)
(C) \(\frac{4\mu_0}{2} a_y \text{ wb/m}^2\)
(D) \(-\mu_0 a_y \text{ wb/m}^2\)

MCQ 4.1.26 The vector magnetic potential at \(z = -2\) will be
(A) \(4\mu_0 a_x \text{ wb/m}\)
(B) \(-4\mu_0 a_x \text{ wb/m}\)
(C) \(2\mu_0 a_x \text{ wb/m}\)
(D) \(-4\mu_0 a_x \text{ wb/m}\)

MCQ 4.1.27 In the free space, magnetic field intensity at any point \((\rho, \phi, z)\) is given by \(H = 2\rho^2 a_\phi \text{ A/m}\). The current density at \(\rho = 2\) m will be
(A) \(12 a_\phi \text{ A/m}\)
(B) \(24 a_\phi \text{ A/m}\)
(C) \(4 a_\phi \text{ A/m}\)
(D) \(0\)

MCQ 4.1.28 The current density that would produce the magnetic vector potential \(A = 2a_\phi\) in cylindrical coordinates is
(A) \(\frac{1}{\mu_0 \rho} a_\phi\)
(B) \(\frac{2\rho^2}{\mu_0} a_\phi\)
(C) \(\frac{2}{\mu_0 \rho} a_\phi\)
(D) \(\frac{2}{\mu_0 \rho^2} a_\phi\)

MCQ 4.1.29 Magnetic field intensity produced due to a current source is given as
\[H = (2z \cos cy)a_y + (4z + c^2)a_z\]
The current density over the \(xz\) plane will be
(A) \(\left(a_x - a_y - a_z\right) \text{ A/m}^2\)
(B) \(-a_x + a_y - a_z\)
(C) \(-2a_x + a_y - 2a_z\)
(D) \(a_x + a_y + a_z\)

MCQ 4.1.30 **Assertion (A):** In a source free region, magnetic field intensity can be expressed as a gradient of scalar function.

**Reason (R):** Current density for a given magnetic field intensity is defined as \(J = \nabla \times H\).

(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) R is true but A is false.
MCQ 4.1.31 Magnetic field intensity produced at a distance \( r \) from an infinite cylindrical wire located along entire \( z \)-axis is \( 3 \rho a_0 \, A/m \). The current density within the conductor will be
(A) \( 6 \rho a_0 \, A/m^2 \)
(B) \( 3a_0 \, A/m^2 \)
(C) \( 6a_0 \, A/m^2 \)
(D) \( 3 \rho a_0 \, A/m^2 \)

MCQ 4.1.32 An electron beam of radius \( a \) travelling in \( a \)-direction, the current density is given as
\[
J = 2 \left( 1 - \frac{\rho}{a} \right) a_z
\]
For \( \rho < a \)
The magnetic field intensity at the surface of the beam will be
(A) \( \frac{a}{3} a_0 \)
(B) \( \frac{a}{6} a_0 \)
(C) \( \frac{2\pi a^2}{3} \)
(D) \( \frac{a^2}{3} a_0 \)

MCQ 4.1.33 In a certain region consider the magnetic vector potential is \( A \) and the current density is \( J \). Which of the following is the correct relation between \( J \) and \( A \)?
(A) \( \nabla A = J \)
(B) \( \nabla^2 A = \mu_0 J \)
(C) \( \nabla \times A = -\mu J \)
(D) \( \nabla^2 A = -\mu_0 J \)

MCQ 4.1.34 A circular loop of wire with radius \( R = 1.5 \, m \) is located in plane \( x = 0 \), centered at origin. If the loop carries a current \( I = 7 \, A \) flowing in clockwise as viewed from negative \( x \)-axis then, its magnetic dipole moment will be
(A) \( 0.12a_0 \, A \cdot m^2 \)
(B) \( -5.5a_0 \, A \cdot m^2 \)
(C) \( 5.5a_0 \, A \cdot m^2 \)
(D) \( 22a_0 \, A \cdot m^2 \)

***********
EXERCISE 4.2

MCQ 4.2.1 In the free space, the positive z-axis carries a filamentary current of $10\ A$ in the $- a_z$ direction. Magnetic field intensity at a point $(0, 3, 2)$ due to the filamentary current will be

(A) $-0.73 a_z, \text{ A/m}$  
(B) $1.46 a_z, \text{ A/m}$  
(C) $0.40 a_z, \text{ A/m}$  
(D) $0.73 a_z, \text{ A/m}$

MCQ 4.2.2 If there is a current filament on the x-axis carrying $4.4\ A$ in $a_x$ direction then what will be the magnetic field intensity at point $(4, 2, 3)$?

(A) $0.1(2) a_x, \text{ A/m}$  
(B) $1.76 a_x - 1.62 a_y, \text{ A/m}$  
(C) $-1.077 a_x + 1.62 a_y, \text{ A/m}$  
(D) $-0.1(2a_x - a_y)$

MCQ 4.2.3 A filamentary conductor is formed into an equilateral triangle of side $2\ m$ that carries a current of $4\ A$ as shown in figure. The magnetic field intensity at the center of the triangle will be

(A) $\frac{9}{\pi} a_z, \text{ A/m}$  
(B) $\frac{2}{\pi} a_y, \text{ A/m}$  
(C) $\frac{6}{\pi} a_x, \text{ A/m}$  
(D) $0$

MCQ 4.2.4 A current sheet $K = 4 a_y, \text{ A/m}$ flows in the region $-2 < z < 2$ in the plane $x = 0$. Magnetic field intensity at point $P(3, 0, 0)$ due to the current sheet will be

(A) $-1.5 a_y, \text{ A/m}$  
(B) $-0.75 a_y, \text{ A/m}$  
(C) $+0.75 a_y, \text{ A/m}$  
(D) $-2.1 a_y, \text{ A/m}$
MCQ 4.2.5 In the plane $z = 0$ a disk of radius $\sqrt{3}$ m, centered at origin carries a uniform surface charge density $\rho_s = 2 \text{ C/m}^2$. If the disk rotates about the $z$-axis at an angular velocity $\omega = 2 \text{ rad/s}$ then the magnetic field intensity at the point $P(0,0,1)$ will be

(A) $a_z \text{ A/m}$  
(B) $2a_z \text{ A/m}$  
(C) $a_y \text{ A/m}$  
(D) $2a_y \text{ A/m}$

MCQ 4.2.6 A square conducting loop of side 1 m carries a steady current of 2 A. Magnetic flux density at the center of the square loop will be.

(A) $17.78 \times 10^{-7} \text{ Wb/m}^2$  
(B) $0.45 \text{ Wb/m}^2$  
(C) $2.26 \times 10^{-6} \text{ Wb/m}^2$  
(D) $4 \times 10^{-7} \text{ Wb/m}^2$

MCQ 4.2.7 A filamentary conductor is formed into a loop $ABCD$ as shown in figure. If it carries a current of 5.2 A then the magnetic field intensity at point $P$ will be

(A) 0.2 A/m  
(B) 0.8 A/m  
(C) 0.26 A/m  
(D) 1.01 A/m

MCQ 4.2.8 The magnetic field intensity at point $P$ due to the steady current configurations shown in figure will be

(A) 0.82 A m  
(B) 0.32 A m  
(C) 0.5 A m  
(D) 0.18 A m

MCQ 4.2.9 In the plane $z = 5$ m a thin ring of radius $a = 3$ m is placed such that $z$-axis passes through it’s center. If the ring carries a current of 50 mA in $a_y$ direction then the magnetic field intensity at point $(0, 0, 1)$ will be

(A) $0.9a_y \text{ mA/m}$  
(B) $1.8a_y \text{ mA/m}$  
(C) $0.6a_y \text{ mA/m}$  
(D) $0.5a_y \text{ A/m}$
MCQ 4.2.10 An infinite solenoid (infinite in both direction) consists of 1000 turns per unit length wrapped around a cylindrical tube. If the solenoid carries a current of 4 mA then the magnetic field intensity at its axis will be
(A) 4 A/m  (B) 0  
(C) 2000 A/m  (D) 0.2 A/m

Common Data for Question 11 - 13 :
The two long coaxial solenoids of radius \( a \) and \( b \) carry current \( I = 6 \) mA but in opposite directions. Solenoids are placed along \( y \)-axis as shown in figure. The inner solenoid has 2000 turns per unit length and outer solenoid has 1000 turns per unit length.

MCQ 4.2.11 Magnetic field intensity inside the inner solenoid will be
(A) \(-3a_y\) A/m  (B) \(+6a_y\) A/m  
(C) \(-6a_y\) A/m  (D) \(+3a_y\) A/m

MCQ 4.2.12 The magnetic field intensity in the region between the two solenoids will be
(A) \(3a_y\) A/m  (B) 0  
(C) \(6a_y\) A/m  (D) \(-3a_y\) A/m

MCQ 4.2.13 The magnetic field outside the outer solenoid will be
(A) \(6a_y\) A/m  (B) \(-3a_y\) A/m  
(C) 0  (D) \(+3a_y\) A/m

Statement for Linked Question 14 - 15 :
In a cartesian system two parallel current sheets of surface current density \( K_1 = 3a_z\) A/m and \( K_2 = -3a_z\) A/m are located at \( x = 2\) m and \( x = -2\) m respectively. The net vector and scalar potential due to the sheets are zero at a point \( P(1,2,5)\).

MCQ 4.2.14 Consider the scalar potential at any point \((x,y,z)\) in the region between the two
planar sheets is \( V_m \). The plot of \( V_m \) versus \( y \) will be

\[ V_m(A) \]

(A) \[
\begin{array}{c}
6 \\
2 \\
2 \\
-6
\end{array}
\]

(B) \[
\begin{array}{c}
2 \\
2 \\
-2 \\
-2
\end{array}
\]

(C) \[
\begin{array}{c}
2 \\
2 \\
2 \\
-2
\end{array}
\]

(D) \[
\begin{array}{c}
2 \\
2 \\
2 \\
2
\end{array}
\]

MCQ 4.2.15 The vector potential at origin will be
(A) \( 3\mu_0 a \) Wb/m
(B) \( -3\mu_0 a \) Wb/m
(C) 0
(D) \( -3 \) Wb/m

MCQ 4.2.16 A long cylindrical wire lying along \( z \)-axis carries a total current \( I_0 = 15 \) mA as shown in the figure. The current density inside the wire at a distance \( \rho \) from it’s axis is given by \( J \propto \rho \).

If the cross sectional radius of the wire is 2 cm then the magnetic flux density at \( \rho = 1 \) cm will be
(A) \( 25 \) nWb/m²
(B) \( 6.25 \times 10^{-4} \) Wb/m²
(C) 1.25 nWb/m²
(D) 12.5 nWb/m²
MCQ 4.2.17  
Magnetic field intensity is given in a certain region as

\[ H = \frac{x^2 y z}{1 + x} a_x + 3x^2 z^2 a_y - \frac{xy^2}{y + 1} a_z \text{ A/m} \]

The total current passes through the surface \( x = 2 \text{ m}, 1 \leq y \leq 4 \text{ m}, 3 \leq z \leq 4 \text{ m} \) in \( a_x \) direction will be

(A) 259 A  
(B) 259 A  
(C) 18.2 A  
(D) \( 8 \times 10^8 \) A

MCQ 4.2.18  
A phonograph record of radius 1m carries a uniform surface charge density \( \rho_s = 20 \text{ C/m}^2 \). If it is rotating with an angular velocity \( \omega = 0.1 \text{ rad/s} \); then the magnetic dipole moment will be

(A) \( 4\pi \text{ A} \cdot \text{m}^2 \)  
(B) \( \pi/2 \text{ A} \cdot \text{m}^2 \)  
(C) \( 2\pi \text{ A} \cdot \text{m}^2 \)  
(D) \( \frac{2\pi}{3} \text{ A} \cdot \text{m}^2 \)

Statement for Linked Question 19 - 20:

A uniformly charged solid sphere of radius \( r \) is spinning with angular velocity \( \omega = 6 \text{ rad/s} \) about the z-axis. The sphere is centered at origin and carries a total charge 5C which is uniformly distributed over it’s volume.

MCQ 4.2.19  
The plot of magnetic dipole moment of the sphere, \( m(r) \) versus the radius of the sphere, \( r \) will be

(A)  
(B)  
(C)  
(D)

MCQ 4.2.20  
The average magnetic field intensity within the sphere will be

(A) \( \frac{2}{\pi r} a_\phi \)  
(B) \( \frac{2}{\pi r} a_z \)  
(C) \( \frac{2}{\pi r} a_\phi \)  
(D) \( \frac{1}{2\pi r} a_z \)
A rectangular coil, lying in the plane \( x + 3y - 1.5z = 3.5 \) carries a current \( 7 \, \text{A} \) such that the magnetic moment of the coil is directed away from the origin. If the area of the rectangular coil is \( 0.1 \, \text{m}^2 \) then the magnetic moment of the coil will be

(A) \(-0.2a_x - 0.2a_y + 0.3a_z \, \text{A} \cdot \text{m}^2\)
(B) \(2a_x + 6a_y - 3a_z \, \text{A} \cdot \text{m}^2\)
(C) \(1.4a_x + 4.2a_y - 2.1a_z \, \text{A} \cdot \text{m}^2\)
(D) \(0.2a_x + 0.6a_y - 0.3a_z \, \text{A} \cdot \text{m}^2\)

Vector magnetic potential in a certain region of free space is \( \mathbf{A} = (6y - 2z) \mathbf{a}_x + 4xz\mathbf{a}_y \).

The electric current density at any point \( (x, y, z) \) will be

(A) \((a_x + 2a_y + 6a_z) \, \text{A/m}^2\)
(B) \((3a_y + a_z) \, \text{A/m}^2\)
(C) \(0 \, \text{A/m}^2\)
(D) \(\frac{1}{6}(8a_x + 2a_y - 6a_z) \, \text{A/m}^2\)

Statement for Linked Question 23 - 24:

A circular toroid with a rectangular cross section of height \( h = 5 \, \text{m} \), carries a current \( I = 10 \, \text{A} \) flowing in \( 10^5 \) turns of closely wound wire around it as shown in figure. The inner and outer radii of toroid are \( a = 1 \, \text{m} \) and \( b = 2 \, \text{m} \) respectively.

MCQ 4.2.23
The total magnetic flux across the circular toroid will be

(A) \(1.39 \, \text{Wb}\)
(B) \(0.14 \, \text{Wb}\)
(C) \(15.1 \, \text{Wb}\)
(D) \(0 \, \text{Wb}\)

MCQ 4.2.24
If the magnetic flux is found by multiplying the cross sectional area by the flux density at the mean radius then what will be the percentage of error?

(A) \(-4.31\%\)
(B) \(-3.14\%\)
(C) \(-4.61\%\)
(D) \(-6.14\%\)

MCQ 4.2.25
Magnetizing force at any point \( P \) on \( z \)-axis due to a semi infinite current element placed along positive \( x \)-axis is \( H \). If one more similar current element is placed along positive \( y \)-axis then the resultant magnetizing force at the point \( P \) will be

(A) \(H/\sqrt{2}\)
(B) \(\sqrt{2}H\)
(C) \(2H\)
(D) \(-\sqrt{2}H\)
MCQ 4.2.26
An infinitely long straight wire carrying current $5\,\text{A}$ and a square loop of side $2\,\text{m}$ are coplanar as shown in the figure. The distance between side $AB$ of square loop and the straight wire is $4\,\text{m}$. What will be the total magnetic flux crossing through the rectangular loop?

![Diagram](image)

(A) $2.55\,\mu\text{Wb}$  
(B) $8.11\,\mu\text{Wb}$  
(C) $0.81\,\mu\text{Wb}$  
(D) $8.11\,\mu\text{Wb}$

MCQ 4.2.27
A $1.5\,\text{m}$ square loop is lying in $x$-$y$ plane such that one of its side is parallel to $y$-axis and the centre of the loop is $0.3\,\text{m}$ away from the $y$-axis. How much current must flow through the entire $y$-axis for which the magnetic flux through the loop is $5 \times 10^{-5}\,\text{Tesla m}^2$?

(A) $417\,\text{A}$  
(B) $834\,\text{A}$  
(C) $208.5\,\text{A}$  
(D) $280\,\text{A}$

MCQ 4.2.28
A $L$-shaped filamentary wire with semi infinite long legs making an angle $90^\circ$ at origin and lying in $y$-$z$ plane as shown in the figure.

![Diagram](image)

If the current flowing in the wire is $I = 4\,\text{A}$ then the magnetic flux density at $(2\,\text{m}, 0, 0)$ will be

(A) $-2 \times 10^{-7}(a_y + a_z)\,\text{Wb/m}^2$  
(B) $2 \times 10^{-7}(a_x - a_z)\,\text{Wb/m}^2$  
(C) $-4 \times 10^{-7}(a_y + a_z)\,\text{Wb/m}^2$  
(D) $4 \times 10^{-7}(a_y + a_z)\,\text{Wb/m}^2$

Common Data for Question 29 - 30:
An infinitely long straight conductor of cylindrical cross section and of radius $R$ carries a current $I$, which is uniformly distributed over the conductor cross section.
MCQ 4.2.29  If the conductor is located along \( z \)-axis then the magnetic flux density at a distance \( \rho > R \) from the cylindrical axis will be

(A) \( \frac{\mu_0 I \rho}{2\pi R^2} \mathbf{a}_\phi \)  
(B) \( \frac{\mu_0 I}{2\pi \rho} \mathbf{a}_\phi \)  
(C) \( \frac{\mu_0 I}{2\pi} \mathbf{a}_\phi \)  
(D) \( \frac{\mu_0 I \rho^2}{R^2} \mathbf{a}_\phi \)

MCQ 4.2.30  Magnetic flux density at a distance \( \rho > R \) from the cylindrical axis will be proportional to

(A) \( \frac{1}{\rho} \)  
(B) \( \frac{1}{\rho^2} \)  
(C) \( \rho \)  
(D) \( \rho^2 \)

MCQ 4.2.31  Consider a filamentary wire is bent to form a square loop of side 3 m lying in the \( x-y \) plane as shown in the figure. If the current flowing in the wire is \( I = 1 \text{ A} \) then the magnetic flux density at the center of the loop will be

(A) \( 2\sqrt{2} \times 10^{-7} \text{ Wb/m}^2 \)  
(B) \( 4\sqrt{2} \times 10^{-7} \text{ Wb/m}^2 \)  
(C) \( 2 \times 10^{-7} \text{ Wb/m}^2 \)  
(D) \( -4\sqrt{2} \times 10^{-7} \text{ Wb/m}^2 \)

MCQ 4.2.32  An infinitely long straight wire carrying a current 20 A and a circular loop of wire carrying a current \( I \) are coplanar as shown in the figure.

The radius of the circular loop is 10 cm and the distance of the centre of the loop from the straight wire is 1 m. If the net magnetic field intensity at the centre of the loop is zero then the current \( I \) is

(A) \( \frac{2}{7} \text{ A} \)  
(B) \( \frac{20}{7} \text{ A} \)  
(C) \( \frac{5}{7} \text{ A} \)  
(D) \( 2\pi \text{ A} \)
MCQ 4.2.33 The magnitude of the magnetic field intensity produced at center of a square loop of side $a$ carrying current $I$ is

(A) $\frac{2\sqrt{2}I}{\pi a}$  \hspace{1cm} (B) $\frac{\sqrt{2}I}{\pi a}$

(C) $\frac{I}{\sqrt{2} \pi a}$  \hspace{1cm} (D) $\frac{8I}{\pi a}$

MCQ 4.2.34 For the single turn loop of current shown in the figure the magnetic field intensity at the center point $P$ of the semi circular portion will be

(A) 5.8 A/m outward  \hspace{1cm} (B) 5.8 A/m inward

(C) 3.8 A/m outward  \hspace{1cm} (D) 3 A/m inward

MCQ 4.2.35 Two perfect conducting infinite parallel sheets separated by a distance 2 m carry uniformly distributed surface currents with equal and opposite densities $4a_x$ and $-4a_x$ respectively as shown in figure.

The medium between the two sheets is free space. What will be the magnetic flux between the sheets per unit length along the direction of current ?

(A) 0  \hspace{1cm} (B) $8\mu_0 a_y$ Wb/m

(C) $-8\mu_0 a_y$ Wb/m  \hspace{1cm} (D) $-4\mu_0 a_y$ Wb/m

************
EXERCISE 4.3

Statement for Linked Question 1 - 2:
An infinitely long uniform solid wire of radius $a$ carries a uniform dc current of density $J$.

MCQ 4.3.1
The magnetic field at a distance $r$ from the center of the wire is proportional to
(A) $r$ for $r < a$ and $1/r^2$ for $r > a$
(B) $0$ for $r < a$ and $1/r$ for $r > a$
(C) $r$ for $r < a$ and $1/r$ for $r > a$
(D) $0$ for $r < a$ and $1/r^2$ for $r > a$

MCQ 4.3.2
A hole of radius $b (b < a)$ is now drilled along the length of the wire at a distance $d$ from the center of the wire as shown below.

The magnetic field inside the hole is
(A) uniform and depends only on $d$
(B) uniform and depends only on $b$
(C) uniform and depends on both $b$ and $d$
(D) non-uniform

MCQ 4.3.3
Two infinitely long wires carrying current are as shown in the figure below. One wire is in the $yz$-plane and parallel to the $y$-axis. The other wire is in the $xy$-plane and parallel to the $x$-axis. Which components of the resulting magnetic field are non-zero at the origin?
MCQ 4.3.4
IES EC 2012
A flux of 2.2 mWb exerts in a magnet having a cross-section of 30 cm². The flux density in tesla is
(A) 4
(B) 0.4
(C) 2.5
(D) 40

MCQ 4.3.5
IES EC 2010
The magnetic flux density $B$ and the vector magnetic potential $A$ are related as
(A) $B = \nabla \times A$
(B) $A = \nabla \times B$
(C) $B = \nabla \cdot A$
(D) $A = \nabla \cdot B$

MCQ 4.3.6
IES EC 2010
Consider the following statements relating to the electrostatic and magnetostatic field:
1. The relative distribution of charges on an isolated conducting body is dependent on the total charge of the body.
2. The magnetic flux through any closed surface is zero.
Which of the above statements is/are correct?
(A) Neither 1 nor 2
(B) 1 only
(C) 2 only
(D) Both 1 and 2

MCQ 4.3.7
IES EC 2008
The line integral of the vector potential $A$ around the boundary of a surface $S$ represents which one of the following?
(A) Flux through the surface $S$
(B) Flux density in the surface $S$
(C) Magnetic field intensity
(D) Current density

MCQ 4.3.8
IES EC 2008
An infinitely long straight conductor located along z-axis carries a current $I$ in the $+ve$ z-direction. The magnetic field at any point $P$ in the $x-y$ plane is in which
direction?
(A) In the positive $z$-direction
(B) In the negative $z$-direction
(C) In the direction perpendicular to the radial line $OP$ (in $x-y$ plane) joining the origin $O$ to the point $P$
(D) Along the radial line $OP$

**MCQ 4.3.9**
IES EC 2008
A 13 A current enters a right circular cylinder of 5 cm radius. What is the linear surface current density at the end surface?
(A) $(50/\pi) \text{A/m}$
(B) $(100/\pi) \text{A/m}$
(C) $(1000/\pi) \text{A/m}$
(D) $(2000/\pi) \text{A/m}$

**MCQ 4.3.10**
IES EC 2007
What is the value of the magnetic vector potential due to an infinitesimally small current element, evaluated at infinite distance from it?
(A) Infinity
(B) Unity
(C) Zero
(D) Any number between zero and infinity depending on the strength of the current element

**MCQ 4.3.11**
IES EC 2007
What is the magnetic field intensity vector $\mathbf{H}$ between two parallel sheets with separation ‘$d$’ along z-axis both sheets carrying surface current $\mathbf{K} = K_y a_y$?
(A) $-k_y a_y$
(B) $+k_y a_y$
(C) $-k_y a_x$
(D) Zero

**MCQ 4.3.12**
IES EC 2005
Current density ($\mathbf{J}$), in cylindrical coordinate system is given as:
$$\mathbf{J}(\rho, \phi, z) = \begin{cases} 0 & \text{for } 0 < \rho < a \\ \rho / a^2 \mathbf{a}_z & \text{for } a < \rho < b \end{cases}$$
where $\mathbf{a}_z$ is the unit vector along $z$-coordinate axis. In the region, $a < \rho < b$, what is the expression for the magnitude of magnetic field intensity ($H$)?
(A) $J_0 (\rho^3 - a^3) / \rho^2$
(B) $J_0 (\rho^3 + a^3) / \rho^2$
(C) $J_0 (\rho^3 - a^3) / 3a^2 \rho$
(D) $J_0 (\rho^3 + a^3) / 2\pi \rho$

**MCQ 4.3.13**
IES EC 2005
Which one of the following concepts is used to find the expression of radiated $\mathbf{E}$ and $\mathbf{H}$ field due to a magnetic current element?
(A) Concept of vector magnetic potential
(B) Concept of scalar electric potential
(C) Concept of scalar magnetic potential
(D) Concept of vector electric potential

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 4.3.14 The circulation of $H$ around the closed contour $C$, shown in the figure is

(A) 0  
(B) 2l  
(C) 4l  
(D) 6l

MCQ 4.3.15 The unit of magnetic flux density is

(A) gauss  
(B) tesla  
(C) bohr  
(D) weber/sec

MCQ 4.3.16 The magnetic flux density created by an infinitely long conductor carrying a current $I$ at a radial distance $R$ is

(A) $\frac{\mu_0 I}{2\pi R}$  
(B) $\frac{1}{2\pi R}$  
(C) $\frac{\mu_0 I}{2\pi R^3}$  
(D) $\frac{q^2 I}{3}$

MCQ 4.3.17 Match List I with List II and select the correct answer using the codes given below the lists.

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Work</td>
<td>1. Ampere/metre</td>
</tr>
<tr>
<td>b. Electric field strength</td>
<td>2. Weber</td>
</tr>
<tr>
<td>c. Magnetic flux</td>
<td>3. Volt/metre</td>
</tr>
<tr>
<td>d. Magnetic field strength</td>
<td>4. Joule</td>
</tr>
</tbody>
</table>

Codes :

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>(B)</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>(C)</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(D)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

MCQ 4.3.18 A long straight wire carries a current $I = 1$ A. At what distance is the magnetic field 1 Am$^{-1}$?

(A) 1.59 m  
(B) 0.159 m  
(C) 0.0159 m  
(D) 0.00159 m
MCQ 4.3.19
How much current must flow in a loop radius 1 m to produce a magnetic field 1 mA/m? 
(A) 1.0 mA  
(B) 1.5 mA  
(C) 2.0 mA  
(D) 2.5 mA

MCQ 4.3.20
Assertion (A): Knowing magnetic vector potential $\mathbf{A}$ at a point, the flux density $\mathbf{B}$ at the point can be obtained.
Reason (R): $\nabla \cdot \mathbf{A} = 0$.
(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true and R is not the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true

MCQ 4.3.21
The magnetic vector potential $\mathbf{A}$ obeys which equations? 
1. $\mathbf{B} = \nabla \times \mathbf{A}$  
2. $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$  
3. $\mathbf{A} = \int \frac{\mu_0 I dl}{4\pi R}$
Select the correct answer using the code given below: 
(A) 1 and 2  
(B) 2 and 3  
(C) 1 and 3  
(D) 1, 2 and 3

MCQ 4.3.22
A long straight wire carries a current $I = 10$ A. At what distance is the magnetic field $H = 1$ Am$^{-1}$? 
(A) 1.19 m  
(B) 1.39 m  
(C) 1.59 m  
(D) 1.79 m

MCQ 4.3.23
What is the magnetic field due to an infinite linear current carrying conductor? 
(A) $H = \frac{\mu I}{2\pi r}$ A/m  
(B) $H = \frac{I}{2\pi r}$ A/m  
(C) $H = \frac{\mu I}{2r}$ A/m  
(D) $H = \frac{I}{r}$ A/m

MCQ 4.3.24
Equation $\nabla \cdot \mathbf{B} = 0$ is based on 
(A) Gauss’s Law  
(B) Lenz’s Law  
(C) Ampere’s Law  
(D) Continuity Equation

MCQ 4.3.25
Plane $y = 0$ carries a uniform current density $30a_z$ mA/m. At (1, 20, -2), what is the magnetic field intensity? 
(A) $-15a_z$ mA/m  
(B) $15a_z$ mA/m  
(C) $18.85a_z$ mA/m  
(D) $25a_z$ mA/m
MCQ 4.3.26
IES EE 2005
Which one of the following is not the valid expression for magnetostatic field vector $B$?

(A) $B = \nabla \cdot A$
(B) $B = \nabla \times A$
(C) $\nabla \cdot B = 0$
(D) $\nabla \times B = \mu_0 J$

MCQ 4.3.27
IES EE 2004
Which one of the following statements is correct? Superconductors are popularly used for

(A) generating very strong magnetic field
(B) reducing $iR^2$ losses
(C) generating electrostatic field
(D) generating regions free from magnetic field

MCQ 4.3.28
IES EE 2002
Assertion (A) : $\int B \cdot dS = 0$ where, $B =$ magnetic flux density, $dS =$ vector with direction normal to surface elements $dS$.

Reason (R) : Tubes of magnetic flux have no sources or sinks.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 4.3.29
IES EE 2001
Plane defined by $z = 0$ carry surface current density $2a_x$ A/m. The magnetic intensity $H_y$ in the two regions $-\alpha < z < 0$ and $0 < z < \alpha$ are respectively

(A) $a_y$ and $-a_y$
(B) $-a_y$ and $a_y$
(C) $a_x$ and $-a_x$
(D) $-a_x$ and $a_x$

***********
SOLUTIONS 4.1

SOL 4.1.1 Option (D) is correct. 
It is not possible to have an isolated magnetic poles (or magnetic charges). If we desire to have an isolated magnetic dipole by dividing a magnetic bar successively into two, we end up with pieces each having north and south poles. So an isolated magnetic charge doesn’t exist. 
That’s why the total flux through a closed surface in a magnetic field must be zero. 
i.e. \( \int B \cdot dS = 0 \) 
or more clear, we can write that for a static magnetic field the total number of flux lines entering a given region is equal to the total number of flux lines leaving the region. 
So, (A) and (R) are both true and R is correct explanation of A.

SOL 4.1.2 Option (B) is correct. 

SOL 4.1.3 Option (D) is correct. 
Since the field intensity exists in a closed surface and lines of field intensity makes a closed curve so the flux lines leaving the spherical surface equal to the total flux entering the surface and So the net flux 
\[ \Phi = \int B \cdot dS = 0 \] 
According to divergence theorem 
\[ \int B \cdot dS = \int ( \nabla \cdot B ) dv \] 
\[ 0 = \int \nabla \cdot B dv \] 
Since volume of the sphere will have certain finite value so, 
\[ \nabla \cdot B = 0 \] 
or 
\[ \nabla \cdot H = 0 \] at all points inside the sphere

SOL 4.1.4 Option (C) is correct. 
The Magnetic field are caused only by current carrying elements and given as 
\[ B = \frac{\mu_0 I dl \times R}{4\pi R^2} \] 
Since an accelerated electron doesn’t form any current element \( I dl \) so it is not a source of magnetic field.

SOL 4.1.5 Option (D) is correct. 
The magnetic field intensity produced due to a small current element \( I dl \) is defined
as
\[ dH = \frac{Idl \times a_R}{4\pi R^2} \]
where \( dl \) is the differential line vector and \( a_R \) is the unit vector directed towards the point where field is to be determined. So for the circular current carrying loop we have
\[ dl = ad\phi a_\phi \]
\[ a_R = -\hat{r} \]
Therefore the magnetic field intensity produced at the centre of the circular loop is
\[ H = \int_{\phi=0}^{2\pi} \frac{Id\phi a_\phi \times (-a_\phi)}{4\pi a^2} = \frac{Ia}{4\pi a^2} \left[ (\hat{\phi})_0 \right] (a_z) = \frac{I}{4a} a_z \text{ A/m} \]

**SOL 4.1.6** Option (B) is correct.

[Diagram of a current-carrying loop with vectors labeled]

Magnetic field intensity at any point \( P \) due to a filamentary current \( I \) is defined as
\[ H = \frac{I}{4\pi \rho} (\cos \alpha_2 - \cos \alpha_1) a_\phi \]
where \( \rho \rightarrow \) distance of point \( P \) from the current filament.
\( \alpha_1 \rightarrow \) angle subtended by the lower end of the element at \( P \).
\( \alpha_2 \rightarrow \) angle subtended by the upper end of the element at \( P \).

From the figure we have
\[ \rho = \sqrt{3^2 + 4^2} = 5 \]
\[ \alpha_1 = \pi/2 \Rightarrow \cos \alpha_1 = 0 \]
and
\[ \cos \alpha_2 = \frac{12}{\sqrt{5^2 + 12^2}} = \frac{12}{13} \]

Now we put these values to get,
\[ H = \frac{5}{4\pi} \times 5 \left( \frac{12}{13} - 0 \right) a_\phi \]
\[ = \frac{3}{13\pi} a_\phi = 0.15 a_\phi \text{ wb/m}^2 \]

**SOL 4.1.7** Option (C) is correct.

According to Biot-savart law, magnetic field intensity at any point \( P \) due to the current element \( Idl \) is defined as
For View Only

\[ H = \int \frac{Idl \times R}{4\pi R^3} \]

where \( R \) is the vector distance of point \( P \) from the current element.

Here current is flowing in \( a_\phi \) direction

So the small current element

\[ Idl = I\rho d\phi a_\phi = 4 \times 2d\phi a_\phi = 8d\phi a_\phi \]

and since the magnetic field to be determined at center of the loop so we have

\[ R = \rho \]
\[ a_R = -a_\rho \]

(pointing towards origin)

Therefore the magnetic field intensity at origin is

\[ H = \int_0^{2\pi} \frac{8d\phi a_\phi}{16\pi} = \int_0^{2\pi} \frac{8}{16\pi} d\phi a_\phi = \frac{a_z}{4\pi[\phi]_{0}^{2\pi}} \]
\[ = 2a_z, \text{ A/m} \]

**SOL 4.1.8** Option (B) is correct.

According to right hand rule if the thumb points in the direction of outward or inward current then rest of the fingers will curl along the direction of magnetic flux lines, This condition is satisfied by the configuration shown in option (C).

**SOL 4.1.9** Option (B) is correct.

According to right hand rule if the thumb points in the direction of current then rest of the fingers will curl along the direction of magnetic field lines. This condition is satisfied by the configuration shown in option (C).

**SOL 4.1.10** Option (D) is correct.

According to Ampere’s circuital law, the line integral of magnetic field intensity \( H \) around a closed path is equal to the net current enclosed by the path.

Since we have to determine the magnetic field intensity due to the infinite line current at \( \rho = 5 \text{ cm} \) so we construct a circular loop around the line current as shown in the figure.

Now from Ampere’s circuital law we have

\[ \oint_C B \cdot dl = \mu_0 I_{enc} \]

or

\[ B(2\pi\rho) = \mu_0 \times 10 \]

\[ (I_{enc} = 10 \text{ A}) \]

Therefore we have the magnetic flux density at \( \rho = 5 \text{ cm} \) as

\[ B = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 5 \times 10^{-2}} = 5 \times 10^{-5} \text{ wb/m}^2 \]
SOL 4.1.11 Option (B) is correct.
According to Ampere’s circuital law the contour integral of magnetic field intensity in a closed path is equal to the current enclosed by the path.

\[ \oint H \cdot dl = I_{enc} \]

Now using right hand rule, we obtain the direction of the magnetic field intensity in the loop as it will be opposite to the direction of \( \mathbf{L} \).

So,

\[ \oint H \cdot dl = -I_{enc} = -20 \, \text{A} \]

(10 A is not inside the loop. So it won’t be considered.)

SOL 4.1.12 Option (C) is correct.
The magnetic field intensity produced at a distance \( r \) from an infinitely long straight wire carrying current \( I \) is defined as

\[ H = \frac{I}{2\pi r} \]

As determined by right hand rule, the direction of magnetic field intensity will be same (in \(-a_y\) direction) due to both the current source. So, at point \( P \) the net magnetic field intensity due to both the current carrying wires will be

\[ H = H_1 + H_2 = \frac{I}{2\pi} (-a_y) + \frac{I}{2\pi} (-a_y) = -\frac{5(I)}{8\pi} a_y = -\frac{4(I)}{8\pi} \]

\( I = 8 \, \text{A} \)

SOL 4.1.13 Option (B) is correct.
As calculated in previous question the magnetic field intensity produced at the centre of the current carrying circular loop is

\[ H = \frac{I}{2a} \]

So by symmetry the semicircular loop will produce the field intensity half to the field intensity produced by complete circular loop.

i.e. Field intensity at the centre of semicircular loop \( \frac{1}{2} H = \frac{I}{4a} \)

SOL 4.1.14 Option (C) is correct.
Since current in the wire is distributed over the outer surface so net enclosed current, \( I_{enc} \) for any Amperian loop inside the wire will be zero.

and as from Ampere’s circuital law we have

\[ \oint H \cdot dl = I_{enc} \]

So

\[ \oint H \cdot dl = 0 \quad (I_{enc} = 0) \]

or

\[ H = 0 \text{ for } r < R \]

SOL 4.1.15 Option (C) is correct.
Consider the cylindrical wire is lying along \( z \)-axis as shown in the figure. As the current \( I \) is distributed over the outer surface of the cylinder so for an Amperian
loop at a distance \( r(>R) \) from the centre axis, enclosed current is equal to the total current flowing in the wire.

Now from Ampere’s circuital law we have,
\[
\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}
\]
\[
B(2\pi r) = \mu_0 I
\]
\[\text{(}\mathcal{I}_{\text{enc}} = I)\]

or
\[
\mathbf{B} = \frac{\mu_0 I}{2\pi r} \hat{a}_\phi
\]

or
\[
B \propto \frac{1}{r}
\]

**SOL 4.1.16**

Option (A) is correct.

Since the current flows from \( Q_1 \) and terminates at \( Q_2 \) and the charge \( Q_2 \) is located at the surface of the contour so the actual current is not enclosed by the closed path and the circulation of the field is given as

\[
\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 [\mathcal{I}_{\text{enc}}]
\]

\[\text{(}\mathcal{I}_{\text{enc}} = 0)\]

and

\[
[I_{\text{enc}}] = \frac{d}{dt} \left[ \int \varepsilon_0 \mathbf{E}_1 \cdot d\mathbf{S} + \int \varepsilon_0 \mathbf{E}_2 \cdot d\mathbf{S} \right]
\]

where \( \mathbf{E}_1 \) is the electric field intensity produced by charge \( Q_1 \) while \( \mathbf{E}_2 \) is the field intensity produced by charge \( Q_2 \).

So,
\[
[I_{\text{enc}}] = \frac{d}{dt} \left[ \varepsilon_0 \left( \frac{Q_1}{8\varepsilon_0} \right) + \varepsilon_0 \left( \frac{Q_2}{2\varepsilon_0} \right) \right] = \frac{1}{8} \frac{dQ_1}{dt} + \frac{1}{2} \frac{dQ_2}{dt}
\]

As the current flows from \( Q_1 \) and terminates at \( Q_2 \) so the rate of change in the net charges is given as
\[
\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{11}{4}
\]

Therefore from equation (1) we have the enclosed displacement current as
\[
[I_{\text{enc}}] = \frac{1}{8} \left(-16\right) + \frac{1}{2} \left(16\right) = 6 \text{ A}
\]

Thus, the circulation of magnetic flux density around the closed loop is
\[
\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 (6)
\]
\[= 8\mu_0 \text{ Wb/m} \]
SOL 4.1.17  Option (D) is correct.

Magnetic field intensity at any point \( P \) due to an infinite current carrying sheet is defined as

\[
H = \frac{1}{2} K \times \mathbf{a}_n
\]

where \( K \) is the current density and \( \mathbf{a}_n \) is the unit vector normal to the current sheet directed toward the point \( P \).

Since we have to determine the magnetic field intensity at origin so from the figure we have

\[
\mathbf{a}_n = - \mathbf{a}_z
\]

Therefore the magnetic field intensity at the origin is

\[
H = \frac{1}{2} (20\mathbf{a}_z) \times (-\mathbf{a}_z) = 5\mathbf{a}_y \text{ A/m} \quad (K = 20\mathbf{a}_z)
\]

SOL 4.1.18  Option (C) is correct.

Magnetic field intensity at any point \( P \) due to an infinite current carrying sheet is defined as

\[
H = \frac{1}{2} K \times \mathbf{a}_n
\]
where $K$ is the current density of the infinite sheet and $a_n$ is the unit vector normal to the current sheet directed toward the point $P$.

Since we have to determine the magnetic field intensity at point $(2, -1, 5)$ which is above the plane sheet as shown in figure, so we have,

$$a_n = +a_z$$

Therefore the magnetic field intensity at the point $(2, -1, 5)$ is

$$H = \frac{1}{2}(20a_x \times a_z) = -\frac{20}{2}a_y = -5a_y \text{ A/m} \quad (K = 20a_x)$$

**SOL 4.1.19**

Option (C) is correct.

Consider one of the sheets carries the current density $K_1$. So, the other sheet will have the current density $-K_1$.

Magnetic flux density produced at any point $P$ due to a current sheet is defined as

$$B = \frac{\mu_0}{2} K \times a_n$$

where $K$ is current density of the sheet and $a_n$ is the unit vector normal to the sheet directed towards point $P$.

So for any point in the space between the sheets normal vector will be opposite in direction for the two sheets as shown in figure i.e.

$$a_n = -a_m$$

Therefore, the resultant magnetic flux density at any point in the space between the two sheets will be

$$B = \frac{\mu_0}{2}[K_1 \times a_m + (-K_1) \times (-a_m)] = K_1 \times a_m$$

Since $a_m$ is unit vector normal to the surface, and $K_1$ is given current density. So the cross product will be a constant.

**SOL 4.1.20**

Option (D) is correct.

The magnetic flux density at any point is equal to the curl of magnetic vector potential $A$ at the point.

i.e.

$$B = \nabla \times A = \nabla \times (12\cos\theta a_\phi)$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (12r\cos\theta) a_\phi = \frac{12\cos\theta}{r} a_\phi$$

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
SOL 4.1.21
Option (C) is correct.
Consider the current sheet shown in the figure.

Magnetic field intensity produced at any point \( P \) due to a current sheet is defined as

\[
\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_n
\]

where \( \mathbf{K} \) is current density of the sheet and \( \mathbf{a}_n \) is the unit vector normal to the sheet directed towards point \( P \).

So, for \( y > 0 \)

\[
\mathbf{H} = \frac{1}{2}(\mathbf{K}\mathbf{a}_z) \times (\mathbf{a}_y) = -\frac{1}{2} \mathbf{K}\mathbf{a}_y \quad (K = Ka, \ A/m, \ a_n = a_y)
\]

and for \( y < 0 \)

\[
\mathbf{H} = \frac{1}{2}(\mathbf{K}\mathbf{a}_z) \times (-\mathbf{a}_y) = \frac{K}{2} \mathbf{a}_x \quad (K = Ka, \ A/m, \ a_n = -a_y)
\]

SOL 4.1.22
Option (D) is correct.
Consider the current density at \( \rho = 8 \text{ cm} \) is \( \mathbf{J} \) directed along \( +\mathbf{a}_z \).
Now the magnetic field for \( \rho > 8 \text{ cm} \) must be zero.

i.e.

\[
\mathbf{H} = 0 \quad \text{ (for } \rho > 8 \text{ cm)}
\]

So from Ampere’s circuital law we have

\[
\int \mathbf{H} \cdot d\mathbf{l} = I_{enc} = 0
\]

Since for the region \( \rho > 8 \text{ cm} \) the Amperian loop will have all the current distributions enclosed inside it.

i.e.

\[
I_{enc} = 14 \times 10^{-2} + 2 \times (2\pi \times 0.5 \times 10^{-2}) - 0.8 \times (2\pi \times 0.25 \times 10^{-2}) + J(2\pi \times 8 \times 10^{-2})
\]

\[
= 6.43 \times 10^{-2} + J(16\pi \times 10^{-2})
\]

So we have

\[
6.43 \times 10^{-2} + J(16\pi \times 10^{-2}) = 0 \quad (I_{enc} = 0)
\]

or

\[
J = \frac{6.43 \times 10^{-2}}{16\pi \times 10^{-2}}
\]

or

\[
J = -0.23a, \ A/m
\]

SOL 4.1.23
Option (C) is correct.
Magnetic flux density is defined as the curl of vector magnetic potential 
\[ B = \nabla \times A \]
\[
= \begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial a_y}{\partial z} & \frac{\partial a_z}{\partial y} & \frac{\partial a_x}{\partial z} \\ 2x^2y & 2y^2 & -8xyz \end{bmatrix} \\
= (-8xz - 0) a_x + (0 + 8yz) a_y + (2y^2 - 2x^2) a_z
\]
So the net magnetic flux density at \((1, -2, 5)\) is 
\[ B = 20a_x + 60a_y + 12a_z \text{ wb/m}^2 \]

**SOL 4.1.24** Option (B) is correct.

Total magnetic flux through a given surface \(S\) is defined as 
\[
\Phi = \int_S B \cdot dS 
\]
where \(dS\) is the differential surface vector having direction normal to the surface 
So, for the given surface \(z = 4, 0 \leq x \leq 1, -1 \leq y \leq 4\) we have 
\[ dS = (dx\,dy) a_z \]
and as calculated in previous question we have 
\[ B = (-8xz - 0) a_x + (0 + 8yz) a_y + (2y^2 - 2x^2) a_z \]
Therefore, the total magnetic flux through the given surface is 
\[ \Phi = \int_{x=0}^{1} \int_{y=-1}^{1} (2y^2 - 2x^2) (dx\,dy) = 2 \times 1 \int_{-1}^{1} y^2 dy - 2 \times 5 \int_{0}^{1} x^2 dx \]
\[ = 2 \left[ \frac{y^3}{3} \right]_{-1}^{1} - 10 \left[ \frac{x^3}{3} \right]_{0}^{1} = 2 \times \frac{65}{3} - \frac{10}{3} = 40 \text{ wb} \]

**SOL 4.1.25** Option (D) is correct.

For determining the magnetic field at any point above the plane \(z = 0\), we draw a rectangular Amperian loop parallel to the \(y-z\) plane and extending an equal distance above and below the surface as shown in the figure.
From Ampere’s circuital law,
\[
\int B \cdot dl = \mu_0 I_{enc}
\]
Since the infinite current sheet is located in the plane \( z = 0 \) so, the \( z \)-component of the magnetic flux density will be cancelled due to symmetry and in the closed Amperian loop the integral will be only along \( y \)-axis. Thus we have

\[
B(2l) = \mu_0 I_{enc}
\]

\[
2Bl = \mu_0 Kl
\]

(\( I_{enc} = Kl \))

As determined by right hand rule, the magnetic flux density above the plane \( z = 0 \) will be in \(-a_y\) direction. So we have the flux density above the current sheet as

\[
B = -\frac{\mu_0}{2} \times \frac{4}{a_y} = -2\mu_0 a_y \text{ wb/m}^2 \quad (K = 4 \text{ A/m})
\]

Alternate Method :

The magnetic flux density produced at any point \( P \) due to an infinite sheet carrying uniform current density \( K \) is defined as

\[
B = \frac{1}{2} \mu_0 (K \times a_n)
\]

where \( a_n \) is the unit vector normal to the sheet directed toward the point \( P \). So, magnetic flux density at any point above the current sheet \( K = 4a_z \) is

\[
B = \frac{1}{2} \mu_0 (4a_y) \times (a_z) = -4\mu_0 a_y \text{ wb/m}^2 \quad (a_n = a_z)
\]

SOL 4.1.26 Option (A) is correct.

Magnetic flux density at a certain point is equal to the curl of magnetic vector potential at the point.

i.e.

\[
B = \nabla \times A
\]

So from the above determined value of magnetic flux density \( B \) we have,

\[
\nabla \times A = -2\mu_0 a_y \text{ wb/m}^2 \quad (1)
\]

Since \( A \) is parallel to \( K \) so the vector potential \( K \) will depend only on \( z \). Hence, we have

\[
A = A(z) a_z
\]

From equation (1) we have,

\[
-2\mu_0 a_y = \begin{vmatrix}
    a_x & a_y & a_z \\
    \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
    A(z) & 0 & 0
\end{vmatrix}
\]

or

\[
A(z) = 2\mu_0 z
\]

So,

\[
A = 2\mu_0 z a_y
\]

Therefore the vector magnetic potential at \( z = -2 \) is

\[
A = -4\mu_0 a_y \text{ wb/m}
\]

SOL 4.1.27 Option (D) is correct.

Current density at any point in a magnetic field is defined as the curl of magnetic field intensity at the point.

i.e.

\[
J = \nabla \times H
\]

Since the magnetic field intensity in the free space is given as
Therefore the current density is
\[ J = \frac{1}{\rho} \frac{\partial}{\partial \rho} (2\rho^3) \mathbf{a}_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} (2\rho) \mathbf{a}_z = 6\rho \mathbf{a}_z = 2a_z \text{ A/m}^2 \]  

\[ \rho = 2 \text{ m} \]

**SOL 4.1.28**  
Option (A) is correct.

The magnetic flux density at any point is equal to the curl of the vector magnetic potential at the point
i.e.,
\[ \mathbf{B} = \nabla \times \mathbf{A} \]
\[ = \frac{1}{\mu_0} \frac{\partial}{\partial \rho} (\rho \mathbf{A}_\phi) \mathbf{a}_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} (2\rho) \mathbf{a}_z = \frac{2}{\rho} \mathbf{a}_z \]

The current density \( J \) in terms of magnetic flux density \( \mathbf{B} \) is defined as
\[ J = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) = \frac{1}{\mu_0} \left[ \frac{\partial}{\partial \rho} \left( \frac{2}{\rho} \right) \right] \mathbf{a}_z = \frac{2}{\mu_0 \rho^2} \mathbf{a}_\phi \]

This current density would produce the required vector potential.

**SOL 4.1.29**  
Option (C) is correct.

The current density for a given magnetic field intensity \( \mathbf{H} \) is defined as
\[ \mathbf{J} = \nabla \times \mathbf{H} \]

Given \( \mathbf{H} = (z \cos ay) \mathbf{a}_y + (z + e^y) \mathbf{a}_z \)

So \( \nabla \times \mathbf{H} = \left[ \begin{array}{ccc} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \end{array} \right] \left[ \begin{array}{c} z \cos ay \\ z \cos ay \\ 0 \\ \end{array} \right] = \left[ \begin{array}{c} -\frac{\partial}{\partial z} (z \cos ay) \mathbf{a}_x - \left( \frac{\partial}{\partial x} z \cos ay - \frac{\partial}{\partial y} (z + e^y) \right) \mathbf{a}_y + \left( \frac{\partial}{\partial x} z \cos ay - \frac{\partial}{\partial y} (z + e^y) \right) \mathbf{a}_z \\ \end{array} \right] \]

or, \[ \mathbf{J} = \nabla \times \mathbf{H} = -\cos ay \mathbf{a}_x + \mathbf{a}_y - e^y \mathbf{a}_z \]

Therefore the current density in the \( x-z \) plane is
\[ \mathbf{J} = -\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z \text{ A/m}^2 \] \( (y = 0 \text{ in } x-z \text{ plane}) \)

**SOL 4.1.30**  
Option (D) is correct.

In a source free region current density, \( \mathbf{J} = 0 \)

The current density at any point is equal to the curl of magnetic field intensity \( \mathbf{H} \).
i.e., \[ \mathbf{J} = \nabla \times \mathbf{H} \]

or \[ \nabla \times \mathbf{H} = 0 \] \( (\mathbf{J} = 0) \)

and since the curl of a given vector field is zero so it can be expressed as the gradient of a scalar field
i.e., \[ \mathbf{H} = \nabla \phi \]

So A and R both are true and R is correct explanation of A.

**SOL 4.1.31**  
Option (B) is correct.

Given that the cylindrical wire located along \( z \)-axis produces a magnetic field intensity, \( \mathbf{H} = 3\rho \mathbf{a}_y \).
So, applying the differential form of Ampere’s circuital law we have the current density with in the conductor as

\[ \mathbf{J} = \nabla \times \mathbf{H} = \frac{1}{\rho} \begin{vmatrix} a_\phi & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho} \begin{vmatrix} a_\phi & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 3 \rho^2 & 0 \end{vmatrix} \\
= \frac{1}{\rho} \frac{\partial}{\partial \rho} (3 \rho^2) a_z = 6 a_z \text{ A/m}^2 \]

SOL 4.1.32
Option (D) is correct.

As the beam is travelling in \( a_z \) direction so the field intensity produced by it will be in \( a_\phi \) direction and using Ampere’s circuital law at it’s surface we have

\[
H_\phi(2\pi a) = I_{ac}
\]

\[
H_\phi(2\pi a) = \int_0^a 2 \left(1 - \frac{\rho}{a}\right) 2 \pi \rho d\rho
\]

\[
H_\phi(2\pi a) = 4 \pi \left[ \frac{\rho^2}{2} - \frac{\rho^3}{3a} \right]_0^a
\]

\[
H_\phi(2\pi a) = 2 \pi a^2 \frac{2}{3}
\]

or,
\[
\mathbf{H} = \frac{a}{4} a_\phi
\]

SOL 4.1.33
Option (A) is correct.

Since the magnetic flux density is defined as

\[
\mathbf{B} = \nabla \times \mathbf{A}
\]

and

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}
\]

Now using the vector identity, we have

\[
\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}
\]

or,

\[
\nabla \times \mathbf{B} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}
\]

or,

\[
\mu_0 \mathbf{J} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}
\]

As the vector potential is always divergence free so we get,

\[
\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}
\]

SOL 4.1.34
Option (B) is correct.

Magnetic dipole moment of a conducting loop carrying current \( I \) is defined as :

\[
m = I \mathbf{S}
\]

where \( S \) is the area enclosed by the conducting loop. So we have

\[
m = 7 \times (\pi \times 0.5^2) = 5.5 \quad (I = 7 \text{ A}, R = 0.5 \text{ m})
\]

The direction of the moment is determined by right hand rule as when the curl of fingers lies along the direction of current, then the thumb indicates the direction of moment.

So,

\[
m = 5.5 a_z \text{ A-m}^2
\]

***********
SOLUTIONS 4.2

SOL 4.2.1

Option (D) is correct.

Since the current is flowing in the $-a_z$ direction

So,

$$Idl = \int_{-a_z} dz$$

Magnetic field intensity at any point $P$ due to a filamentary current $I$ is defined as

$$H = \frac{I}{4\pi \rho} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_z$$

where $ho$ → distance of point $P$ from the current filament.

$\alpha_1$ → angle subtended by the lower end of the element at $P$.

$\alpha_2$ → angle subtended by the upper end of the element at $P$.

Now from the figure we have,

$\rho = 2$

$\alpha_2 = \pi - \theta$

or

$\cos \alpha_2 = \cos (\pi - \theta)$

$= -\cos \theta = -\frac{3}{\sqrt{2^2 + 3^2}} = -\frac{3}{\sqrt{13}}$

and

$\alpha_1 = 0$ (angle subtended by end $z = \infty$)

or

$\cos \alpha_1 = \cos 0 = 1$

So,

$$H = \frac{I}{4\pi \rho} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_z$$

$$= \frac{I}{4\pi} \times 2 \left[ 1 - \left( -\frac{3}{\sqrt{13}} \right) \right] a_z = \frac{10}{8\pi} \times \left( 2 + \frac{3}{\sqrt{13}} \right) a_z$$

Now the direction of magnetic field intensity is defined as
where \( \mathbf{a}_i \) is unit vector along the line current and \( \mathbf{a}_\rho \) is the unit vector normal to the line current directed toward the point \( P \).

So we have \( \mathbf{a}_\rho = ( - \mathbf{a}_z ) \times ( \mathbf{a}_\rho ) = \mathbf{a}_z \)

Therefore, \( \mathbf{H} = \frac{10}{8\pi} \left( 1 + \frac{3}{\sqrt{13}} \right) ( \mathbf{a}_z ) \)
\[ = 1.73 \mathbf{a}_z \text{ A/m} \]

**SOL 4.2.2**

Option (D) is correct.

According to Biot-savart law, magnetic field intensity at any point \( P \) due to the current element \( Idl \) is defined as

\[ \mathbf{H} = \int \frac{Idl \times \mathbf{R}}{4\pi R^3} \]

where \( \mathbf{R} \) is the vector distance of point \( P \) from the current element.

Now the current element carries a current of 4.4 A in \( +\mathbf{a}_z \) direction.

So we have, \( \mathbf{R} = (4\mathbf{a}_r + 2\mathbf{a}_y + 3\mathbf{a}_z) - (x\mathbf{a}_z) \)

(Since on \( x \)-axis \( y \)- and \( z \)-component will be zero)

\[ \mathbf{R} = (4 - x) \mathbf{a}_r + 2\mathbf{a}_y + 3\mathbf{a}_z \]

or

\[ R = \sqrt{(4 - x)^2 + 2^2 + 3^2} \]
\[ = \sqrt{x^2 - 8x + 29} \]

and

\[ Idl = 4.4dx\mathbf{a}_z \]

(filament lies from \( x = -\infty \) to \( x = \infty \))

Therefore the magnetic field intensity is

\[ \mathbf{H} = \int_{-\infty}^{\infty} \frac{(4.4\mathbf{a}_z) \times [(4 - x) \mathbf{a}_r + 2\mathbf{a}_y + 3\mathbf{a}_z]}{4\pi (x^2 - 8x + 29)^{3/2}} \ dx \]
\[ = \frac{4.4}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 - 8x + 29)^{3/2}} \]
\[ = \frac{4.4}{20\pi} (2\mathbf{a}_z - 3\mathbf{a}_y) \]
\[ = 0.1077 \mathbf{a}_z - 0.162 \mathbf{a}_y = 0.1 \mathbf{a}_z - 0.2 \mathbf{a}_y \text{ A/m} \]
Alternate Method:

According to Ampere’s circuital law, the line integral of magnetic field intensity $H$ around a closed path is equal to the net current enclosed by the path. Since we have to determine the magnetic field intensity at point (4, 2, 3) so we construct a circular loop around the infinite current element that passes through the point (4, 2, 3) as shown in the figure.

Now from Ampere’s circuital law we have,

$$
\int H \cdot dl = I_{enc}
$$

or

$$
(2\pi r) H = 4.4 \quad (I_{enc} = 4.4 \text{ A})
$$

Now direction of the magnetic field intensity is defined as

$$
\mathbf{a}_\phi = \mathbf{a}_l \times \mathbf{a}_v
$$

where $\mathbf{a}_l$ unit vector in the direction of flow of current

$\mathbf{a}_v$ unit vector normal to the line current directed toward the point.

So we have,

$$
\mathbf{a}_\phi = \mathbf{a}_x \times \frac{[(4\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z) - (4\mathbf{a}_x)]}{\sqrt{(4 - 4)^2 + 2^2 + 3^2}} = \frac{2\mathbf{a}_x - 3\mathbf{a}_y}{\sqrt{13}}
$$

Therefore the magnetic field intensity at the point (4, 2, 3) is

$$
H = \frac{4.4}{2\pi \sqrt{13}} \frac{(2\mathbf{a}_x - 3\mathbf{a}_y)}{\sqrt{13}} = \frac{4.4}{26\pi} (2\mathbf{a}_x - 3\mathbf{a}_y)
$$

$$
= 1.5\mathbf{a}_x - 2.5\mathbf{a}_y \text{ A/m}
$$

SOL 4.2.3 Option (D) is correct.

As the magnetic field intensity at the center of the triangle produced by all the
three sides will be exactly equal so we consider only one side lying along $x$-axis that carries $4 \ A$ current flowing in $+a_x$ direction as shown in the figure.

Now the magnetic field intensity at any point $P$ due to a filamentary current $I$ is defined as

$$H = \frac{I}{4\pi \rho} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_\phi$$

where $\rho \rightarrow$ distance of point $P$ from the current filament.

$\alpha_1 \rightarrow$ angle subtended by the lower end of the element at $P$.

$\alpha_2 \rightarrow$ angle subtended by the upper end of the element at $P$.

From the figure we have

$$\tan 30^\circ = \frac{\rho}{1} \Rightarrow \rho = \frac{1}{\sqrt{3}}$$

$$\alpha_1 = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \Rightarrow \cos \alpha_1 = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

and

$$\alpha_2 = 30^\circ \Rightarrow \cos \alpha_2 = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

So the magnetic field intensity produced by one side of the triangle at centre of the triangle is

$$H_1 = \frac{4}{4\pi} \frac{1}{\sqrt{3}} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_\phi$$

$$= \frac{\sqrt{3}}{\pi} \left[ \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right] a_\phi = \frac{5}{\pi} a_\phi$$

Now the direction of magnetic field intensity is determined as

$$a_\phi = a_i \times a_\phi$$

where $a_i$ is unit vector along the line current and $a_\phi$ is the unit vector normal to the line current directed toward the point $P$.

and since the line current is along $x$-axis so we have

$$a_\phi = a_x \times a_y = a_z$$  \hspace{1cm} (a_i = a_x, \ a_p = a_y)

Therefore the net magnetic field intensity due to all the three sides of triangle is

$$H = 3H_1 = 3 \times \left( \frac{3}{\pi} \right) a_i = \frac{9}{\pi} a_i \ A/m$$  \hspace{1cm} (a_i = a_x)
Option (C) is correct.

According to Biot-savart law, magnetic field intensity at any point $P$ due to the current sheet element $KdS$ is defined as

$$ H = \frac{\int KdS \times a_R}{4\pi R^2} $$

where $R$ is the vector distance of point $P$ from the current element.

Now we consider a point $(0, y, z)$ on the current carrying sheet, from which we have the vector distance of point $(3, 0, 0)$

$$ R = (3a_x + 0a_y + 0a_z) - (0a_x + ya_y + za_z) = (3a_x - ya_y - za_z) $$

or

$$ a_R = \frac{3a_x - ya_y - za_z}{\sqrt{9 + y^2 + z^2}} = \frac{3a_x - ya_y - za_z}{\sqrt{9 + y^2 + z^2}} $$

Therefore the magnetic field intensity due to the current sheet is

$$ H = \int_{-2}^{2} \int_{-\infty}^{\infty} \frac{4a_x \times (3a_x - ya_x - za_z)}{4\pi (9 + y^2 + z^2)^{3/2}} dy dz \quad (K = 4a_x) $$

We note that the $x$ component is anti symmetric in $z$ about the origin (odd parity).

Since the limits are symmetric, the integral of the $x$ component over $z$ is zero. So we are left with

$$ H = \int_{-2}^{2} \int_{-\infty}^{\infty} \frac{-12a_x}{4\pi (9 + y^2 + z^2)^{3/2}} dy dz $$

$$ = -\frac{3}{\pi} a_x \left[ \frac{y}{\sqrt{(z^2 + 9)(9 + y^2 + z^2)}} \right]_{-\infty}^{\infty} dz $$

$$ = -\frac{3}{\pi} a_x \left[ \frac{z}{2z^2 + 9} \right]_{-2}^{2} $$

$$ = -\frac{6}{\pi} a_x \left[ \frac{1}{3} \tan^{-1}\left(\frac{z}{3}\right) \right]_{-2}^{2} $$

$$ = -\frac{2}{\pi} \times (2) \times (0.59) a_x = -0.14 \text{ A/m} $$

Option (D) is correct.

Since the uniformly charged disk is rotating with an angular velocity $\omega = 2 \text{ rad/s}$
about the z-axis so we have the current density

\[ K = \rho_0 \times (\text{angular velocity}) = \rho_0 (\omega \rho) = 2 \times 2 \times \rho \]

or

\[ K = 4\rho \alpha_0 \]

According to Biot-savart law, magnetic field intensity at any point \( P \) due to the current sheet element \( KdS \) is defined as

\[ H = \frac{\int_{\phi} KdS \times a_R}{4\pi R^2} \]

where \( R \) is the vector distance of point \( P \) from the current element.

Now from the figure we have

\[ R = a_z - \rho \alpha_0 \]

or

\[ R = \sqrt{1 + \rho^2} \]

and

\[ a_R = \frac{a_z - \rho \alpha_0}{\sqrt{1 + \rho^2}} \]

So the magnetic field intensity due to a small current element \( KdS \) at point \( P \) is

\[ dH = \frac{KdS \times a_R}{4\pi R^2} = \frac{(4\rho \alpha_0) \times (a_z - \rho \alpha_0)}{4\pi (\rho^2 + 1)^{3/2}} = \frac{4\rho (a_z + \rho \alpha_0)}{4\pi (\rho^2 + 1)^{3/2}} \]

On integrating the above over \( \phi \) around the complete circle, the \( a_\phi \) components get cancelled by symmetry, leaving us with

\[ H(z) = \int_0^{2\pi} \int_0^{\sqrt{3}} \frac{4\rho^2 a_z}{4\pi (\rho^2 + 1)^{3/2}} (\rho d\rho d\phi) \]

\[ = 2 \int_0^{\sqrt{3}} \frac{\rho^4}{(\rho^2 + 1)^{3/2}} d\rho a_z = 2 \left[ \sqrt{\rho^2 + 1} + \frac{1}{\sqrt{\rho^2 + 1}} \right] a_z \]

\[ = 2 \left[ 3 + 2 (1 - \sqrt{1 + 3}) \right] a_z = 5a_z \text{ A/m} \]

SOL 4.2.6 Option (B) is correct.

---

As all the four sides of current carrying square loop produces the same magnetic field at the center so we consider only the line current \( AB \) for which we determine the magnetic field intensity at the center.

Now the magnetic field intensity at any point \( P \) due to a filamentary current \( I \) is defined as

\[ H = \frac{I}{4\pi \rho} [\cos \alpha_2 - \cos \alpha_1] a_\phi \]

where \( \rho \rightarrow \) distance of point \( P \) from the current filament.
\[ \alpha_1 \to \text{angle subtended by the lower end of the filament at } P. \]
\[ \alpha_2 \to \text{angle subtended by the upper end of the filament at } P. \]

From the figure, we have
\[ \rho = \frac{1}{2} \text{ m, } \alpha_2 = 45^\circ \text{ and } \alpha_1 = 180^\circ - 45^\circ \]

So the magnetic field intensity at the centre \( O \) due to the line current \( AB \) is
\[ H_1 = \frac{I}{2\pi R} \left[ \cos \alpha_2 - \cos \alpha_1 \right] \]
\[ = \frac{1}{2\pi} \times (1/2) \left[ \cos 45^\circ - \cos(180^\circ - 45^\circ) \right] \]
\[ = \frac{1}{\pi} \times \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{\pi} \text{ A/m} \]

and the magnetic flux density produced by the line current \( AB \) is
\[ B_1 = \mu_0 H_1 = 4\pi \times 10^{-7} \times \frac{\sqrt{2}}{\pi} \]
\[ = 5.66 \times 10^{-7} \text{ wb/m}^2 \]

Therefore the net magnetic flux density due to the complete square loop will be
four times of \( B_1 \)
i.e.
\[ B = 4B_1 = 4 \times (5.66 \times 10^{-7}) \]
\[ = 3.26 \times 10^{-6} \text{ wb/m}^2 \]

**SOL 4.2.7**

Option (D) is correct.

According to Biot-savart law, magnetic field intensity at any point \( P \) due to the current element \( Idl \) is defined as
\[ H = \frac{\int Idl \times a_R}{4\pi R^2} \]

where \( R \) is the vector distance of point \( P \) from the current element.

As the cross product of two parallel lines is always zero so the straight segments will produce no field at \( P \). Therefore the net magnetic field produced at point \( P \) will be only due to the two circular section.
i.e.
\[ H = H_{CD} + H_{AB} \]

or
\[ H = \left[ \int_0^{\pi/2} \frac{I d\phi a_\phi}{4\pi \rho^2} \right]_{\text{at } \rho = 1 \text{ m}} + \left[ \int_0^{\pi/2} \frac{I d\phi (-a_\phi) \times (-a_\theta)}{4\pi \rho^2} \right]_{\text{at } \rho = 2 \text{ m}} \]
\[ = \frac{Ia_\phi}{4\pi} \left( \frac{1}{1} \right) - \frac{Ia_\phi}{4\pi} \left( \frac{2}{2} \right) \]
\[ = \frac{3.2}{4\pi} \times \left[ 1 - \frac{1}{2} \right] \times (\pi) a_\phi = 0.2 \text{ A/m} \]

**Alternate Method:**
The magnetic field intensity produced at the center of a circular loop of radius \( R \) carrying current \( I \) is defined as
\[ H = \frac{I}{2R} \]
and since the straight line will not produce any field at point $P$ so due to the two quarter circles having current in opposite direction, magnetic field at the center will be

$$H = \frac{1}{4} \left[ \frac{I}{2a} - \frac{I}{2b} \right]$$

where $a \rightarrow$ inner radius

$b \rightarrow$ outer radius

$$H = \frac{1}{4} \left[ \frac{3(2)}{2 \times 1} - \frac{3(2)}{2 \times 2} \right] = 0.2 \text{ A/m}$$

SOL 4.2.8 Option (D) is correct.

The magnetic field intensity at any point $P$ due to an infinite filamentary current $I$ is defined as

$$H = \frac{I}{2\pi \rho}$$

where $\rho$ is the distance of point $P$ from the infinite current filament.

Now the two semi infinite lines will be in combination treated as a single infinite line for which magnetic field intensity at point $P$ will be

$$H_1 = \frac{I}{2\pi R} \quad (R \text{ is the length of point } P \text{ from line current})$$

$$H_1 = \frac{4}{2\pi \times 2} = \frac{1}{\pi} \quad (I = 4 \text{ A, } R = 2 \text{ m})$$

As the magnetic field intensity produced at the center of a circular loop of radius $R$ carrying current $I$ is defined as

$$H = \frac{I}{2R}$$

So magnetic field produced at point $P$ due to the semi circular segment is

$$H_2 = \frac{1}{2} \times \frac{I}{2R} = \frac{1}{2}$$

Therefore net magnetic field intensity produced at point $P$ is

$$H = H_1 + H_2 = \frac{1}{\pi} + \frac{1}{2}$$

$$= 0.82 \text{ A/m}$$

SOL 4.2.9 Option (C) is correct.
Magnetic field intensity produced at any point \( P \) on the axis of the circular loop carrying current \( I \) is defined as

\[
H = \frac{I\rho^2}{2(\rho^2 + h^2)^{3/2}}
\]

where \( h \) is the distance of point \( P \) from the centre of circular loop and \( \rho \) is the radius of the circular loop.

From the figure we have

\[
\rho = 3\, \text{m} \quad \text{and} \quad h = 5 - 1 = 4\, \text{m}
\]

and using right hand rule we conclude that the magnetic field intensity is directed along \( +a_z \). So the magnetic field intensity produced at point \( P \) is

\[
H = \frac{50 \times 10^{-3} (3)^2}{2(3^2 + 4^2)^{3/2}} a_z = \frac{9 \times 50 \times 10^{-3}}{2 \times 125} a_z = 2.8 a_z \, \text{mA/m}
\]

SOL 4.2.10 Option (D) is correct.

Let the cylindrical tube is of radius \( a \) for which we have to determine the magnetic field intensity at the axis of solenoid.

Now we consider a small ring (small section of solenoid) of the width \( dz \) at a distance \( z \) from point \( P \) lying on the axis of the solenoid as shown in the figure.

The total current flowing in the loop of the ring will be

\[
dI = nIdz
\]

where \( n \) is the no of turns per unit length.
Since magnetic field intensity produced at any point \( P \) on the axis of the circular loop carrying current \( I \) is defined as

\[
H = \frac{I\rho^2}{2(\rho^2 + h^2)^{3/2}}
\]

where \( h \) is the distance of point \( P \) from the centre of circular loop and \( \rho \) is the radius of the circular loop.

So we have the magnetic field intensity due to the ring as

\[
dH = \frac{(nIdz) a^2}{2(a^2 + z^2)^{3/2}}
\]

\((\rho = a, h = z)\)

From the figure we have

\[
z = a \cot \theta \Rightarrow dz = -\frac{a}{\sin^2 \theta} d\theta
\]

and

\[
\sin \theta = \frac{a}{\sqrt{a^2 + z^2}} \Rightarrow \frac{1}{(a^2 + z^2)^{3/2}} = \frac{\sin^3 \theta}{a^3}
\]

The total magnetic field intensity produced at point \( P \) due to the solenoid is

\[
H = \int_{z=-\infty}^{z=\infty} \frac{(nIdz) a^2}{2(a^2 + z^2)^{3/2}} = \frac{nI}{2} \int_{0}^{\pi} a^2 \sin^3 \theta (-d\theta) = \frac{nI}{2} (\cos 0 - \cos \pi) = nI = 1000 \times 6 \times 10^{-3} = 6 \text{ A/m} \quad (n = 1000, I = 4 \text{ mA})
\]

**SOL 4.2.11** Option (D) is correct.

As calculated in the previous question the magnetic field intensity inside a long solenoid carrying current \( I \) is defined as

\[
H = nI
\]

where \( n \) is no. of turns per unit length and since using right hand rule we conclude that the direction of magnetic field intensity will be right wards \((+a_y)\) due to outer solenoid and left wards \((-a_y)\) due to inner solenoid. So the resultant magnetic field intensity produced inside the inner solenoid will be

\[
H = H_1 + H_2 = n_1 I(-a_y) + n_2 Ia_y
\]

where \( n_1 \) and \( n_2 \) are the no. of turns per unit length of the inner and outer solenoids respectively.

So

\[
H = - (3 \times 10^{-3}) (2000) a_y + (3 \times 10^{-3}) (1000) a_y = 3 \times 10^{-3} (-1000) a_y = -3a_y \text{ A/m}
\]

**SOL 4.2.12** Option (D) is correct.

Since no any magnetic field is produced at any point out side a solenoid so in the region between the two solenoids field will be produced only due to the outer solenoid.

\[
H = n_2 Ia_y = 100 \times 4 \times 10^{-4} a_y = 4a_y \text{ A/m}
\]

**SOL 4.2.13** Option (B) is correct.

Since no any magnetic field is produced at any point out side a solenoid so, at any point outside the outer solenoid, the net magnetic field intensity produced due to
the two solenoids will be zero.

SOL 4.2.14  
Option (B) is correct.
The magnetic field intensity produced at any point $P$ due to an infinite sheet carrying uniform current density $K$ is defined as

$$H = \frac{1}{2}(K \times a_n)$$

where $a_n$ is the unit vector normal to the sheet directed toward the point $P$.

So the field intensity produced between the two sheets due to the sheet $K_1 = 3a_z$ located at $x = 2\text{ m}$ is

$$H_1 = \frac{1}{2}(3a_z) \times (-a_z) = -\frac{3}{2}a_y \text{ A/m} \quad (a_n = -a_x)$$

and the field intensity produced between the two sheets due to the sheet $K_2 = -3a_z$ located at $x = -2\text{ m}$ is

$$H_2 = \frac{1}{2}(-3a_z) \times (a_z) = -\frac{3}{2}a_y \text{ A/m} \quad (a_n = a_z)$$

Therefore the net magnetic field intensity produced at any point between the two sheets is

$$H = H_1 + H_2 = -3a_y$$

Since the magnetic field intensity at any point is the equal to the negative gradient of scalar potential at the point

$$H = -\nabla V_m$$

i.e.

So for the field $H = -3a_y$ in the region between the two current carrying sheets, we have

$$-3a_y = -\frac{dV_m}{dy}a_y \quad (\text{the field has a single component in } a_y \text{ direction})$$

or

$$V_m = 3y + C_1 \quad \text{where } C_1 \text{ is constant}$$

Putting $V_m = 0$ for point $P(1,2,5)$ (given), we have

$$0 = 3(2) + C_1$$

or

$$C_1 = -6$$

Thus,

$$V_m = (3y - 6) \text{ A}$$

and the graph of $V_m$ versus $y$ will be as plotted below.

![Graph](attachment:image.png)
Option (C) is correct.

The magnetic flux density at any point is equal to the curl of the vector magnetic potential at the point

\[ B = \nabla \times A \]  \hspace{1cm} (1)

Since \[ B = \mu_0 H = -3\mu_0 a_y \text{ Wb/m}^2 \] (calculated in previous question)

As the magnetic flux density is in \( a_y \) direction so \( A \) is expected to be \( z \)-directed.

Therefore from eq (1) we have

\[ -\frac{\partial A_z}{\partial x} = -3\mu_0 \]

or

\[ A_z = 3\mu_0 x + C_2 \]

Putting \( A_z = 0 \) at point \( P(1,2,5) \) (given), we have

\[ 0 = 3\mu_0 + C_2 \]

or

\[ C_2 = -3\mu_0 \]

So,

\[ A_z = 3\mu_0(x - 1) = -3\mu_0 \]

at origin \( (0,0,0) \)

Thus, the magnetic vector potential at origin is

\[ A = -8\mu_0 a_y \text{ Wb/m} \]

Option (B) is correct.

Since the current density inside the wire is given by

\[ J \propto \rho \]

So we have,

\[ J = k\rho \]

where \( k \) is a constant.

and the total current flowing in the wire is given by

\[ I_0 = \int J \cdot dS \]

or

\[ 5 \times 10^{-3} = \int_0^{2\pi} k\rho (2\pi \rho) d\rho \]

\[ 5 \times 10^{-3} = \frac{2\pi k(2 \times 10^{-2})^3}{3} \]

So we have

\[ k = \frac{3 \times 5 \times 10^{-3}}{2\pi \times 8 \times 10^{-9}} = \frac{15}{16\pi} \times 10^{3} \]

Now for the Amperian loop at \( \rho = 1 \text{ cm} \) enclosed current is

\[ I_{enc} = \int J \cdot dS = \int_{\rho=0}^{1} k\rho (2\pi \rho) d\rho = \left( \frac{15}{16\pi} \times 10^{3} \right) \times 2\pi \left[ \frac{\rho^3}{3} \right]_{0}^{1} \]

\[ I_{enc} = \frac{15}{8} \times \frac{1}{3} \times 10^{-3} = \frac{15}{24} \times 10^{-3} \]

So from Ampere’s circuital law we have

\[ \oint B \cdot dl = \mu_0 I_{enc} \]

\[ B(2\pi \rho) = \frac{\mu_0 15}{24} \times 10^{-3} \]

Therefore the magnetic flux density at \( \rho = 1 \text{ cm} \) is

\[ B = \frac{15}{24} \times \frac{10^{-3}}{2\pi (1 \times 10^{-2})} \times 4\pi \times 10^{-7} \]

\[ = 1.25 \times 10^{-8} = 12.5 \text{ nWb/m}^2 \]
Current density at any point in a magnetic field is defined as the curl of magnetic field intensity at the point.

\[ \mathbf{J} = \nabla \times \mathbf{H} \]

So the current density component in \( \mathbf{a}_x \) direction is

\[ J_x = (\nabla \times \mathbf{H})_x = \frac{\partial H_y}{\partial y} - \frac{\partial H_z}{\partial z} = -\left(\frac{x^2}{(y+1)^2}\right) a_x \]

Therefore the total current passing through the surface \( x = 2 \text{ m}, \, 1 \leq y \leq 4 \text{ m}, \, 3 \leq z \leq 4 \text{ m} \) is

\[
I = \int_S \mathbf{J}_x \cdot d\mathbf{S} = -\int_3^4 \int_{y=1}^4 \left(\frac{x^2}{(y+1)^2} + 6x^2 y\right) dy dz (d\mathbf{S} = dydz \mathbf{a}_x)
\]

\[
= -\int_3^4 \int_{y=1}^4 \left(2\frac{z^2}{(y+1)^2} + 24y\right) dy dz (x = 2 \text{ m})
\]

\[
= -\int_3^4 \left[\frac{z^2}{y+1} + 24zy\right]_1^4 dz = -\int_3^4 \left(\frac{2z^2}{y+1} + 72y\right) dz
\]

\[
= -145 \text{ A}
\]

Magnetic dipole moment of a conducting loop carrying current \( I \) is defined as:

\[ m = I S \]

where \( S \) is the area enclosed by the conducting loop. So for a ring of radius \( r \), magnetic dipole moment

\[ m = I(\pi r^2) \]

Now as the charged disk(charge density, \( \rho_s = 20 \text{ C/m}^2 \)) is rotating with angular velocity \( \omega = 0.1 \text{ rad/s} \) so, the current in the loop is given as

\[ dI = \rho_s \omega rdr \]

Therefore the magnetic dipole moment is

\[
m = \int dI(\pi r^2) = \int_{r=0}^1 (\rho_s \omega rdr)(\pi r^2) = \rho_s \omega \pi \int_0^1 r^3 dr
\]

\[
= 20 \times 0.1 \times \pi \left[\frac{r^4}{4}\right]_0^1 = \frac{\pi}{3} \text{ A} \cdot \text{m}^2
\]
SOL 4.2.19 Option (B) is correct.
Magnetic dipole moment of a spherical shell of radius \( r \) having surface charge density \( \rho_s \) is given as

\[
m = \frac{4\pi}{3} \rho_s \omega r^4
\]

where \( \omega \) is angular velocity.

Since the total charge of 5 C is distributed over the volume of the sphere so, the magnetic dipole moment of the sphere is given as

\[
m(r) = \int \frac{4\pi}{3} (\rho_v dr) \omega r^4
\]

where \( \rho_v \) is uniformly distributed volume charge density of the sphere. Therefore, we have

\[
m(r) = \frac{4\pi}{3} \rho_v \omega r^5 = \frac{1}{5} Q \omega r^2
\]

\[
= \frac{1}{5} \times (5) \times (4) \times r^2 = 4r^2 \text{ A-m}^2
\]

\( (Q = 5 \text{ C}, \omega = 4 \text{ rad/s}) \)

\[
m(r)(\text{A-m}^2)
\]

\[r(\text{m})\]

SOL 4.2.20 Option (C) is correct.
The average magnetic field intensity over a sphere of radius \( r \), due to steady currents within the sphere is defined as

\[
H_{\text{ave}} = \frac{1}{4\pi} \frac{2m}{r^3} = \frac{1}{4\pi} \frac{2 \times 4r^2}{r^3} = \frac{2}{r} \quad (m = 4r^2)
\]

As the sphere is spinning about the \( z \)-axis so, the produced magnetic field will be in \( a_z \) direction as determined by right hand rule. Thus, we have

\[
H_{\text{ave}} = \frac{3}{\pi r} a_z
\]

SOL 4.2.21 Option (A) is correct.
Magnetic dipole moment of a conducting loop carrying current \( I \) is defined as:

\[
m = I S a_n
\]

where \( S \) is the area enclosed by the conducting loop and \( a_n \) is normal vector to the surface. So we have

\[
m = (7)(0.1) a_n \quad (I = 7 \text{ A}, S = 0.1 \text{ m}^2)
\]

Now the given plane is

\[x + 3y - 1.5z = 3.5\]

For which we have the function

\[f = x + 2y - 1.5z\]
and the normal unit vector to the plane is,
\[ a_\text{n} = \frac{\nabla f}{|\nabla f|} = \frac{a_x + 3a_y - 1.5a_z}{\sqrt{1^2 + 3^2 + (-1.5)^2}} \]
So the magnetic dipole moment of the coil is
\[ m = (0.7) \left( a_x + 3a_y - 1.5a_z \right) \]
\[ = 1.2a_x + 0.6a_y - 0.3a_z \text{ A·m}^2 \]

SOL 4.2.22 Option (B) is correct.
The magnetic field intensity, \( \mathbf{H} \) in the terms of magnetic vector potential, \( \mathbf{A} \) is defined as
\[ \mathbf{H} = \frac{1}{\mu_0} (\nabla \times \mathbf{A}) = \frac{1}{\mu_0} \left( \nabla \times (6y - 2z) a_x + 4xz a_y \right) \]
\[ = \frac{1}{\mu_0} [-3a_x - 2a_y + 4a_z] \]
Since the electric current density at any point is equal to the curl of magnetic field intensity at that point.
i.e.
\[ \mathbf{J} = \nabla \times \mathbf{H} \]
So, we have the electric current density in the free space as
\[ \mathbf{J} = \nabla \times \left( \frac{1}{\mu_0} [-8a_x - 2a_y + 6a_z] \right) = 0 \]

SOL 4.2.23 Option (D) is correct.
Magnetic flux density across the toroid at a distance \( r \) from its center is defined as
\[ \mathbf{B} = \frac{\mu_0 NI}{2\pi r} a_\text{r} \]
where \( N \to \) Total no. of turns
\( I \to \) Current flowing in the toroid
So, the total magnetic flux across the toroid is given by the surface integral of the flux density
i.e.
\[ \phi = \int_S \mathbf{B} \cdot d\mathbf{S} \]
where \( d\mathbf{S} \) is differential surface area vector.
Consider a width \( dr \) of toroid at a distance \( r \) from its center as shown in figure

So we have the total magnetic flux across the toroid as
\[ \phi = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left( \frac{\mu_0 NI}{2\pi r} a_\text{r} \right) (hdr)a_\text{r} \]
\( (d\mathbf{S} = hdr a_\text{r}) \)
SOL 4.2.24 Option (B) is correct.
As determined in previous question the magnetic flux density across the toroid at a distance \( r \) from it’s center is
\[
B = \frac{\mu_0 NI}{2\pi r} a_\phi
\]
So at the mean radius,
\[
r = \frac{a + b}{2} = 1.5 \text{ m}
\]
we have,
\[
B = \frac{\mu_0 NI}{3\pi} a_\phi \quad \text{ \( (r = 1.5 \text{ m}) \)}
\]
Therefore the total magnetic flux is
\[
\phi' = \int B \cdot dS = \int_{r=1}^{r=2} \left( \frac{\mu_0 NI}{3\pi} a_\phi \right) (hdra_\phi)
\]
\[
= \frac{4\pi \times 10^{-7} \times 10^5 \times 10 \times 10}{3\pi} [r^2]
\]
\[
= 1.33 \text{ Wb}
\]
Thus, the percentage of error is
\[
\% \text{ error} = \frac{\phi' - \phi}{\phi} \times 100\% \quad \text{(\( \phi = 1.39 \text{ wb as calculated above} \))}
\]
\[
= \frac{1.33 - 1.39}{1.39} \times 100\% = -1.39\% \quad \text{\( -1\frac{3}{4}\% \)}
\]

SOL 4.2.25 Option (C) is correct.

Consider the point \( P \) on \( z \)-axis is \( (0,0,h) \) and current flowing in the current element is \( I \) in \( a_x \) direction. Since the magnetic field intensity at any point \( P \) due to a current element \( I \) is defined as
\[
H = \frac{I}{4\pi \rho} [\cos \alpha_2 - \cos \alpha_1] a_\phi
\]
where \( \rho \rightarrow \) distance of point \( P \) from the current element. 
\( \alpha_1 \rightarrow \) angle subtended by the lower end of the element at \( P \).
\( \alpha_2 \rightarrow \) angle subtended by the upper end of the element at \( P \).

So for the given current element along positive \( x \)-axis we have
\[ \alpha_1 = 90^\circ \]
\[ \alpha_2 = 0^\circ \]

Therefore,
\[ H = \frac{I}{4\pi \rho} a_\phi \]

Now the direction of magnetic field intensity is defined as
\[ a_\phi = a_i \times a_\rho \]
where \( a_i \) is unit vector along the line current and \( a_\rho \) is the unit vector normal to the line current directed toward the point \( P \).

So,
\[ a_\phi = a_i \times a_\rho = -a_\rho \]

Therefore magnetizing force is
\[ H = \frac{I}{4\pi \rho} (-a_\rho) \]
or
\[ H = \frac{I}{4\pi \rho} \]

...(1)

Now consider the current flowing in the current element introduced along the positive \( y \)-axis is \( I \) in \( a_y \) direction. So, the magnetic field intensity produced at point \( P \) due to the current element along the positive \( y \)-axis is
\[ H = \frac{I}{4\pi \rho} [\cos \alpha_2 - \cos \alpha_1] a_\phi \]
\[ = \frac{I}{4\pi \rho} [\cos 90^\circ - \cos 0^\circ] a_\phi \]
\[ = \frac{I}{4\pi \rho} a_\phi \]

Therefore the resultant magnetic field intensity produced at point \( P \) due to both the current elements will be
\[ H_{net} = \frac{I}{4\pi \rho} (-a_\rho + a_z) \]
or,
\[ H_{net} = \frac{I}{4\pi \rho} \sqrt{2} \]

Thus, from equation (1) we have
\[ H_{net} = \frac{I}{2\pi \rho} \]

Option (B) is correct.

The magnetic flux density produced at a distance \( \rho \) from a straight wire carrying current \( I \) is defined as
\[ B = \frac{\mu_0 I}{2\pi \rho} \]

Now consider a strip of width \( d\rho \) of the square loop at a distance \( \rho \) from the straight wire as shown in the figure.
Total magnetic flux crossing the strip is

\[ d\psi_m = B(2d\rho) \quad \text{(area of strip = } 2d\rho) \]

So, the flux crossing the complete square loop is

\[ \psi_m = \int d\psi_m = \int_{\rho=0}^{6} \frac{\mu_0 I}{2\pi \rho} (2d\rho) = \frac{\mu_0 I}{\pi} \left[ \ln(6) - \ln(4) \right] = 8.11 \times 10^{-7} \text{ Weber} \]

\[ = 2.42 \mu \text{Wb} \]

**SOL 4.2.27** Option (B) is correct.

As calculated in previous question the total flux crossing through the square loop due to the straight conducting element is

\[ \psi_m = \int_{\rho=a}^{L} \frac{\mu_0 I}{2\pi \rho} (1) \]

where \( I \) is the current carried by the conductor, \( L \) is the side of the square loop and \( a, b \) are the distance of the two sides of square loop from the conductor.

So we have

\[ L = 0.5 \text{ m} \]

\[ a = 0.3 - \frac{0.5}{2} = 0.05 \text{ m} \]

\[ b = 0.3 + \frac{0.5}{2} = 0.55 \text{ m} \]

Thus,

\[ \psi_m = \int_{\rho=0.05}^{0.55} \frac{\mu_0 I}{2\pi \rho} (0.5d\rho) = \frac{\mu_0 I}{4\pi} \left[ \ln(0.55) - \ln(0.05) \right] = \frac{\mu_0 I}{4\pi} (\ln 11) \]

Therefore the current that produces the net flux \( \psi_m = 5 \times 10^{-5} \text{Tm}^2 \) is

\[ I = \frac{4\pi}{\mu_0 (\ln(11))} \times 5 \times 10^{-5} \]

\[ = 238.5 \text{ A} \]

**SOL 4.2.28** Option (D) is correct.

Consider the flux density at the given point due to semi infinite wire along \( y \)-axis is \( B_1 \) and the flux density due to wire along \( z \)-axis is \( B_2 \).

The magnetic flux density \( B \) produced at any point \( P \) due to a straight wire carrying current \( I \) is defined as

\[ B = \frac{\mu_0 I}{4\pi \rho} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_\phi \]

---

**GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia**
where \( \rho \) → distance of point \( P \) from the straight wire.
\( \alpha_1 \) → angle subtended by the lower end of the wire at \( P \).
\( \alpha_2 \) → angle subtended by the upper end of the wire at \( P \).
and the direction of the magnetic flux density is given as
\[
\mathbf{a}_{\phi} = \mathbf{a}_l \times \mathbf{a}_r
\]
where \( \mathbf{a}_l \) is unit vector along the line current and \( \mathbf{a}_r \) is the unit vector normal to the line current directed toward the point \( P \). So, we have
\[
\rho = 2 \text{ m} \\
\mathbf{a}_{\phi} = \mathbf{a}_y \times (\mathbf{a}_x) = -\mathbf{a}_z \\
\alpha_1 = \frac{\pi}{2}, \quad \alpha_2 = 0
\]
(\( y \) tends to \( \infty \))
Therefore the magnetic flux density produced at point \( P \) due to the semi infinite wire along y-axis is
\[
B_1 = \frac{\mu_0}{4\pi(2)}(\cos \theta - \cos \frac{\pi}{2})(-\mathbf{a}_z) = -\frac{\mu_0}{2\pi} \mathbf{a}_z
\]
Similarly we have the magnetic flux density produced at point \( P \) due to semi infinite wire along z-axis as
\[
B_2 = \frac{\mu_0}{2\pi} \mathbf{a}_z
\]
Thus, the net magnetic flux density produced at point \( P \) due to the L-shaped filamentary wire is
\[
B = -3(\mathbf{a}_y + 3\mathbf{a}_z) \times 10^{-4} \text{ Wb/m}^2
\]
Option (D) is correct.
Using amperes’ circuital law we have
\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}
\]
As the conductor carries current \( I \) which is uniformly distribute over the conductor cross section so, the current density inside the conductor is
\[
J = \frac{I}{\pi R^2}
\]
We construct an Amperian loop of radius \( \rho(< R) \) inside the cylindrical wire for
which the enclosed current is
\[ I_{enc} = \frac{1}{\pi R^2} \pi R^2 = \frac{I}{R^2} \]
and since the current is flowing along z-axis so using right hand rule we get the
direction of magnetic flux density along \( +a_\phi \).
Thus, from Amperes circuital law, we have
\[ (B_\phi)(2\pi \rho) = I_{enc} \]
or
\[ B_\phi = \frac{\mu_0 I}{2\pi \rho} \]
or
\[ B = \frac{\mu_0 I}{6\pi \rho^2 a_\phi} \]

**SOL 4.2.30** Option (D) is correct.
Similarly as calculated above we construct an Amperian loop of radius \( \rho (> R) \)
outside the cylinder for which the entire current flowing in the wire will be enclosed.
i.e.,
\[ I_{enc} = I \]
and from Ampere’s circuital law we get,
\[ B_\phi (2\pi \rho) = \mu_0 I \]
\[ B_\phi = \frac{\mu_0 I}{2\pi \rho} \]
So
\[ B \propto \frac{1}{\rho} \]

**SOL 4.2.31** Option (C) is correct.
We consider only the half side of the loop to determine the flux density at the
center as shown in the figure:

The magnetic flux density \( B \) produced at any point \( P \) due to a straight wire
carrying current \( I \) is defined as
\[ B = \frac{\mu_0 I}{4\pi \rho} (\cos \alpha_2 - \cos \alpha_1) a_\phi \]
where
\[ \rho \rightarrow \text{distance of point } P \text{ from the straight wire.} \]
\[ \alpha_1 \rightarrow \text{angle subtended by the lower end of the wire at } P. \]
\[ \alpha_2 \rightarrow \text{angle subtended by the upper end of the wire at } P. \]
and the direction of the magnetic flux density is given as
\[ \mathbf{a}_\phi = \mathbf{a}_l \times \mathbf{a}_r \]
where \( \mathbf{a}_l \) is unit vector along the line current and \( \mathbf{a}_r \) is the unit vector normal to the line current directed toward the point \( P \).

Therefore, the magnetic flux density produced at centre \( O \) due to the half side of the square loop is
\[ B_1 = \frac{\mu_0 I}{4\pi \rho} (\cos \alpha_2 - \cos \alpha_1) \mathbf{a}_\phi \]
where \( \rho = 1 \text{ m} \), \( \alpha_1 = \frac{\pi}{2} \) and \( \cos \alpha_2 = \frac{1}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} \).

Thus,
\[ B_1 = \frac{(4\pi \times 10^{-7})(1)}{4\pi(1)} \left( \frac{1}{\sqrt{2}} - 0 \right) \mathbf{a}_z \]
\[ = \frac{10^{-4}}{\sqrt{2}} \mathbf{a}_z \text{ Wb/m}^2 \]

As all the half sides of the loop will produce the same magnetic flux density at the centre so, the net magnetic flux density produced at the centre due to whole square loop will be
\[ B = 8B_1 = 4\sqrt{2} \times 10^{-7} \mathbf{a}_z \text{ Wb/m}^2 \]

**SOL 4.2.32**

Option (D) is correct.

Using right hand rule we conclude that the field intensity produced at centre of the loop by the loop wire and the straight wire are opposing each other, so, the field intensity at the centre of the loop will be zero if
\[ H_{\text{wire}} = H_{\text{loop}} \quad \text{...(1)} \]
where \( H_{\text{wire}} \) is the field intensity produced at the center of loop due to the straight wire and \( H_{\text{loop}} \) is the field intensity produced at the center of loop due to the current in the circular loop.

Since the magnetic field intensity produced at a distance \( \rho \) from an infinitely long straight wire carrying current \( I \) is defined as
\[ H = \frac{I}{2\pi \rho} \]
So we have
\[ H_{\text{wire}} = \frac{I}{2\pi(1)} = \frac{20}{2\pi} = \frac{10}{\pi} \]
and as calculated in Q.59 the field intensity produced by circular loop at its center is
\[ H_{\text{loop}} = \frac{I}{2a} \]
where \( a \) is the radius of the loop

or,
\[ H_{\text{loop}} = \frac{I}{2(10 \times 10^{-2})} = \frac{10I}{2} = 5I \]

So putting the values in eq. (1) we get
\[ \frac{10}{\pi} = 5I \]

Thus,
\[ I = \frac{6}{\pi} \text{ A} \]
SOL 4.2.33  Option (D) is correct.
Consider one half side of the square loop to determine the magnetic field intensity at the centre \( O \) as shown in the figure.

![Diagram of a square loop with vectors](image)

The magnetic field intensity \( H \) produced at any point \( P \) due to a straight wire carrying current \( I \) is defined as

\[
H = \frac{I}{4\pi \rho} [\cos \alpha_2 - \cos \alpha_1]
\]

where 
- \( \rho \) → distance of point \( P \) from the straight wire.
- \( \alpha_1 \) → angle subtended by the lower end of the wire at \( P \).
- \( \alpha_2 \) → angle subtended by the upper end of the wire at \( P \).

So we have 

\[
\rho = a/2 \\
\alpha_1 = \pi/2 \\
\alpha_2 = \pi/4
\]

Therefore the magnetic field intensity produced at centre \( O \) due to the half side of the square loop is

\[
H_1 = \frac{I}{2\pi} \left( \cos \frac{\pi}{4} - \cos \frac{\pi}{2} \right) = \frac{I}{\sqrt{2} \pi a}
\]

As all the eight half sides produces same field intensity at the centre of the loop so, net field intensity produced at the center due to the complete square loop is

\[
H_{\text{net}} = 8 \left( \frac{I}{\sqrt{2} \pi a} \right) = \frac{2\sqrt{2} I}{\pi a}
\]

SOL 4.2.34  Option (A) is correct.
For the shown current loop we divide the loop in two segments as shown in figure

![Diagram of a current loop divided into segments](image)

Now the field intensity due to segment (1) (Semicircular loop) at point \( P \) can be given directly as calculated in Que.60
i.e. \[ H_1 = \frac{I}{4a} \]
where \( a \) is radius of semicircular loop

or

\[ H_1 = \frac{8}{4(1)} = 2 \text{ A/m} \quad (a = 1 \text{ m}) \]

again for determining the field intensity due to segment (2) we consider it as the half portion of a complete square loop of side 2 m and since the field intensity due to a complete square loop of side a carrying current \( I \) can be directly given from previous question.

i.e. \[ H = \frac{2\sqrt{2} I}{\pi a} \]

so the field intensity due to the half portion of square loop is

\[ H_2 = \frac{1}{2} H = \frac{\sqrt{2} I}{\pi a} \]

or

\[ H_2 = \frac{\sqrt{2}(8)}{\pi(2)} = \frac{4\sqrt{2}}{\pi} \quad (I = 8 \text{ A, } a = 2 \text{ m}) \]

As determined by right hand rule the direction of field intensity produced at point \( P \) due to the two segments will be same (inward) therefore, the net magnetic field intensity produced at point \( P \) will be

\[ H_{net} = H_1 + H_2 = 2 + \frac{4\sqrt{2}}{\pi} = 3.8 \text{ A/m inward.} \]

SOL 4.2.35

Option (B) is correct.

The flux density due to infinite current carrying sheet is defined as

\[ B = \frac{\mu_0}{2} K \times a_n \]

where \( K \) is surface current density and \( a_n \) is unit vector normal to the surface directed toward the point where flux density is to be determined.

So, for the sheet in \( z = 0 \) plane

\[ B_1 = \frac{\mu_0}{2}(4a_x) \times (a_z) = -2\mu_0 a_y \quad (a_n = a_z) \]

and for the sheet in \( z = 2 \text{ m} \) plane

\[ B_2 = \frac{\mu_0}{2}(-4a_x) \times (-a_z) = -2\mu_0 a_y \quad (a_n = -a_z) \]

Therefore, the net flux density between the sheets is

\[ B = B_1 + B_2 = -4\mu_0 a_y \]

Thus the magnetic flux per unit length along the direction of current is

\[ \psi_m/l = B \times \text{(Distance between the plates)} \]

\[ = -8\mu_0 a_y \text{ Wb/m} \]

***********
SOLUTIONS 4.3

SOL 4.3.1 Option (B) is correct.
For \( r > a \),
\[
    I_{\text{enclosed}} = (\pi a^2) J
    \]
\[
    \oint H \cdot dl = I_{\text{enclosed}}
    \]
\[
    H(2\pi r) = (\pi a^2) J
    \]
\[
    H = \frac{I_o}{2\pi r}
    \]
\[
    L_o = (\pi a^2) J
    \]
i.e.
\[
    H \propto \frac{1}{r}, \quad \text{for} \ r > a
    \]
For \( r < a \),
\[
    I_{\text{enclosed}} = \frac{J(\pi r^2)}{\pi a^2} = \frac{Jr^2}{a^2}
    \]
So,
\[
    \oint H \cdot dl = I_{\text{enclosed}}
    \]
\[
    H(2\pi r) = \frac{Jr^2}{a^2}
    \]
\[
    H = \frac{Jr^2}{2\pi a^2}
    \Rightarrow \quad H \propto \frac{1}{r}, \quad \text{for} \ r < a
    \]

SOL 4.3.2 Option (C) is correct.
Assume that the cross section of the wire lies in the \( x\)-\( y \) plane as shown in figure below:

Since, the hole is drilled along the length of wire. So, it can be assumed that the
drilled portion carries current density of \(-J\).

Now, for the wire without hole, magnetic field intensity at point \(P\) will be given as

\[
H_{o1}(2\pi R) = J(\pi R^2) \quad \Rightarrow \quad H_{o1} = \frac{JR}{2}
\]

Since, point \(O\) is at origin and the cross section of the wire located in \(x-y\) plane. So, in vector form the field intensity due to the current carrying wire without considering the hole is given as

\[
H_1 = \frac{J}{2}(xa_x + ya_y)
\]  

Again, only due to the hole magnetic field intensity at point \(P\) will be given as

\[
(H_{o2})(2\pi r) = -J(\pi r^2)
\]

\[
H_{o2} = -\frac{Jr}{2}
\]

Again, if we take \(O'\) at origin then in vector form

\[
H_2 = -\frac{J}{2}(x'a_x + y'a_y)
\]

where \(x'\) and \(y'\) denotes point ‘\(P\)’ in the new co-ordinate system.

Now the relation between two co-ordinate system will be

\[
x = x' + d \quad \text{and} \quad y = y'
\]

So, putting it into equation (2) we have

\[
H_2 = -\frac{J}{2}[(x - d)a_x + y'a_y]
\]

Therefore, the net magnetic field intensity at point \(P\) is

\[
H_{net} = H_1 + H_2 = \frac{J}{4}da_a
\]

i.e. the magnetic field inside the hole will depend only on \(d\).

**SOL 4.3.3**
Option (A) is correct.

Due to 1 A current wire in \(x-y\) plane, magnetic field be at origin will be in \(x\) direction as determined by right hand rule.

Due to 1 A current wire in \(y-z\) plane, magnetic field be at origin will be in \(z\) direction as determined by right hand rule.

Thus, \(x\) and \(z\)-component is non-zero at origin.

**SOL 4.3.4**
Option (D) is correct.

The total flux,

\[
\Phi = 1.2 \text{ mWb} = 1.2 \times 10^{-3} \text{ Wb}
\]

Cross sectional area,

\[
A = 30 \text{ cm}^2 = 30 \times 10^{-4} \text{ m}^2
\]

So, the flux density is given as

\[
B = \frac{\Phi}{A} = \frac{1.2 \times 10^{-3}}{30 \times 10^{-4}} = 0.4 \text{ Tesla}
\]

**SOL 4.3.5**
Option (D) is correct.

The relation between magnetic flux density \(B\) and vector potential \(A\) is given as

\[
B = \nabla \times A
\]

**SOL 4.3.6**
Option (B) is correct.
For an isolated body the charge is distributed over its region which depends on the total change and the curvature of the body. Thus Statement 1 is correct. Since the magnetic flux lines form loop so the total magnetic flux through any closed surface is zero. Thus Statement 2 is correct.

**SOL 4.3.7**
Option (D) is correct.

The magnetic flux density in terms of vector potential is defined as

\[
B = \nabla \times A
\]

\[
\int \mathbf{B} \cdot d\mathbf{S} = \oint (\mathbf{\nabla} \times \mathbf{A}) d\mathbf{S}
\]

\[
\Phi = \oint \mathbf{A} \cdot dl
\]

i.e. the line integral of vector potential \( \mathbf{A} \) around the boundary of a surface \( \mathbf{S} \) is equal to the flux through the surface \( \mathbf{S} \).

**SOL 4.3.8**
Option (B) is correct.
Consider the current element along \( z \)-axis as shown in the figure.

Using right hand rule we get the direction of magnetic field directing normal to radial line \( OP \).

**SOL 4.3.9**
Option (D) is correct.
For the given circular cylinder, consider the surface current density is \( K \). So, the total current \( I \) through the cylinder is given as

\[
K(2\pi r) = I
\]

where \( r \) is radius of circular cylinder.

So,

\[
K = \frac{I}{2\pi r} = \frac{5}{2\pi(5 \times 10^{-2})} = \frac{100}{\pi} \text{ A/m}
\]

**SOL 4.3.10**
Option (B) is correct.
Magnetic vector potential of an infinitesimally small current element is defined as

\[
\mathbf{A} = \int \frac{\mu_0}{4\pi} \frac{I dl}{R}
\]

where \( R \) is the distance from current element. Given that \( R \rightarrow \infty \)

So

\[
\mathbf{A} = 0
\]

**SOL 4.3.11**
Option (A) is correct.
Consider the two parallel sheets are separated by a distance ‘\( d \)’ as shown in the figure below.
The two sheets carries surface currents
\[ K = K_y a_y \]
At any point between them the magnetic field intensity is given as
\[ H = \frac{1}{2} K \times (a_{nu} + a_{nl}) \]
where \( a_{nu} \) is the normal vector to the upper plate and \( a_{nl} \) is normal vector to the lower plate both directs toward the point between them
i.e.
\[ a_{nu} = -a_z \quad \text{and} \quad a_{nl} = a_z \]
So,
\[ H = \frac{1}{2} K_y a_y \times (-a_z + a_z) = 0 \]

SOL 4.3.12 Option (B) is correct.
For the given current distribution, the current enclosed inside the cylindrical surface of radius \( \rho \) for \( a < \rho < b \) is
\[ I_{enc} = \int_a^b (J_0 \rho J_0 \rho d\rho) = 2\pi J_0 J_0 (\rho^3 - a^3) \]
and the magnetic field intensity is given as
\[ H = \frac{2\pi J_0 J_0 (\rho^3 - a^3)}{3a^2 \rho} \]

SOL 4.3.13 Option (D) is correct.
The radiated \( E \) and \( H \) field are determined by following steps
1. Determine magnetic field intensity \( H \) from the expression
\[ B = \mu H = \nabla \times A \]
2. then determine \( E \) from the expression
\[ \nabla \times H = \frac{\partial E}{\partial t} \]
So, the concept of vector magnetic potential is used to find the expression of radiated \( E \) and \( H \) field.
Option (B) is correct.
Using right hand rule, we conclude that the direction of field intensity is same as determined for the two correct elements $3I$ and $2I$ while it is opposite for the current element $I$. Therefore, from the ampere’s circuital law, we get the circulation of $\mathbf{H}$ around the closed contour as

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enclosed}} = 2I + 3I - I = 4I$$

Option (C) is correct.
The unit of magnetic flux density ($\mathbf{B}$) is Tesla or $\text{Wb/m}^2$.

Option (D) is correct.
From Ampere’s circuital law, the circulation of magnetic field intensity in a closed path is equal to the current enclosed by the path, i.e.,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}}$$

So, for the current $I$ the circulation at a radial distance $R$ is given as

$$H(2\pi R) = I$$

or

$$H = \frac{I}{2\pi R}$$

Therefore, the magnetic flux density at the radial distance $R$ is

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi R}$$

Option (A) is correct.
Unit of work is Joule.
Unit of electric field strength ($\mathbf{E}$) is volt/meter.
Unit of magnetic flux is Weber.
Unit of magnetic field strength is Ampere/meter.
So, in the match list we get, $A \rightarrow 4$, $B \rightarrow 3$, $C \rightarrow 2$, $D \rightarrow 1$.

Option (D) is correct.
Magnetic field intensity at a distance $r$ from a long straight wire carrying current $I$ is defined as

$$H = \frac{I}{2\pi r}$$

So, the current that must flow in the loop to produce the magnetic field $H = 1\, \text{mA/m}$ is

$$r = \frac{1}{2\pi} = 1.59\, \text{m}$$
I = 2rH = 2 \times 1 \times 1 = 2 \text{mA}

SOL 4.3.20 Option (C) is correct.
Magnetic flux density $B$ in terms of vector potential $A$ is defined as

$$B = \nabla \times A$$

So, $B$ can be easily obtained from $A$ also we know $\nabla \cdot A = 0$ but it is not the explanation of Assertion (A).

i.e. A and R both are true but R is not the correct explanation of A.

SOL 4.3.21 Option (A) is correct.
(1) Magnetic flux density in terms of vector potential is given as

$$B = \nabla \times A$$

(2) Poisson’s equation for magnetic vector potential is

$$\nabla^2 A = -\mu_0 J$$

(3) Magnetic vector potential for a line current is defined as

$$A = \int \frac{\mu_0 I dl}{2\pi r}$$

So, all the statements are correct.

SOL 4.3.22 Option (B) is correct.
Magnetic field intensity due to a long straight wire carrying current $I$ at a distance $r$ from it is defined as

$$H = \frac{I}{2\pi r}$$

1 = \frac{10}{2\pi r} \quad r = \frac{10}{2\pi} = 1.59 \text{m}

SOL 4.3.23 Option (C) is correct.
Magnetic field intensity due to an infinite linear current carrying conductor is defined as

$$\int H \cdot dl = I_{enc} \quad H(2\pi r) = I \quad \Rightarrow \quad H = \frac{I}{2\pi r}$$

SOL 4.3.24 Option (D) is correct.
The net outward magnetic flux through a closed surface is always zero as magnetic flux lines has no source or sinks.

i.e. $\int B \cdot dS = 0$

Now, from Gauss’s law we have

$$\int (\nabla \cdot B) dv = \int B \cdot dS = 0$$

So, comparing the equation (1) and (2) we get

$$\nabla \cdot B = 0$$
SOL 4.3.25 Option (D) is correct.

Given, the plane \( y = 0 \) carries a uniform current density 30 \( a_z \) mA/m and since the point \( A \) is located at \( (1, 20, -2) \) so, unit vector normal to the current sheet is \( a_n = a_y \)

Therefore, the magnetic field intensity

\[
H = \frac{1}{2} K \times a_n = \frac{1}{2} (30a_z) \times (a_y) = -15a_z \text{ mA/m} \quad (K = 30a_z \text{ mA/m})
\]

SOL 4.3.26 Option (D) is correct.

The magnetic flux density at any point is curl of the magnetic vector potential at that point.

i.e. \( B = \nabla \times A \)

From the Maxwell’s equation, the divergence of magnetic flux density is zero.

i.e. \( \nabla \cdot B = 0 \)

Again from the Maxwell’s equation, the curl of the magnetic field intensity is equal to the current density.

i.e. \( \nabla \times H = J \)

or, \( \nabla \times B = \mu_0 J \) \( (B = \mu_0 H) \)

The expression given in option (A) is incorrect

i.e. \( B \neq \nabla \cdot A \)

SOL 4.3.27 Option (D) is correct.

Superconductors are popularly used for generating very strong magnetic field.

SOL 4.3.28 Option (D) is correct.

As the magnetic flux lines have no source or sinks i.e. it forms a loop. So the total outward flux through a closed surface is zero.

i.e. \( \oint B \cdot dS = 0 \)

SOL 4.3.29 Option (D) is correct.

The magnetic field intensity due a surface current density \( K \) is defined as

\[
H = \frac{1}{2} K \times a_n
\]

Where \( a_n \) is unit normal vector to the current carrying surface directed toward the point of interest.

Given that, \( K = 2a_z \).

and since the surface carrying current is in plane \( z = 0 \).

So, for \(-\alpha < z < 0\) \( a_n = -a_z \)

and \( H = \frac{1}{2}(2a_z) \times (-a_z) = a_y \)

For \( 0 < z < x \), \( a_n = a_z \)

and \( H = \frac{1}{4}(2a_z) \times (a_z) = -\frac{1}{2}a_z \)

**********
CHAPTER 5

MAGNETOSTATIC FIELDS IN MATTER
EXERCISE 5.1

MCQ 5.1.1 Path of a charged particle \( A \) that enters in a uniform magnetic field \( B \) (pointing into the page) is shown in the figure.

The deflection in the path of the particle shows that the particle is
(A) positive charged  
(B) negatively charged  
(C) uncharged  
(D) can’t be determined

MCQ 5.1.2 Unit of a magnetic point charge is
(A) Ampere meter  
(B) coulomb meter  
(C) Ampere meter square  
(D) doesn’t exist

MCQ 5.1.3 Assertion (A) : Both the electric force and magnetic force are produced when a charged particle moves at a constant velocity.
Reason (R) : Electric force is an accelerating force where as magnetic force is a purely deflecting force.
(A) Both A and R are true and R is correct explanation of A.  
(B) Both A and R are true but R is not the correct explanation of A.  
(C) A is true but R is false.  
(D) A is false but R is true.

MCQ 5.1.4 An electron beam is passed through a uniform crossed electric and magnetic fields \( \mathbf{E} = 15 \mathbf{a}_x \text{ V/m} \) and \( \mathbf{B} = 23 \mathbf{a}_z \text{ wb/m}^2 \). \( \mathbf{E} \) and \( \mathbf{B} \) are mutually perpendicular and both of them perpendicular to the beam). If the beam passes the field without any deflection then the velocity of the beam will be
(A) 5 m/s  
(B) 45 m/s  
(C) 30 m/s  
(D) 18 m/s
**MCQ 5.1.5**

An electron is moving in the combined fields $E = 0.1a_x - 0.2a_y + 0.3a_z \text{ kV/m}$ and $B = -a_x + 3a_y - a_z \text{ Tesla}$. If the velocity of the electron at $t = 0$ is $V(0) = (200a_x - 300a_y - 400a_z) \text{ m/s}$ then the acceleration of the electron at $t = 0$ will be

(charge on electron, $e = 1.6 \times 10^{-19} \text{ C}$; mass of electron, $m_e = 9.1 \times 10^{-31} \text{ kg}$)

(A) $1.75 \times 10^{13}(1.1a_x + 1.4a_y - 0.5a_z) \text{ m/s}^2$

(B) $2.1 \times 10^4(a_x + a_y - a_z) \text{ m/s}^2$

(C) $3.5 \times 10^{13}(6a_x + 6a_y - a_z) \text{ m/s}^2$

(D) $3.19 \times 10^{-17}(6a_x + 6a_y - a_z) \text{ m/s}^2$

**MCQ 5.1.6**

A current element of length placed along $z$-axis carries a current of $I = 3 \text{ mA}$ in the $+a_z$ direction. In a uniform magnetic flux density of $B = a_x + 7a_y \text{ wb/m}^2$ is present in the space then what will be the force on the current element in the presence of the magnetic flux density?

(A) $6a_x - 18a_y \text{ mN}$

(B) $-18a_x + 6a_y \text{ mN}$

(C) $18a_x - 6a_y \text{ mN}$

(D) $-1.8a_x + 6a_y \text{ mN}$

**MCQ 5.1.7**

Consider two current loops $C_1$ and $C_2$ carrying current $I_1$ and $I_2$, separated by a distance $R$. If the force experienced by the current loop $C_1$ due to the current loop $C_2$ is $F$, then the force experienced by current loop $C_1$ due to the current loop $C_2$ will be

(A) $-F$

(B) $F$

(C) $-F \left( \frac{I_1}{I_2} \right)$

(D) $F \left( \frac{I_1}{I_2} \right)$

**MCQ 5.1.8**

Which of the following statements is correct for a current free interface between two different magnetic media?

(A) Normal component of magnetic field intensity will be continuous.

(B) Tangential component of magnetic flux density will be continuous.

(C) Magnetic scalar potential will be same in both the medium.

(D) None of these

**Statement for Linked Question 9 - 10:**

In the free space three uniform current sheets with surface current densities $K_1 = 4a_x$, $K_2 = -4a_x$, $K_3 = -2a_x$ are located in the plane $z = 0$, $z = 1$ and $z = -1$ respectively.

**MCQ 5.1.9**

Net magnetic field intensity produced between the sheets located at $z = 0$ and $z = 1$ will be

(A) $2a_y \text{ A/m}$

(B) $4a_y \text{ A/m}$

(C) $-2a_y \text{ A/m}$

(D) $0$
MCQ 5.1.10 If a conducting filament located along the line \( y = 0, z = 0.2 \text{ m} \) carries 7 A current in \( +a_x \) direction then what will be the force per unit length exerted on it?

(A) \(-14a, \text{ N/m}\)
(B) \(14\mu_0 a, \text{ N/m}\)
(C) \(14\mu_0 a, \text{ N/m}\)
(D) \(-14\mu_0 a, \text{ N/m}\)

MCQ 5.1.11 A rectangular coil of area 1 m\(^2\) carrying a current of 5 A lies in the plane \(2x + 6y - 3z = 4\). Such that magnetic moment is directed away from origin. If the coil is surrounded by a uniform magnetic field \( B = 2a_x + 4a_y + 5a_z \) wb/m\(^2\) then the torque on the coil will be

(A) \(3a_x - 20a_y - 20a_z, \text{ N.m}\)
(B) \(30a_x - 20a_y - 20a_z, \text{ N.m}\)
(C) \(21a_x - 14a_y - 14a_z, \text{ N.m}\)
(D) \(6a_x - 4a_y - 4a_z, \text{ N.m}\)

MCQ 5.1.12 A circular current loop of radius 1 m is located in the plane \( z = 0 \) and centered at origin. What will be the torque acting on the loop in presence of magnetic field \( B = 4a_x - 4a_y - 2a_z \) wb/m\(^2\), if a uniform current of 10 A is flowing in the loop?

(A) \(20\pi (2a_x - a_z)\)
(B) \(40\pi (a_x + a_y)\)
(C) \(4\pi (a_x + a_y)\)
(D) \(40\pi (a_x - a_y)\)

MCQ 5.1.13 List I shows the type of magnetic materials and List-II shows their criterions. Match List I with List II and select the correct answer using the codes given below:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Ferromagnetic</td>
<td>(\chi_m = 0, \mu_r = 1)</td>
</tr>
<tr>
<td>b. Diamagnetic</td>
<td>(\chi_m &gt; 0, \mu_r \geq 1)</td>
</tr>
<tr>
<td>c. Non-magnetic</td>
<td>(\chi_m &lt; 0, \mu_r \leq 1)</td>
</tr>
<tr>
<td>d. Paramagnetic</td>
<td>(\chi_m &gt;&gt; 0, \mu_r &gt;&gt; 1)</td>
</tr>
</tbody>
</table>

Codes:

- (A) 2 3 1 4
- (B) 4 3 1 2
- (C) 4 1 3 2
- (D) 1 3 4 2

MCQ 5.1.14 Which of the following is a diamagnetic material?

(A) copper
(B) sodium
(C) carbon
(D) aluminum

MCQ 5.1.15 **Assertion (A):** Aluminium is a paramagnetic material.

**Reason (R):** A paramagnetic material have an odd no. of electrons.

(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
MCQ 5.1.16 Magnetic flux density inside a medium is $6a_s$ mwb/m². If the relative permeability of the medium is 2.3 then the magnetization inside the medium will be

(A) 3979 A/m  (B) 2249 A/m
(C) 9151 A/m  (D) 8650 A/m

MCQ 5.1.17 Magnetic flux density inside a magnetic material is $B$. If the permeability of the material is $\mu = 3\mu_0$ then the vector magnetization of the material will be

(A) $\frac{B}{3\mu_0}$  (B) $\frac{3B}{2\mu_0}$
(C) $\frac{B}{2\mu_0}$  (D) $\frac{2B}{3\mu_0}$

MCQ 5.1.18 A portion of $H$ curve for a ferromagnetic material can be approximated by the analytical expression $B = \mu_0 kH$. The magnetization vector $M$ inside the material is

(A) $(\mu_0 k - 1)H$  (B) $kH$
(C) $(k + 1)H$  (D) $(k - 1)H$

MCQ 5.1.19 A magnetic material of relative permeability $\mu_r = 4/\pi$ is placed in a magnetic field having strength $H = 2\rho a_s A/m$. The magnetization of the material at $\rho = 2$ will be

(A) 8.19$a_s$ A/m  (B) 1.10$a_s$ A/m
(C) 0.546$a_s$ A/m  (D) 2.19$a_s$ A/m

MCQ 5.1.20 A metallic bar of cross sectional area $2 m^2$ is placed in a magnetizing field $H = 8 A/m$. If the field causes a total magnetic flux of $\Phi = 4.2 mWb$ in the bar then the susceptibility of the bar will be

(A) 22.87  (B) 23.87
(C) 46.74  (D) $3 \times 10^{-5}$

MCQ 5.1.21 A large piece of magnetic material carries a uniform magnetization $M$ and magnetic field intensity $H_0$ inside it. The magnetic flux density inside the material is given by

$$B_0 = \mu_0 (2H_0 + M)$$

If a small spherical cavity is hollowed out of the material then the magnetic field intensity $H$ at the center of the cavity will be

(A) $2H_0$  (B) $H_0 + \frac{M}{3}$
(C) $H_0 - \frac{2M}{3}$  (D) $H_0 - \frac{M}{3}$
Statement for Linked Question 22 - 23:
A nonuniform magnetic field $B$ inside a medium with magnetic susceptibility $\chi_m = 2$ is given as $B = 3z\mathbf{a}_z$ Tesla

**MCQ 5.1.22** Bound current density inside the medium will be
- (A) $\frac{3\mu_0}{8}a_y$ A/m$^2$
- (B) $\frac{3\mu_0}{4}a_y$ A/m$^2$
- (C) $\frac{4}{3\mu_0}a_y$ A/m$^2$
- (D) $\frac{16}{3\mu_0}a_y$ A/m$^2$

**MCQ 5.1.23** Total current density inside the medium will be
- (A) $\frac{4}{\mu_0}a_x$ A/m$^2$
- (B) $\frac{4}{3\mu_0}a_y$ A/m$^2$
- (C) $\frac{8}{3\mu_0}a_y$ A/m$^2$
- (D) $4\mu_0a_y$ A/m$^2$

**MCQ 5.1.24** A uniformly magnetized circular cylinder of infinite length has magnetization $M$ along its axis. The magnetic field intensity outside the cylinder will be
- (A) non uniform
- (B) uniform and depend on the radius of circular cylinder
- (C) zero
- (D) none of these

**MCQ 5.1.25** An infinite circular cylinder is located along $z$-axis that carries a uniform magnetization $M = 1.2a_y$ A/m. The magnetic flux density due to it inside the cylinder will be
- (A) $2.2 \times 10^{-7}a_y$
- (B) $0.7a_y$
- (C) $8.8 \times 10^{-7}a_y$
- (D) $4.4a_y$

**MCQ 5.1.26** Magnetic flux lines are passing from a nickel material to the free space. If the incident of the flux line makes an angle $\alpha_1 = 75^\circ$ to the normal of the boundary in the nickel side as shown in figure then what will be the angle $\alpha_2$ with normal of the flux when it comes out in free space? (relative permeability of Nickel = 600)

- (A) 15°
- (B) 1.23°
- (C) 2.7°
- (D) 0.356°
Statement for Linked Question 27 - 28:
The two homogenous, linear and isotropic medium is defined in a Cartesian system
such that medium 1 with relative permeability $\mu_1 = 8$ is located in the region
$y \leq 0$ and medium 2 with relative permeability $\mu_2 = 6$ is in the region $y > 0$.

**MCQ 5.1.27**
The magnetic field intensity in the 1st medium is $\mathbf{H}_1 = 13\mathbf{a}_x + 16\mathbf{a}_y - 10\mathbf{a}_z$. What
will be the magnetic field intensity in the 2nd medium?
(A) $9\mathbf{a}_x - 18.67\mathbf{a}_y + 10\mathbf{a}_z \text{ A/m}$
(B) $9\mathbf{a}_x + 2.63\mathbf{a}_y - 10\mathbf{a}_z \text{ A/m}$
(C) $9\mathbf{a}_x + 18\mathbf{a}_y - 10\mathbf{a}_z \text{ A/m}$
(D) $18.67\mathbf{a}_x - 9\mathbf{a}_y + 10\mathbf{a}_z \text{ A/m}$

**MCQ 5.1.28**
Magnetic flux density in medium 2 will be
(A) $(6.8\mathbf{a}_x - 14.1\mathbf{a}_y + 7.5\mathbf{a}_z) \times 10^{-5} \text{ wb/m}^2$
(B) $(6.8\mathbf{a}_x + 14.1\mathbf{a}_y - 7.5\mathbf{a}_z) \times 10^{-5} \text{ wb/m}^2$
(C) $(14.1\mathbf{a}_x - 6.8\mathbf{a}_y + 7.5\mathbf{a}_z) \times 10^{-5} \text{ wb/m}^2$
(D) $(54\mathbf{a}_x + 117\mathbf{a}_y - 60\mathbf{a}_z) \text{ wb/m}^2$

**MCQ 5.1.29**
The magnetic flux density in the region $z < 0$ is given as $\mathbf{B} = 4\mathbf{a}_x + 8\mathbf{a}_z \text{ Wb/m}^2$. If
the plane $z = 0$ carries a surface current density $\mathbf{K} = 4\mathbf{a}_x \text{ A/m}$, then the magnetic
flux density in the region $z > 0$ will be
(A) $4\mathbf{a}_x + 3(1 + \mu_0)\mathbf{a}_x \text{ Wb/m}^2$
(B) $4\mathbf{a}_x + \mu_0\mathbf{a}_y + 3\mathbf{a}_z$
(C) $(4\mathbf{a}_x + 3\mathbf{a}_y)(1 + \mu_0) \text{ Wb/m}^2$
(D) $4\mathbf{a}_x + 4\mu_0\mathbf{a}_y + 3\mathbf{a}_z \text{ Wb/m}^2$

**MCQ 5.1.30**
An infinite plane magnetic material slab of thickness $d$ and relative permeability $\mu$, occupies the region $0 < x < d$. An uniform magnetic field $\mathbf{B} = B_0\mathbf{a}_z$ is applied in
free space (outside the magnetic material). The field intensity $H_m$ and flux density $B_m$ inside the material will be respectively
(A) $\mu_0B_0$ and $\mu_0B_0$
(B) $\frac{B_0}{\mu_0}$ and $B_0$
(C) $\mu_0B_0$ and $\frac{B_0}{\mu_0}$
(D) $\frac{B_0}{\mu_0}$ and $\mu_0B_0$

**MCQ 5.1.31**
Two infinite plane conducting sheets are located in the plane $z = 0$ and $z = 2m$.
The medium between the plates is a magnetic material of uniform permeability $\mu = 4\mu_0$. If in the region between the plates a uniform magnetic flux density is defined as $\mathbf{B} = (12\mathbf{a}_x + 4\mathbf{a}_y) \times 10^{-3} \text{ wb/m}^2$ then the magnetic energy stored per
unit area of the plates will be
(A) $2.5 \text{ J/m}^2$
(B) $4.1 \text{ J/m}^2$
(C) $5 \text{ J/m}^2$
(D) $7.8 \text{ J/m}^2$
MCQ 5.1.32
In the two different mediums of permeability $\mu_1$ and $\mu_2$, the magnetic fields are ($B_1, H_1$) and ($B_2, H_2$) respectively as shown in the figure.

If the interface carries no current then the correct relation for the angle $\theta_1$ and $\theta_2$ is
(A) $B_1 \cos \theta_1 = B_2 \cos \theta_2$
(B) $H_1 \cos \theta_1 = H_2 \cos \theta_2$
(C) $B_1 \sin \theta_1 = B_2 \sin \theta_2$
(D) Both (B) and (C)

MCQ 5.1.33
In a three layer medium shown in the figure below, Magnetic flux impinges at an angle $\theta_1$ on the interface between regions 1 and 2. The permeability of three regions are $\mu_1$, $\mu_2$ and $\mu_3$. So the angle of emergence will be independent of

(A) $\mu_1$ and $\mu_2$ both
(B) $\mu_2$ only
(C) All $\mu_1$, $\mu_2$ and $\mu_3$
(D) $\mu_1$ only

MCQ 5.1.34
A conducting wire is bent to form a circular loop of mean radius 20 cm. If cross sectional radius of the wire is $a$, such that $a << 20$ cm then the internal inductance of the loop will be
(A) 125 H
(B) 785 nH
(C) 157.1 nH
(D) 250 nH
MCQ 5.1.35  The magnetic circuit shown in the figure has \( N \) turns of coil. Electrical analog for the magnetic circuit shown in the figure is

(A) \[ N I_0 \quad \phi \quad R \]

(B) \[ I_0 \quad \phi \quad NR \]

(C) \[ I_0 \quad \phi \quad R \]

(D) \[ N I_0 \quad \phi \quad R \]

MCQ 5.1.36  The coil of the magnetic circuit shown in figure has 100 turns.

Which of the following is correct electrical analog for the magnetic circuit?

(A) 

(B) 

(C) 

(D)
MCQ 5.1.37  A 200 turns of a coil is wound over a magnetic core of length 15 cm that has the relative permeability of 150. The current that must flow through the coil to produce 0.4 Tesla of flux density in the core is 

(A) 320 A  
(B) 1.6 A  
(C) 20.1 A  
(D) 0.63 A  

***********
EXERCISE 5.2

MCQ 5.2.1 In the free space the magnetic flux density $B$ points in the $a_z$ direction and electric field $E$ points in the $a_y$ direction as shown in the figure. If a charged particle at rest is released from the origin, then what path will it follow?

MCQ 5.2.2 A point charge $+2$ C of mass $m = 2$ kg is injected with a velocity $v_0 = 2a_x$ m/s into the region $y > 0$, where the magnetic field is given by $B = 3a_x$ wb/m². If the point charge is located at origin at the time of injection then in the region $y > 0$ the point charge will follow
(A) a circular path centered at (0,0, −2)
(B) an elliptical path centered at origin
(C) a circular path centered at (1,2,0)
(C) a parabolic path passing through origin

Statement for Linked Question 3 - 4:
A filamentary conductor is formed into a rectangle such that its corners lie on points \( P(1,1,0), Q(1,3,1), R(3,4,0), S(4,1,0) \). An infinite straight wire lying on entire \( x \)-axis carries a current of 5 A in \( a_x \) direction.

**MCQ 5.2.3** If the filamentary conductor carries a current of 3 A flowing in \( +a_x \) direction from \( Q \) to \( R \) then the force exerted by wire on the side \( QR \) of rectangle will be
(A) \(-3 \times 10^6 a_y \) N
(B) \(-2 \times 10^6 a_y \) N
(C) \(-6 \times 10^6 a_y \) N
(D) \(3 \times 10^6 a_y \) N

**MCQ 5.2.4** The total force exerted on the conducting loop by the straight wire will be
(A) \(-6 \times 10^6 a_y \) N
(B) \(12 \times 10^6 a_y \) N
(C) \(6 \times 10^6 a_y \) N
(D) \(-12 \times 10^6 a_y \) N

**MCQ 5.2.5** Two filamentary currents of \(-5a_y \) and \(5a_y \) A are located along the lines \( y = 0, z = -1 \) m and \( y = 0, z = 1 \) m respectively. If the vector force per unit length exerted on the third filamentary current of \(10a_z \) A located at \( y = k, z = 0 \) be \( F \) then the plot of \( F \) versus \( k \) will be

![Graphs A, B, C, D showing plots of force vs. distance](https://www.gatehelp.com)
A current filament placed on $x$-axis carries a current $I = 10 \text{ A}$ in $+a_z$ direction. If a conducting current strip having surface current density $K = 3a_z \text{ A/m}$ is located in the plane $y = 0$ between $z = 0.5$ and $z = 1.5 \text{ m}$ then what will be the force per unit meter on the filament exerted by the strip?

(A) $6.6a_z \mu \text{N/m}$ \hspace{1cm} (B) $6.6a_z \mu \text{N/m}$

(C) $6a_z \mu \text{N/m}$ \hspace{1cm} (D) $0$

A conducting current strip of $5 \text{ m}$ length is located in the plane $x = 0$ between $y = 1$ and $y = 3$. If surface current density of the strip is $K = 6a_z \text{ A/m}$ then the force exerted on it by a current filament placed on $z$-axis that carries a current $I = \text{ A}$ in $+a_z$ direction will be

(A) $-16.4a_y \mu \text{N}$ \hspace{1cm} (B) $-4.8a_y \mu \text{N}$

(C) $-26.4a_y \mu \text{N}$ \hspace{1cm} (D) $26.4a_z \mu \text{N}$

A thick slab extending from $y = -a$ to $y = +a$ carries a uniform current density $J = J_0 a_z$.

Plot of magnetizing factor $H$ at any point in the space (inside or outside slab) versus $y$ will be

(A) \hspace{1cm} (B) \hspace{1cm} (C) \hspace{1cm} (D)
MCQ 5.2.9 If a magnetic dipole of moment $m = m_0 a_z$ is placed at the origin then the force exerted on it due to the slab will be

(A) $0 \text{ N}$  
(B) $m_0 \mu_0 J_0 y a_z$
(C) $m_0 \mu_0 J_0 a_z$
(D) $-m_0 \mu_0 J_0 y a_z$

Statement for Linked Question 10 - 11:
A long circular cylinder placed along $z$-axis carries a magnetization $M = 2 \rho^2 a_z$.

MCQ 5.2.10 The volume current density $J$ at any point inside the cylinder is proportional to

(A) $\rho$
(B) $\rho^2$
(C) $\rho \sin \phi$
(D) $\rho$

MCQ 5.2.11 The plot of the magnetic flux density $B$ inside the cylinder versus $\rho$ will be

(A) 
(B) 
(C) 
(D)

MCQ 5.2.12 A short cylinder placed along $z$-axis carries a “frozen-in” uniform magnetization $M$ in $+a_z$ direction. If length of the cylinder is equal to its cross sectional diameter then pattern of its surface current density $K$ will be as

(A) 
(B)
MCQ 5.2.13 Magnetization of a long circular cylinder is $M$ along its axis. Which of the following gives the correct pattern of magnetic field lines ($B$).

Statement for Linked Question 14 - 15:
A conducting rod of square cross section of side $2\text{ cm}$ carries a uniform magnetization $M = 4 \text{ A/m}$ along its axis. Length of the rod is $L >> 4 \text{ cm}$.

MCQ 5.2.14 If the rod is bent around it into a complete circular ring then magnetic flux density inside the circular ring will be
(A) $4 \text{ wb/m}^2$  
(B) $4\mu_0 \text{ wb/m}^2$  
(C) $2\pi\mu_0 \text{ wb/m}^2$  
(D) $\mu_0 \text{ wb/m}^2$

MCQ 5.2.15 Assume that there remains a narrow gap of width $0.1 \text{ mm}$ between the ends of the rod when it is formed into a circular ring. The net magnetic flux density at the center of the gap will be
(A) $50.04 \times 10^{-7} \text{ wb/m}^2$  
(B) $49.88 \times 10^{-6} \text{ wb/m}^2$  
(C) $51.23 \times 10^{-6} \text{ wb/m}^2$  
(D) $34.66 \times 10^{-6} \text{ wb/m}^2$
MCQ 5.2.16 Magnetic flux density $B$ inside a sphere that carries a uniform magnetization $M$ will be

(A) 0  
(B) $\frac{1}{2}\mu_0 M$  
(C) $\frac{\mu_0 M}{2}$  
(D) $\frac{2}{3}\mu_0 M$

MCQ 5.2.17 A short cylinder of length equals to it’s diameter carries a uniform magnetization $M$ as shown in the figure.

The correct sketch for the magnetic field intensity $H$ inside the cylinder is

MCQ 5.2.18 Mutual inductance between an infinite current filament placed along $y$-axis and rectangular coil of 1500 turns placed in $x$-$y$ plane as shown in figure will be
For View Only

MCQ 5.2.19
An infinitely long straight wire of radius $a$, carries a uniform current $I$. The energy stored per unit length in the internal magnetic field will be:
(A) uniform and depends on $I$ only
(B) non uniform
(C) uniform and depends on $a$ only
(D) uniform and depends on both $I$ and $a$

MCQ 5.2.20
A planar transmission line consists of two conducting plates of 2 m width placed along $x$-$z$ plane such that the current in one plate is flowing in $+a_z$ direction. While in the other it is flowing in $-a_z$ direction. If both the plate carries 4 A current and there is a very small separation between them then what will be force of repulsion per meter between the two plates?
(A) $16\mu_0$
(B) $4\mu_0$
(C) $8\mu_0$
(D) $\mu_0/4$

MCQ 5.2.21
A very long solenoid having 20000 turns per meter. The core of solenoid is formed of iron. If the cross sectional area of solenoid is 0.04 m$^2$ and it carries a current $I = 100$ mA then what will be the energy stored per meter in its field?

(relative Permeability of iron, $\mu_r = 100$)
(A) 8.1 J/m
(B) 20.11 J/m
(C) 100.5 J/m
(D) 10.05 J/m

MCQ 5.2.22
A mass spectrograph is a device for separating charged particles having different masses. Consider two particles of same charges $Q$ but different masses $m$ and $2m$ injected into the region of a uniform field $B$ with a velocity $v$ normal to the magnetic field as shown in the figure. When the particles will be releasing out of the spectrograph the separation between them will be

\[ \frac{2mv}{Bq} \]
(A) \( \frac{2mv}{Bq} \)
(B) \( \frac{mv}{2Bq} \)
(C) \( \frac{mv}{Bq} \)
(D) 0

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
Statement for Linked Question 23 - 24:
Consider a conducting filamentary wire of length 1 meter and mass 0.3 kg oriented in east-west direction, situated in the earth’s magnetic field at the magnetic equator. (Assume the magnetic field at equator has a value of $0.2 \times 10^{-4}$ Wb/m² and directed northward)

MCQ 5.2.23 The current that required to counteract the earth’s gravitational force on the wire must flow from
(A) west to east  
(B) east to west  
(C) any of (A) and (B)  
(D) none of these

MCQ 5.2.24 What will be the magnitude of the current flowing in the wire as to counteract the gravitational force ?
(A) 49 kA  
(B) 24.5 kA  
(C) 98 kA  
(D) 4.9 kA

MCQ 5.2.25 A rigid loop of wire in the form of a square is hung by pivoting one of it’s side along the $x$-axis as shown in the figure. The loop is free to swing about it’s pivoted side without friction. The mass of the wire is 0.2 kg/m and carries a current 2 A. If the wire is situated in a uniform magnetic field $B = 2.96$ Wb/m² then the angle by which the loop swings from the vertical is

(A) $3\pi/4$  
(B) $\pi/4$  
(C) $\pi/6$  
(D) $\pi/2$

MCQ 5.2.26 Electron beams are injected normally to the plane edge of a uniform magnetic field $H = H_0 a_z$ as shown in figure.
For View Only

The path of the electrons ejected out of the field will be in
(A) $+ a_y$ direction  (B) $- a_y$ direction
(C) $(a_y + a_x)$ direction  (D) $(a_y - a_x)$ direction

MCQ 5.2.27

The medium between the two infinite plane parallel sheets carrying current densities $4a_x$ and $-8a_x$ A/m, consists of two magnetic material slabs of thickness 1 m and 2 m having permeabilities $\mu_1 = 2\mu_0$ and $\mu_2 = 4\mu_0$ respectively as shown in the figure.

$$K = -4a_x$$

$$\mu_2 = 4\mu_0$$

$$\mu_1 = 2\mu_0$$

$$K = 4a_x$$

What will be the magnetic flux per unit length between the current sheets along the direction of flow of current?
(A) $-24\mu_0 a_y$ Wb/m  (B) $-16\mu_0 a_y$ Wb/m
(C) $-4\mu_0 a_y$ Wb/m  (D) $-40\mu_0 a_y$ Wb/m

MCQ 5.2.28

Two perfectly conducting, infinite plane parallel sheets separated by a distance $d$ carry uniformly distributed surface currents with equal and opposite densities $K$ and $-K$ respectively. The medium between the two plates is a magnetic material of non uniform permeability which varies linearly from a value of $\mu_1$ near one plate to a value of $\mu_2$ near the second plate. What will be the magnetic flux between the current sheets per unit length along the direction of flow of the current?
(A) $\left(\frac{\mu_1 + \mu_2}{2}\right)Kd$  (B) $(\mu_1 + \mu_2)Kd$
(C) $\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)Kd$  (D) $\left(\frac{\mu_2 - \mu_1}{2}\right)Kd$

Statement for Linked Question 29 - 30:

The magnetic field intensity inside an infinite plane magnetic material slab is given as $H = 12a_x + 24a_y$. The permeability of the magnetic material is $\mu = 2\mu_0$

MCQ 5.2.29

If the magnetic material slab occupies the region $0 < z < 2$ m then the magnetization surface current densities at the surfaces $z = 0$ and $z = 2$ m will be respectively
(A) $(-4a_y + 2a_x)$ and $(4a_y - 2a_x)$  (B) $(-2a_x + 4a_y)$ and $(2a_x - 4a_y)$
(C) $(4a_x + 4a_y)$ and $(2a_x - 4a_y)$  (D) $(2a_x + 4a_y)$ and $(-2a_x + 4a_y)$
MCQ 5.2.30  The magnetization volume current density \( J_m \) will be
(A) 0  
(B) \( 4a_x + 2a_y \)  
(C) \( 8a_x + 4a_y \)  
(D) \( -4a_x - 2a_y \)

MCQ 5.2.31  \( B \cdot H \) curve for a ferromagnetic material is given as \( B = 2\mu_0 H H \). What will be the work done per unit volume in magnetizing the material from zero to a certain value \( B_0 = 2\mu_0 H_0^2 \)?
(A) \( 4\mu_0 H_0^3 \)  
(B) \( 4\mu_0 H_0^3 \)  
(C) \( 4\mu_0 H_0^2 \)  
(D) \( H_0^5 / 3 \)

MCQ 5.2.32  Two infinitely long straight wire and a third wire of length \( l \) are parallel to each other located as shown in the figure.

![Diagram](image)

Infinitely long wire carries a current \( I \) while the wire of length \( l \) shown at the top carries a current \( 2I \). The magnitude of the force experienced by the top wire is
(A) \( \frac{\mu I^2}{\pi} \)  
(B) \( \mu \pi I^2 \)  
(C) \( \frac{\mu I^2}{2\pi l} \)  
(D) \( \frac{\mu I^2}{2\pi l} \)

MCQ 5.2.33  Two infinite plane conducting sheets lying in the plane \( x = 0 \) and \( x = 5 \text{ cm} \) carry surface current densities \( +10 \text{ mA/m} \ a_x \) and \( -20 \text{ mA/m} \ a_y \) respectively. If the medium between the plates is a magnetic material of uniform permeability \( \mu = 2\mu_0 \) then what will be the energy stored per unit area of the plates?
(A) \( 800\pi \text{ J/m}^2 \)  
(B) \( 160\pi \text{ J/m}^2 \)  
(C) \( 8\pi \text{ J/m}^2 \)  
(D) \( 1.6\pi \text{ J/m}^2 \)

MCQ 5.2.34  Medium 1 comprising the region \( z > 0 \) is a magnetic material with permeability \( \mu_1 = 4\mu_0 \) where as the medium 2, comprising the region \( z < 0 \) is a magnetic material with permeability \( \mu_2 = 2\mu_0 \). Magnetic flux density in medium 1 is given by
\[ B_1 = (0.4a_x + 0.8a_y + a_z) \text{ Wb/m}^2 \]
If the boundary $z = 0$ between the two media carries a surface current of density $K$ given by

$$K = \frac{1}{\mu_0}(0.2\mathbf{a}_x - 0.4\mathbf{a}_z) \text{ A/m}$$

then the magnetic flux density in medium 2 will be

(A) $(0.8\mathbf{a}_x + 0.8\mathbf{a}_z) \text{ Wb/m}^2$

(B) $(-\mathbf{a}_z + 0.8\mathbf{a}_y - \mathbf{a}_x) \text{ Wb/m}^2$

(C) $(\mathbf{a}_x + 0.8\mathbf{a}_y + \mathbf{a}_z) \text{ Wb/m}^2$

(D) $(\mathbf{a}_z + 0.8\mathbf{a}_y) \text{ Wb/m}^2$

**MCQ 5.2.35** A square loop of a conductor lying in the $yz$ plane is bisected by an infinitely long straight wire carrying current $2 \text{ A}$ as shown in the figure. If the current in the square loop is $4 \text{ A}$ then the force experienced by the loop will be

(A) $3\mathbf{a}_y \mu \text{ N}$

(B) $6.4\mathbf{a}_y \mu \text{ N}$

(C) $3.2\mathbf{a}_y \mu \text{ N}$

(D) $0.64\mathbf{a}_y \mu \text{ N}$

**MCQ 5.2.36** A certain region $z < 0$ comprises a magnetic medium with permeability $\mu = 25\mu_0$. The magnetic flux density in free space ($z > 0$) makes an angle $\theta_1$ with the interface whereas in medium 2 flux density makes an angle $\theta_2$ as shown in the figure.

If $B_2 = 1.2\mathbf{a}_x + 0.8\mathbf{a}_z$ then what will be the angular deflection $(\theta_1 - \theta_2)$?

(A) $50.6^\circ$

(B) $19.47^\circ$

(C) $31.15^\circ$

(D) $12.06^\circ$
Statement for Linked Question 37 - 38:
Consider the magnetic circuit shown in figure.

The cross sectional area of the section on which coil is wound is \( S_1 \) where as all the rest of the section has the cross sectional area \( S_2 \). Magnetic core has the permeability \( \mu = 500\mu_0 \).

**MCQ 5.2.37** If \( S_1 = 5 \text{ cm}^2 \) and \( S_2 = 10 \text{ cm}^2 \) then the total reluctance of the circuit will be
(A) \( 1/100\mu_0 \)  
(B) \( 9/20\mu_0 \)  
(C) \( 20\mu_0/9 \)  
(D) \( 39/100\mu_0 \)

**MCQ 5.2.38** If the no. of turn of the coil is 100 then the equivalent self inductance of the coil is
(A) \( 22.22 \text{ kH} \)  
(B) \( 1.41 \text{ mH} \)  
(C) \( 27.9 \text{ mH} \)  
(D) \( 4.5 \text{ kH} \)

**MCQ 5.2.39** The coil of a magnetic circuit has 50 turns. The core of the circuit has a relative permeability of 600 and length of the core is \( 0.6 \text{ m} \). What must be the core cross section of the magnetic circuit so that the coil may have a \( 0.2 \text{ mH} \) inductance ?
(A) \( 6.4 \text{ cm}^2 \)  
(B) \( 0.64 \text{ cm}^2 \)  
(C) \( 11.94 \text{ cm}^2 \)  
(D) \( 1.56 \text{ cm}^2 \)

Statement for Linked Question 40 - 41:
A System of three coils on an ideal core is shown in figure.
The cross sectional area of all the segments of the core is \( S = 450 \text{ cm}^2 \).

**MCQ 5.2.40** If \( N_1 = 500 \) then what will be the self inductance of the coil having \( N_1 \) turns?
(A) 0.125 mH (B) 62.8 mH  
(C) 31.41 mH (D) 6.28 mH

**MCQ 5.2.41** If \( N_2 = 250 \) then the self inductance of the coil \( N_2 \) will be
(A) 2.6 mH (B) 23.6 mH  
(C) 70.7 mH (D) 2.36 mH

**MCQ 5.2.42** A system of three coils on an ideal core that has two air gaps is shown in the figure.

All the segments of core has the uniform cross sectional area 2500 \( \text{ mm}^2 \).
What will be the mutual inductance between \( N_1 \) and \( N_2 \) ?
(A) 39.27 mH (B) 52.36 mH  
(C) 18 mH (D) 78.54 mH

**MCQ 5.2.43** The mutual inductance between \( N_2 \) and \( N_3 \) will be
(A) 0 (B) 78.54 mH  
(C) 52.36 mH (D) 39.27 mH

**MCQ 5.2.44** The magnetization curve for an iron alloy is approximately given by
\[
B = \frac{1}{3} H + H^2 \mu \text{Wb/m}^2
\]
If \( H \) increases from 0 to 210 A/m, the energy stored per unit volume in the alloy is
(A) 6.2 MJ/m³ (B) 1.3 MJ/m³  
(C) 2.3 kJ/m³ (D) 2.9 kJ/m³
EXERCISE 5.3

MCQ 5.3.1
GATE 2011
A current sheet $J = 5a_y$ A/m lies on the dielectric interface $x = 0$ between two dielectric media with $\varepsilon_1 = 5$, $\mu_1 = 1$ in Region-1 ($x < 0$) and $\varepsilon_2 = 2$, $\mu_2 = 2$ in Region-2 ($x > 0$). If the magnetic field in Region-1 at $x = 0^-$ is $H_1 = 30a_y$ A/m, the magnetic field in Region-2 at $x = 0^+$ is

- (A) $H_2 = 1.5a_x + 30a_y - 10a_z$ A/m
- (B) $H_2 = 3a_x + 30a_y - 10a_z$ A/m
- (C) $H_2 = 1.5a_x + 40a_y$ A/m
- (D) $H_2 = 3a_x + 30a_y + 10a_z$ A/m

MCQ 5.3.2
IES EC 2012
A bar magnet made of steel has a magnetic moment of 2.5 A·m² and a mass of $6.6 \times 10^{-3}$ kg. If the density of steel is $7.9 \times 10^3$ kg/m³, the intensity of magnetization is

- (A) $8.3 \times 10^{-7}$ A/m
- (B) $3 \times 10^6$ A/m
- (C) $6.3 \times 10^{-7}$ A/m
- (D) $8.2 \times 10^6$ A/m

MCQ 5.3.3
IES EC 2011
If the current element represented by $2 \times 10^{-4} a_y$ Amp-m is placed in a magnetic field of $H = 5a_y/\mu$ A/m, the force on the current element is

- (A) $-2.0a_z$ mN
- (B) $2.0a_z$ mN
- (C) $-2.0a_z$ N
- (D) $2.0a_z$ N

MCQ 5.3.4
IES EC 2011
Match List I with List II and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. MMF</td>
<td>1. Conductivity</td>
</tr>
<tr>
<td>b. Magnetic flux</td>
<td>2. Electric current</td>
</tr>
<tr>
<td>c. Reluctance</td>
<td>3. EMF</td>
</tr>
<tr>
<td>d. Permeability</td>
<td>4. Resistance</td>
</tr>
</tbody>
</table>

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 5.3.5
IES EC 2008
Consider the following statements associated with boundary conditions between two media:
1. Normal component of B is continuous at the surface of discontinuity.
2. Normal component of D may or may not be continuous.
Which of the statement(s) given above is/are correct?
(A) 1 only 
(B) 2 only 
(C) Both 1 and 2 
(D) Neither 1 nor 2

MCQ 5.3.6
IES EC 2007
Magnetic current is composed of which of the following?
(A) Only conduction component
(B) Only displacement component
(C) Both conduction and displacement components
(D) Neither conduction component nor displacement component

MCQ 5.3.7
IES EC 2006
Which one of the following is the correct expression for torque on a loop in magnetic field \( \mathbf{B} \)? (Here \( \mathbf{M} \) is the loop moment)
(A) \( \mathbf{T} = \nabla \cdot \mathbf{B} \)
(B) \( \mathbf{T} = \mathbf{M} \cdot \mathbf{B} \)
(C) \( \mathbf{T} = \mathbf{M} \times \mathbf{B} \)
(D) \( \mathbf{T} = \mathbf{B} \times \mathbf{M} \)

MCQ 5.3.8
IES EC 2006
Match List I with List II and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Line charge</td>
<td>1. Maxwell</td>
</tr>
<tr>
<td>b. Magnetic flux density</td>
<td>2. Poynting vector</td>
</tr>
<tr>
<td>c. Displacement current</td>
<td>3. Biot-Savart’s law</td>
</tr>
<tr>
<td>d. Power flow</td>
<td>4. Gauss’s law</td>
</tr>
</tbody>
</table>

Codes:

(A) 1 2 4 3
(B) 4 3 1 2
(C) 1 3 4 2
(D) 4 2 1 3
MCQ 5.3.9
IES EE 2009
IES EC 2006
What does the expression $\frac{1}{2} \mathbf{J} \cdot \mathbf{A}$ represent?
(A) Electric energy density  
(B) Magnetic energy density  
(C) Power density  
(D) Radiation resistance

MCQ 5.3.10
IES EC 2003
Two thin parallel wires are carrying current along the same direction. The force experienced by one due to the other is
(A) Parallel to the lines  
(B) Perpendicular to the lines and attractive  
(C) Perpendicular to the lines and repulsive  
(D) Zero

MCQ 5.3.11
IES EC 2001
A boundary separates two magnetic materials of permeability $\mu_1$ and $\mu_2$. The magnetic field vector in $\mu_1$ is $H_1$ with a normal component $H_{1n}$ and tangential component $H_{1t}$ while that in $\mu_2$ is $H_2$ with a normal component $H_{2n}$ and a tangential component $H_{2t}$. Then the derived conditions would be
(A) $H_1 = H_2$ and $H_{1n} = H_{2n}$  
(B) $H_{1n} = H_{2n}$ and $\mu_1 H_{1t} = \mu_2 H_{2t}$  
(C) $H_1 = H_2$ and $\mu_1 H_{1n} = \mu_2 H_{2n}$  
(D) $H_1 = H_2$, $H_{1t} = H_{2t}$ and $\mu_1 H_{1n} = \mu_2 H_{2n}$

MCQ 5.3.12
IES EE 2012
The dependence of $B$ (flux density) on $H$ (magnetic field intensity) for different types of material is

MCQ 5.3.13
IES EE 2012
Statement I: Polarization is due to the application of an electric field to dielectric materials.
Statement II: When the dipoles are created, the dielectric is said to be polarized or in a state of polarization.

(A) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)

(B) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)

(C) Statement (I) is true but Statement (II) is false

(D) Statement (I) is false but Statement (II) is true

MCQ 5.3.14

The following equation is not valid for magneto-static field in inhomogenous magnetic materials

(A) \( \nabla \cdot B = 0 \)

(B) \( \nabla \cdot H = 0 \)

(C) \( \nabla \times A = B \)

(D) \( \nabla \times H = J \)

MCQ 5.3.15

Assertion (A): Superconductors cannot be used as coils for production of strong magnetic fields.

Reason (R): Superconductivity in a wire may be destroyed if the current in the wire exceeds a critical value.

(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)

(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)

(C) Assertion (A) is true but Reason (R) is false

(D) Assertion (A) is false but Reason (R) is true

MCQ 5.3.16

A conductor 2 metre long lies along the z-axis with a current of 10 A in \( a_z \) direction. If the magnetic field is \( B = 0.25 a_z \) T, the force on the conductor is

(A) 4.0 \( a_y \) N

(B) 1.0 \( a_y \) N

(C) 1.0 \( a_y \) N

(D) 3.0 \( a_y \) N

MCQ 5.3.17

The force on a charge moving with velocity \( v \) under the influence of electric and magnetic fields is given by which one of the following?

(A) \( q(E + B \times v) \)

(B) \( q(E + v \times H) \)

(C) \( q(H + v \times E) \)

(D) \( q(E + v \times B) \)

MCQ 5.3.18

If a very flexible wire is laid out in the shape of a hairpin with its two ends secured, what shape will the wire tend to assume if a current is passed through it?

(A) Parabolic

(B) Straight line

(C) Circle

(D) Ellipse
MCQ 5.3.19  
IES EE 2007.  
Consider the following:  
Lorentz force \( F = e(v \times B) \) where \( e, v \) and \( B \) are respectively the charge of the particle, velocity of the particle and flux density of uniform magnetic field. Which one of the following statements is not correct?  
(A) Acceleration is normal to the plane containing the particle path and \( B \)  
(B) If the direction of the particle path is normal to \( B \), the acceleration is maximum  
(C) If the particle is at rest, the field will deflect the particle  
(D) If the particle path is in the same direction of \( B \), there will be no acceleration

MCQ 5.3.20  
IES EE 2006  
What is the force on a unit charge moving with velocity \( v \) in presence of electric field \( E \) and magnetic field \( B \)?  
(A) \( E - v \cdot B \)  
(B) \( E + v \cdot B \)  
(C) \( E + B \times v \)  
(D) \( E + v \times B \)

MCQ 5.3.21  
IES EE 2004  
What is the force experienced per unit length by a conductor carrying 5 A current in positive \( z \)-direction and placed in a magnetic field \( B = (3a_x + 4a_y) \)?  
(A) \( 15a_x + 20a_y \) N/m  
(B) \( -a_x + 15a_y \) N/m  
(C) \( 20a_x - 15a_y \) N/m  
(D) \( -20a_x - 20a_y \) N/m

MCQ 5.3.22  
IES EE 2003  
Which one of the following formulae is not correct for the boundary between two magnetic materials?  
(A) \( B_{n1} = B_{n2} \)  
(B) \( B_z = \sqrt{B_{z1}^2 + B_{z2}^2} \)  
(C) \( H_z = H_{n1} + H_{n2} \)  
(D) \( a_{n2} \times (H_1 - H_2) = K \) where \( a_{n2} \) is a unit vector normal to the interface and directed from region 2 to region 1.

MCQ 5.3.23  
IES EE 2003  
Interface of two regions of two magnetic materials is current-free. The region 1, for which relative permeability \( \mu_{r1} = 2 \) is defined by \( z < 0 \), and region 2, \( z > 0 \) has \( \mu_{r2} = 1 \). If \( B_1 = 1.4a_x + 2.2a_y + 1.4a_z \) T; then \( H_2 \) is  
(A) \( 1/\mu_0[0.6a_x + 0.4a_y + 0.4a_z] \) A/m  
(B) \( 1/\mu_0[1.2a_x + 0.8a_y + 0.8a_z] \) A/m  
(C) \( 1/\mu_0[1.2a_x + 0.4a_y + 0.4a_z] \) A/m  
(D) \( 1/\mu_0[0.6a_x + 0.4a_y + 0.8a_z] \) A/m

MCQ 5.3.24  
IES EE 2002  
If \( A \) and \( J \) are the vector potential and current density vectors associated with a coil, then \( \int A \cdot J \, dv \) has the units of  
(A) flux-linkage  
(B) power  
(C) energy  
(D) inductance

**********
**SOLUTIONS 5.1**

**SOL 5.1.1** Option (A) is correct.
Force applied by a magnetic field \( B \) on a moving charge with velocity \( v \) is defined as

\[
F = v \times B
\]

Since the direction of velocity \( v \) and \( B \) are perpendicular to each other as obtained from the shown figure so the resultant force will be perpendicular to both of them. i.e. the force on the moving charged particle will be in upward direction. and as the particle is also deflected in upward direction with the applied force so it gives the conclusion that the particle will be positively charged.

**SOL 5.1.2** Option (D) is correct.
Since a magnet bar must have south and north pole i.e. a single pole charge can’t exist. So, a magnetic point charge doesn’t exit.

**SOL 5.1.3** Option (B) is correct.
Force applied on a moving charge in the presence of electric and magnetic field is defined as

\[
F = F_e + F_m = q(E + v \times B)
\]

where \( F_e \) and \( F_m \) are the electric and magnetic forces applied on the charge so it is clear that the moving charge experiences both the electric and magnetic forces. The electric force is applied in a uniform direction (in direction of electric field) i.e. it is an accelerating force while, the magnetic force is applied in the normal direction of both the magnetic field and velocity of the charged particle i.e. it is a deflecting force.

Therefore, both the options are correct but R is not the correct explanation of A.

**SOL 5.1.4** Option (A) is correct.
For a moving charge \( Q \) in the presence of both electric and magnetic fields, the total force on the charge is given by

\[
F = Q[E + (v \times B)]
\]

where

- \( E \rightarrow \) electric field
- \( v \rightarrow \) velocity of the charge
- \( B \rightarrow \) magnetic flux density

Since the electron beam follows its path without any deflection so the net force applied by the field will be zero.
For View Only

\[ Q\left[ E + (v \times B) \right] = 0 \]
\[ 15a_x + v \times 3a_y = 0 \]

As the electric field is directed along \(a_x\) and magnetic field is directed along \(a_y\) so the velocity of beam will be in \(a_z\) direction (perpendicular to both of the field).

Consider the velocity of the beam is \(V = ka_z\)

So we have
\[ 15a_x + ka_y \times 3a_z = 0 \]
\[ 15a_x - 3ka_y = 0 \]
\[ k = \frac{15}{3} = 5 \text{ m/s} \]

So, the velocity of the beam will be 5 m/s along the \(x\)-axis.

SOL 5.1.5
Option (C) is correct.

For a moving charge \(Q\) in the presence of both electric and magnetic fields, the total force on the charge is given by
\[ F = Q\left[ E + (v \times B) \right] \]

where \(E\) \(\rightarrow\) electric field 
\(v\) \(\rightarrow\) velocity of the charged particle 
\(B\) \(\rightarrow\) magnetic flux density

So, at time \(t = 0\) total force applied on the electron is
\[ F(0) = e\left[ E + (V(0) \times B) \right] \]

Now we have
\[ V(0) \times B = (200a_x - 300a_y - 400a_z) \times (-3a_x + 2a_y - a_z) \]
\[ = 1100a_x + 1400a_y - 500a_z \]

therefore the applied force on the electron is
\[ F(0) = 1.6 \times 10^{-19}\left[ (0.1a_x - 0.2a_y + 0.3a_z) \times 10^3 + 1100a_x + 1400a_y - 500a_z \right] \]
\[ m_e a(0) = 1.6 \times 10^{-19}\left[ (100 + 1100)a_x + (1400 - 200)a_y + (300 - 500)a_z \right] \]
\[ (F(0) = m_e a(0), \text{ where } a(0) \text{ is acceleration of electron at } t = 0) \]
\[ a(0) = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-30}} \times 200(6a_x + 6a_y - a_z) \]
\[ = 6.5 \times 10^{13}(7a_x + 2a_y - a_z) \text{ m/s}^2 \]

SOL 5.1.6
Option (B) is correct.

Force \(F\) applied on a current element in the presence of magnetic flux density \(B\) is defined as
\[ F = I(L \times B) \]

where \(I\) \(\rightarrow\) current flowing in the element 
\(L\) \(\rightarrow\) vector length of current element in the direction of current flowing

So,
\[ F = 3 \times 10^{-3}\left[ 2a_x \times (a_x + 3a_y) \right] \]
\[ = 6 \times 10^{-3}\left[ a_x - 3a_y \right] = -14a_x + 16a_y \text{ mN} \]

SOL 5.1.7
Option (A) is correct.

The magnitude of the force experienced by either of the loops will be same but the direction will be opposite.

So the force experienced by \(C_1\) due to \(C_2\) will be \(-F\).

SOL 5.1.8
Option (C) is correct.
The boundary condition for the current interface holds the following results.

(i) normal component of magnetic flux density is continuous.

\[ B_{n1} = B_{n2} \]

(ii) Tangential component of magnetic field intensity is continuous.

\[ H_{t1} = H_{t2} \]

So, (A) and (B) are wrong statement. Now, we check the statement (C).

Consider the magnetic field intensity in 1st medium is \( H_1 \) and magnetic field intensity in 2nd medium is \( H_2 \). So, it’s tangential component will be equal

\[ H_{t1} = H_{t2} \]

(tangential component)

Since scalar magnetic potential difference is defined as the line integral of magnetic field intensity

\[ V_1 - V_2 = \int H \cdot dl = I \]

and since there is no current density at boundary.

So, we have

\[ V_1 - V_2 = 0 \]

\[ V_1 = V_2 \]

i.e. magnetic scalar potential will be same in both medium.

**SOL 5.1.9**

Option (C) is correct.

The magnetic field intensity produced at any point in the free space will be the vector sum of the field intensity produced by all the current sheets.

Since, the magnetic field intensity produced at any point \( P \) due to an infinite sheet carrying uniform current density \( K \) is defined as

\[ H = \frac{1}{2} (K \times a) \]

where \( a \) is the unit vector normal to the sheet directed toward the point \( P \). So in the region \( 0 < z < 1 \) magnetic field intensity due to \( K_2 \) and \( K_3 \) will be cancelled as the unit normal vector to the two sheets will be opposite to each other.

Therefore in this region magnetic field intensity will be produced only due to the current density \( K_1 = 4a \), which is given as

\[ H = \frac{1}{2} K_1 \times a = \frac{1}{2} (4a_z) \times (a_z) \]

\[ = -4a_z \text{ A/m} \]

**SOL 5.1.10**

Option (D) is correct.

As the conducting filament is located along the line \( y = 0, z = 0.2 \text{ m} \) which is in the region \( 0 < z < 1 \text{ m} \), so, the net magnetic field intensity produced on the conducting filament by the current sheets is

\[ H = -2a_y \text{ A/m} \] (as determined in previous question)

or,

\[ B = \mu_0 H = -2\mu_0 a_y \]

Now the force experienced by a current element \( Idl \) in the presence of magnetic flux density \( B \) is defined as

\[ dF = Idl \times B \]

where \( I \) is the current flowing in the element and \( dl \) is the differential vector length
of the current element in the direction of flow of current.
So force per unit length experienced by the conducting filament is
\[ \frac{dF}{dl} = 7a_x \times (-2\mu_0 a_y) \quad (I = 7 \text{ A}, \ \ dl = dlla_z) \]
\[ = -14\mu_0 a_z \text{ N/m} \]

SOL 5.1.11 Option (B) is correct.
Magnetic dipole moment of a coil carrying current \( I \) and having area \( S \) is given by
\[ m = ISa_n \]
where \( a_n \) is normal vector to the surface of the loop.
Since the coil is lying in the plane \( 2x + 6y - 3z = 4 \) so the unit vector normal to the plane of the coil is given as.

So,
\[ a_n = \nabla f = \left[ \frac{2a_x + 6a_y - 3a_z}{\sqrt{2^2 + 6^2 + (-3)^2}} \right] \quad (f = 2x + 6y - 3z) \]

Therefore the magnetic dipole moment of the coil is
\[ m = \left( 5 \right) \left( \frac{2a_x + 6a_y - 3a_z}{7} \right) \quad (I = 5 \text{ A}, \ S = 1 \text{ m}^2) \]
\[ = \frac{5(2a_x + 6a_y - 3a_z)}{7} \]

As the torque a magnetic field \( B \) on the loop having magnetic moment \( m \) is defined as
\[ T = m \times B \]
So the torque on the given coil is
\[ T = \left[ \frac{5(2a_x + 6a_y - 3a_z)}{7} \right] \times (6a_x + 4a_y + 5a_z) \]
\[ = 3a_x - 5a_y - 8a_z \text{ N-m} \]

SOL 5.1.12 Option (B) is correct.
Magnetic dipole moment of a coil of area \( S \) carrying current \( I \) is defined as
\[ m = ISa_n \]
where \( a_n \) is the unit vector normal to the surface of the loop.
and since from the given data we have
\[ I = 10 \text{ A} \]
\[ S = \pi r^2 = \pi \times 1^2 = \pi \]
\[ a_n = a_z \quad \text{(normal vector to the surface \( z = 0 \))} \]
So the magnetic moment of the circular current loop lying in the plane \( z = 0 \) is
\[ m = 10\pi a_z \]

Now the torque on an element having magnetic moment \( m \) in the presence of magnetic flux density \( B \) is defined as
\[ T = m \times B \]
Therefore, the torque acting on the circular loop is
\[ T = (10\pi a_z) \times (4a_x - 4a_y - 2a_z) \quad (B = 4a_x - 4a_y - 2a_z) \]
\[ = 10\pi(4a_y + 4a_z) = 30\pi(a_y + a_z) \]
For View Only

SOL 5.1.13  Option (B) is correct.

SOL 5.1.14  Option (C) is correct.
A diamagnetic material carries even no. of electrons inside it’s atom.
Number of electron in carbon atom is six.
Which is even so it is a diamagnetic material rest of the material having odd no.
of electrons.

SOL 5.1.15  Option (A) is correct.
A paramagnetic material have an odd no. of electrons and since atomic no. of Al is
13, which is odd. So, it is a paramagnetic material.
So, A and R both true and A is correct explanation of R.

SOL 5.1.16  Option (B) is correct.
In a magnetic medium the magnetization in terms of magnetic field intensity is
defined as
\[ M = \chi_m H \]
where \( \chi_m \) is magnetic susceptibility given as
\[ \chi_m = \mu_r - 1 = 1.3 \]
(relative permeability, \( \mu_r = 2.3 \))
and since the magnetic field intensity in terms of magnetic flux density is given as
\[ H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} \]
\[ = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 2.3} a_0 = 1730 a_0 \text{ A/m} \]
\[ (\mu_r = 2.3) \]
So the magnetization inside the medium is
\[ M = \chi_m H = 2249 \text{ A/m} \]

SOL 5.1.17  Option (D) is correct.
Given the permeability, \( \mu = 3 \mu_0 \) and magnetic flux density = \( B \)
So the field intensity inside the material will be
\[ H = \frac{B}{\mu} = \frac{B}{3\mu_0} \]
Since the magnetization of a magnetic material is defined as
\[ M = \frac{B}{\mu_0} - \frac{B}{3\mu_0} \]
where \( B \) and \( H \) are the flux density and field intensity inside the material. So we get
\[ M = \frac{B}{\mu_0} - \frac{B}{3\mu_0} = \frac{2B}{3\mu_0} \]

SOL 5.1.18  Option (D) is correct.
As the magnetic flux density and magnetic field intensity inside a magnetic material
are related as
\[ B = \mu_r \mu_0 H \]
So, comparing it with given expression for magnetic flux density we get the relative
permeability as
\[ \mu_r = k = k - 1 \]
Therefore, the magnetization vector inside the material is given as 
\[ \mathbf{M} = (\mu_r - 1) \mathbf{H} = (k - 1) \mathbf{H} \]

**SOL 5.1.19** Option (D) is correct.

Magnetic flux density in a medium in terms of magnetic field intensity is defined as 
\[ \mathbf{B} = \mu_0 \mathbf{H} = \mu_r \mu_0 \mathbf{H} \]
\[ = (4/\pi)(4\pi \times 10^{-7})(2\rho^2 \mathbf{a_o}) \quad (\mu_r = 4/\pi, \quad \mathbf{H} = 2\rho^2 \mathbf{a_o} \text{ A/m}) \]
\[ = 16 \times 10^{-7} \rho^2 \mathbf{a_o} \]
Again the magnetic flux density inside a magnetizing material is defined as 
\[ \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \]
where \( \mathbf{M} \) is the magnetization of the material. So, we have
\[ \mathbf{M} = \mu_0 \mathbf{B} \]
\[ = \frac{32 \times 10^{-7} \rho^2 - 2\rho^2 \mathbf{a_o}}{4\pi \times 10^{-7} - 2\rho^2 \mathbf{a_o}} \]
\[ = 2\rho^2 \left[ \frac{4}{\pi} - 1 \right] \]
At \( \rho = 2 \)
\[ \mathbf{M} = 4.14 \mathbf{a_o} \text{ A/m} \]

**SOL 5.1.20** Option (A) is correct.

Given
Magnetic field intensity, \( \mathbf{H} = 70 \text{ A/m} \)
Total magnetic flux in the bar, \( \Phi = 4.2 \text{ mWb} \)
Cross sectional area of bar, \( S = 2 \text{ m}^2 \)
So we have the magnetic flux density in the bar
\[ \mathbf{B} = \frac{\Phi}{S} = \frac{4.2 \times 10^{-3}}{2} \]
\[ = 2.1 \text{ mWb/m}^2 \]
Since the magnetic field intensity and magnetic flux density are related as 
\[ \mathbf{B} = \mu_0(1 + \chi_m) \mathbf{H} \]
So, we have
\[ 2.1 \times 10^{-3} = \frac{(2.1 \times 10^{-3})}{70(4\pi \times 10^{-7})} \]
\[ (1 + \chi_m) = \frac{(2.1 \times 10^{-3})}{70(4\pi \times 10^{-7})} \]
\[ \chi_m = \left( \frac{3 \times 10^{-5}}{4\pi \times 10^{-7}} - 1 \right) \]
\[ = (23.87 - 1) = 22.97 \]

**SOL 5.1.21** Option (B) is correct.

For the spherical cavity of magnetization \( \mathbf{M} \), the flux density is given by
\[ \mathbf{B}_{\text{cavity}} = \frac{2}{3} \mu_0 \mathbf{M} \]
Since the cavity is hollowed. So not magnetic flux density at the center of cavity is
\[ \mathbf{B}_{\text{net}} = \mathbf{B}_0 - \mathbf{B}_{\text{cavity}} = \mathbf{B}_0 - \frac{2}{3} \mu_0 \mathbf{M} \]
and so the net magnetic field intensity at the center of cavity is
\[ \mathbf{H}_{\text{net}} = \frac{1}{\mu_0} \mathbf{B}_{\text{net}} = \frac{1}{\mu_0} \left[ \mathbf{B}_0 - \frac{2}{3} \mu_0 \mathbf{M} \right] \]
\[ = \frac{1}{\mu_0} \left[ \mu_0 \mathbf{H}_0 + \mu_0 \mathbf{M} - \frac{2}{3} \mu_0 \mathbf{M} \right] = \left[ \mathbf{H}_0 + \frac{\mathbf{M}}{3} \right] \quad (\mathbf{B}_0 = \mu_0(\mathbf{H}_0 + \mathbf{M})) \]
Option (B) is correct.

In a magnetic medium the magnetic field intensity and magnetic flux density are related as

\[ B = \mu_0 (1 + \chi_m) H \]

So the magnetic flux density inside the medium is

\[ H = \frac{B}{(1 + \chi_m) \mu_0} = \frac{4z}{3\mu_0} a_x \]

(B = 4z\(a_x\) T, \(\chi_m = 2\))

Now the magnetization of a magnetic medium having magnetic field intensity \(H\) is given as

\[ M = \chi_m H \]

\[ M = 2\left(\frac{4z}{3\mu_0}\right) a_x = \frac{8z}{3\mu_0} a_x \]

The bound current density inside a medium having magnetization \(M\) is given as

\[ J_b = \nabla \times M \]

\[ = \nabla \times \left(\frac{8z}{3\mu_0} a_x\right) = \frac{12}{3\mu_0} a_y \text{ A/m}^2 \]

Option (A) is correct.

Total current density inside a medium having magnetic flux density \(B\) is given as

\[ J_T = \frac{\nabla \times B}{\mu_0} = \frac{1}{\mu_0} \left[ \frac{\partial (4z)}{\partial z}\right] a_y \]

\[ = 2a_y \text{ A/m}^2 \]

(B = 4z\(a_x\) T)

Option (C) is correct.

Volume current density inside a material is equal to the curl of magnetization \(M\) i.e.

\[ J = \nabla \times M \]

and the surface current density in terms of magnetization is defined as

\[ K = M \times a_n \]

where \(a_n\) is unit vector normal to the surface. Consider the cylinder is placed along \(z\)-axis

So, \[ a_n = a_p \]

and \[ M = Ma_z \]

Therefore the volume current density inside the cylinder is

\[ J = \nabla \times (Ma_z) = 0 \]

\((M\) is not the function of \(z\))
and the surface current density of the cylinder is

\[ K = M a_z \times a_\theta = M a_\phi \]

So the current flowing in cylinder is just similar to a solenoid and the field intensity produced due to a solenoid at any point outside it is zero. Thus we have the magnetic field intensity outside the cylinder as

\[ H_{\text{outside}} = 0 \]

**SOL 5.1.25** Option (C) is correct.

Volume current density inside a material is equal to the curl of magnetization \( M \) i.e.

\[ J = \nabla \times M \]

So the volume current density inside the cylinder is

\[ J = \nabla \times (0.7a_z) = 0 \quad (M = 0.7a_z \text{ A/m}) \]

and since the surface current density in terms of magnetization is defined as

\[ K = M \times a_n \quad \text{where} \quad a_n \text{ is unit vector normal to the surface.} \]

So the surface current density of the cylinder is

\[ K = (0.7a_z) \times a_\theta = 0.7a_\phi \quad (M = 0.7a_z \text{ A/m}, \ a_n = a_\phi) \]

Therefore the current flowing in cylinder is just similar to a solenoid and the field intensity produced due to a solenoid at any point inside it is given as

\[ B = \mu_0 K = \mu_0 nI \]

where \( n \) is the no. of turns per unit length of the solenoid and \( I \) is the current flowing in the solenoid.

Thus, the magnetic flux density inside the cylinder is (direction is determined by right hand rule)

\[ B = 0.7\mu_0 a_z = 2.8 \times 10^{-7} a_z \quad (K = 0.7) \]

**SOL 5.1.26** Option (D) is correct.

From Snells law we have the relation between the incidence and refracted angle of magnetic flux lines as

\[ \frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_1}{\mu_2} \]

where \( \mu_1 \) and \( \mu_2 \) are relative permeability of the two medium.
\[
\frac{\tan 75^\circ}{\tan \alpha_2} = \frac{600}{1} \quad \text{(relative permeability of air } = 1) \\
\tan \alpha_2 = \frac{\tan 75^\circ}{600} \\
\alpha_2 = \tan^{-1} \left( \frac{\tan 75^\circ}{600} \right) = 0.356^\circ 
\]

**SOL 5.1.27**
Option (C) is correct.

Magnetic field intensity in 1st medium is given
\[
H_1 = 9a_x + 16a_y - 10a_z = H_{1t} + H_{1n}
\]
where \(H_{1t}\) and \(H_{1n}\) are respectively the tangential and normal components of the magnetic field intensity to the boundary interface in medium 1.

From boundary condition we have
\[
H_{1t} = H_{2t} \\
\mu_2 H_{2n} = \mu_1 H_{1n}
\]
where \(H_{2t}\) and \(H_{2n}\) are respectively the tangential and normal component of magnetic field intensity in medium 2. So we get the components in medium 2 as
\[
H_{2t} = 9a_x - 10a_z \\
H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = \frac{\mu_1 + \mu_0}{\mu_2 + \mu_0} H_{1n}
\]

Therefore, the net magnetic field intensity in medium 2 is
\[
H_2 = H_{2t} + H_{2n} = 12a_x + 13.67a_y - 15a_z \text{ A/m}
\]

**SOL 5.1.28**
Option (B) is correct.

Magnetic flux density in any medium in terms of magnetic field intensity is defined as
\[
B = \mu H
\]
where \(\mu\) is the permeability of the medium. So, the magnetic flux density in medium 2 is given as
\[
B_2 = \mu_2 H_2 = \frac{\mu_2 \mu_0}{\mu_2 + \mu_0} H_2
\]

\[
= 6 \times (4\pi \times 10^{-7}) \times (9a_x + 18.67a_y - 10a_z) \quad (\mu_2 = 6)
\]

\[
= (6.8a_x + 14.1a_y - 7.5a_z) \times 10^{-5} \text{ wb/m}^2
\]

**SOL 5.1.29**
Option (D) is correct.

The magnetic flux density in region \(z < 0\) is given as
\[
B = 4a_x + 3a_z \text{ Wb/m}^2
\]
Now we consider the flux density in region 1 is $B_1$. So, we have
\[ B_1 = 4a_x + 3a_z. \]
Therefore the tangential component $B_{1t}$ and normal component $B_{1n}$ of the magnetic flux density in region 1 are
\[ B_{1t} = 4a_x \]
and
\[ B_{1n} = 3a_z. \]
From the boundary condition the tangential and normal components of magnetic flux density in two mediums are related as
\[ B_{1n} = B_{2n} \]
\[ B_{2t} - B_{1t} = \mu_0 K \]
where $B_{2t}$ and $B_{2n}$ are respectively the tangential and normal components of the magnetic flux density in region 2 and $K$ is the current density at the boundary interface.
So, we get
\[ B_{2n} = B_{1n} = 3a_z \]
\[ B_{2t} = 4a_x + \mu_0 (4a_z) \quad (B_{1t} = 4a_x, K = 4a_x \text{ A/m}) \]
Therefore the net flux density in region 2 ($z > 0$) is
\[ B_2 = B_{2t} + B_{2n} = 4\mu_0 a_x + 2a_y + 5a_z. \]

SOL 5.1.30 Option (D) is correct.

As the surface boundary of the slab is parallel to $yz$-plane so the given magnetic flux density will be tangential to the surface.
i.e.
\[ B_{1o} = B_0 \]
and
\[ H_{1o} = \frac{B_{1o}}{\mu_0} = \frac{P}{I_{1o}}. \]
Since the tangential component of magnetic field intensity is uniform at the boundary of the magnetic material So, magnetic field intensity inside the material is
\[ H_{in} = H_{1o} = \frac{B_0}{\mu_0} \]
Therefore, the flux density inside the material is
\[ B_{in} = \mu H_{in} = \mu_x \frac{B_0}{\mu_0} = \mu_x B_0. \]
The magnetic stored energy per unit volume of the plate for a given uniform flux density (uniform permeability) is defined as

\[ w_m = \frac{1}{2} H \cdot B \]

Given

\[ B = (3a_x + 4a_y) \times 10^{-3} \text{ Wb/m}^2 \]

So we have,

\[ H = \frac{B}{\mu} = \left( \frac{3a_x + 4a_y}{4\mu_0} \right) \times 10^{-3} \text{ A/m} \]

and therefore

\[ w_m = \frac{1}{2} H \cdot B = \frac{1}{2} \left( \frac{9 + 16}{4\mu_0} \right) \times 10^{-6} = \frac{25 \times 10^{-6}}{8 \times 4\pi \times 10^{-7}} = 2.49 \text{ J/m}^3 \]

Now the separation between the plates is given as \( d = 2 \text{ m} \)

Thus magnetic energy stored per unit area of the plate is

\[ W_m/A = w_m \times d = (2.49) \times 2 = 5.1 \text{ J/m}^2 \]

Option (C) is correct.

From boundary condition the normal component of flux density is uniform at boundary

\[ B_{1n} = B_{2n} \]

\[ B_{1n} \sin \theta_1 = B_{2n} \sin \theta_2 \]

and the tangential component of field intensity is uniform

\[ H_{1t} = H_{2t} \]

\[ H_{1t} \cos \theta_1 = H_{2t} \cos \theta_2 \]

Option (D) is correct.

Relation between \( \theta_1 \) and \( \theta_2 \) at boundary of region (1) and region (2) as

\[ \mu_1 \tan \theta_1 = \mu_2 \tan \theta_2 \]

and at the interface between region (2) and region (3) is

\[ \mu_2 \tan \theta_2 = \mu_3 \tan \theta_3; \quad \text{since} \quad \theta_2 = \theta_3 \]

So, combining the two eq. we get,

\[ \mu_1 \tan \theta_1 = \mu_3 \tan \theta_2 \]

Thus, \( \theta_1 \) will be independent of \( \mu_2 \) only.

Option (B) is correct.

Internal inductance of a loop of radius \( r \) is defined as

\[ L_in = \frac{\mu_0}{8\pi} \frac{(2\pi r)^2}{8\pi} = \frac{4\pi \times 10^{-7} \times 2\pi \times 50 \times 10^{-2}}{8\pi} = 157.1 \text{ nH} \]

Option (A) is correct.

From the analogy between electrical and magnetic circuits, we have the following relations,

\[ F \quad \text{(magnetomotive force)} \rightarrow V \quad \text{(voltage)} \]

\[ \phi \quad \text{(magnetic flux)} \rightarrow I \quad \text{(current)} \]
\( \mathbf{\mathcal{R}} \) (Reluctance) \( \rightarrow \) \( R \) (Resistance)

now, magnetomotive force,

\[ \mathcal{F} = \frac{\eta I}{\mathcal{R}} \]

and so, the electrical analog of the magnetic circuit is

\[ = \]

**SOL 5.1.36** Option (A) is correct.

For drawing the electrical analog replace the coil by a source (magnetomotive force) and each section of the core by a reluctance. In the shown magnetic material there are 9 sections so we draw the reluctance for each of them and we get the magnetomotive force as

\[ \mathcal{F} = 1000I \]  
\( (N = 1000) \)

So the equivalent circuit is

\[ = \]

**SOL 5.1.37** Option (B) is correct.

Given that,

- magnetic flux density, \( B = 0.4 \text{ T} \)
- no. of turns of coil, \( N = 200 \)
- length of magnetic core, \( l = 15 \text{ cm} = 15 \times 10^{-2} \)
- permeability of the core, \( \mu_r = 150 \)

So, current required to produce the given magnetic field is

\[ i = \frac{Bl}{\mu N} = \frac{(0.4)(15 \times 10^{-2})}{(150\mu_0)(200)} \]

\[ = 2.6 \text{ A} \]

************
SOL 5.2.1 Option (A) is correct.
Consider the particle carries a total charge $Q$.
Since for a moving charge $Q$ in the presence of both electric and magnetic fields, the total force on the charge is given by
\[ F = Q[E + (v \times B)] \]
where $E \rightarrow$ electric field
$v \rightarrow$ velocity of the charged particle
$B \rightarrow$ magnetic flux density
So initially the magnetic force on the particle will be zero as the particle is released at rest ($v = 0$). Therefore the electric field will accelerate the particle in $y$-direction and as it picks up speed (consider the velocity is $v = ka_y$, $k$ is very small) a magnetic force develops which will be given by
\[ F = v \times B \]
since the magnetic field is in $a_z$ direction while the beam has the velocity in $a_y$ direction so the magnetic force will be in $a_x (a_y \times a_z)$ direction.
Therefore the magnetic force will pull the charged particle around to the right and as the magnetic force will be always perpendicular to both the velocity of particle and electric field. So the particle will initially goes up in the $y$-direction and then following a curve path lowers down towards the $x$-axis.

SOL 5.2.2 Option (A) is correct.
For a moving charge $Q$ in the presence of both electric and magnetic fields, the total force on the charge is given by
\[ F = Q[E + (v \times B)] \]
where $E \rightarrow$ electric field
$v \rightarrow$ velocity of the charged particle
Since initially the velocity of the charge (at the time of injection) is $v_0 = 2a_y \text{ m/s}$ and for the region $y > 0$ magnetic flux density is $B = 3a_x \text{ wb/m}^2$, so there will be no any velocity component in $+a_x$ direction caused by the field (since the magnetic field is in $a_x$ direction).

\[ v_x = 0 \]

So we consider the velocity of the point charge in the region $y > 0$ at a particular time $t$ as

\[ v = v_y a_y + v_z a_z \]

Therefore we have the force applied by the field on the charge particle at time $t$ as

\[ F = Q[(v_y a_y + v_z a_z) \times (3a_z)] \]

\[ m \left[ \frac{dv_y}{dt} a_y + \frac{dv_z}{dt} a_z \right] = Q[-3v_y a_x + 3v_z a_y] \]

So, we get

\[ \frac{dv_y}{dt} = \frac{3Q}{m} v_z \]

and

\[ \frac{dv_z}{dt} = -\frac{3Q}{m} v_y \]

From the two relations we have

\[ \frac{d^2 v_z}{dt^2} + \left( \frac{3Q}{m} \right)^2 v_z = 0 \]

\[ v_z = A_1 \cos \left( \frac{3Q}{m} t \right) + B_1 \sin \left( \frac{3Q}{m} t \right) \]

where $A_1$ and $B_1$ are constants, and since at $t = 0$, $v_z = 0$ (since charge was injected with a velocity in $a_y$ direction)

Putting the condition in the expression we get $A_1 = 0$

and so we have

\[ v_z = B_1 \sin \left( \frac{3Q}{m} t \right) = B_1 \sin t \]

$Q = 2\text{ C}, m = 6\text{ kg}$

Again,

\[ \frac{dv_z}{dt} = \frac{3Q}{m} v_y \]

so

\[ v_y = -\left( \frac{m}{3Q} \right) \frac{dv_z}{dt} = -B_1 \cos \left( \frac{3Q}{m} t \right) = -B_1 \cos t \]

and since at $t = 0$, $v_y = 2\text{ m/s}$

Putting the condition in the expression we get,

\[ 2 = -B_1 \cos 0 \]

\[ B_1 = -2 \]

So, we have,

\[ v_z = -2 \sin t \rightarrow \frac{dz}{dt} = -2 \sin t \]

\[ v_y = 2 \cos t \rightarrow \frac{dy}{dt} = 2 \cos t \]

Solving the equations we get,

\[ z = 2 \cos t + C_2 \]

and

\[ y = 2 \sin t + C_3 \]

and since at $t = 0$, $y = z = 0$ (charge is located at origin at the time of injection)
Putting the condition in the expression we get

\[ C_2 = -2 \quad \text{and} \quad C_3 = 0 \]

So we have

\[ z = 2 \cos t - 2 \Rightarrow z + 2 = 2 \cos t \]

and

\[ y = 2 \sin t \]

Therefore the equation of the path that the charged particle will follow is

\[ y^2 + (z + 2)^2 = 4 \]

This is the equation of a circle centred at \((0, 0, -5)\).

**SOL 5.2.3** Option (A) is correct.

The magnetic flux density produced at a distance \(\rho\) from an infinitely long straight wire carrying current \(I\) is defined as

\[ B = \frac{\mu_0 I}{2\pi \rho} \]

So the magnetic flux density produced by the straight wire at side \(QR\) of the loop is (direction of magnetic flux density is determined by right hand rule)

\[ B_{QR} = \frac{\mu_0 I}{2\pi \rho} a_z \]

\[ = \frac{5\mu_0}{6\pi} a_z \quad (\rho = 3) \]

\[ I = 5 \text{ A} \]

Force experienced by a current element \(Idl\) in the presence of magnetic flux density \(B\) is defined as

\[ dF = Idl \times B \]

where \(I\) is the current flowing in the element and \(dl\) is the differential vector length of the current element in the direction of flow of current.

So the force exerted by wire on the side \(QR\) of the square loop is

\[ F_{QR} = \int_Q^I 2dl \times B_{QR} \]

where \(I_2\) is the current flowing in the square loop as shown in the figure. So, we get

\[ F_{QR} = \int_{-1}^{1} (3dx a_x) \times \left( \frac{5\mu_0 a_z}{6\pi} \right) \]

\[ = \frac{5\mu_0}{2\pi} [4 - 1] (-a_y) = -5 \times 4\pi \times 10^{-7} \times 3 a_y \]

\[ = -3 \times 10^{-6} a_y \text{ N} \]
Option (C) is correct.

Total force on the loop will be the vector sum of the forces applied by the straight wire on all the sides of the loop. The forces on sides $PQ$ and $RS$ will be equal and opposite due to symmetry and so we have

$$\textbf{F}_{PQ} + \textbf{F}_{RS} = 0$$

Therefore the total force exerted on the conducting loop by the straight wire is

$$\textbf{F}_{\text{total}} = \textbf{F}_{QR} + \textbf{F}_{SP} \quad (1)$$

where $\textbf{F}_{QR}$ and $\textbf{F}_{SP}$ are the forces exerted by the straight wire on the sides $QR$ and $SP$ of the conducting loop respectively.

As calculated in previous question we have

$$\textbf{F}_{QR} = -3 \times 10^{-6} \textbf{a}_y \text{ N}$$

Similarly we get the force exerted by the wire on the side $SP$ of the loop as

$$\textbf{F}_{SP} = \int_{S} I_2 \, dl \times \textbf{B}_{SP}$$

where $\textbf{B}_{SP}$ is the magnetic flux density produced by the wire on the side $SP$. So, we get

$$\textbf{B}_{SP} = \frac{\mu_0 I_1}{2\pi(1)} \textbf{a}_z \quad (\rho = 1)$$

$$= \frac{5\mu_0}{2\pi} \textbf{a}_z \quad (I_1 = 5 \text{ A})$$

$$\textbf{F}_{SP} = \int_{S} 3(- \, dx\textbf{a}_x) \times \frac{5\mu_0}{2\pi} \textbf{a}_z \quad (I_2 = 3 \text{ A}, \, dl = - \, dx\textbf{a}_x)$$

$$= 9 \times 10^{-6} \textbf{a}_y \text{ N}$$

Thus, from equation (1), the total force exerted by the straight wire on the conducting loop is

$$\textbf{F}_{\text{total}} = -3 \times 10^{-6} \textbf{a}_y + 9 \times 10^{-6} \textbf{a}_y$$

$$= 12 \times 10^{-6} \textbf{a}_y \text{ N}$$

Option (A) is correct.

Net magnetic flux density arising from the two current filaments $-5\textbf{a}_z$ and $5\textbf{a}_z \text{ A}$
at the location of third filament is given by

\[ B = B_1 + B_2 \]  

(1)

where \( B_1 \) and \( B_2 \) are the magnetic flux density produced by the current filaments \( 5a_x \) and \(-5a_x \) respectively. Since the magnetic flux density produced at a distance \( \rho \) from a straight wire carrying current \( I \) is defined as

\[ B = \frac{\mu_0 I}{2\pi \rho} a_\phi \]

and the direction of the magnetic flux density is given as

\[ a_\phi = a_x \times a_\rho \]

where \( a_x \) is unit vector along the line current and \( a_\phi \) is the unit vector normal to the line current directed toward the point \( P \). So, the magnetic flux density produced by the current filament \( 5a_x \) is

\[ B_1 = \frac{5\mu_0}{2\pi (\sqrt{1 + k^2})} \left[ a_x \times \left( \frac{ka_x - a_\phi}{\sqrt{1 + k^2}} \right) \right] \]

\[ = \frac{5\mu_0}{2\pi (1 + k^2)} (ka_x + a_\phi) \]

Similarly the magnetic flux density produced by the current filament \( -5a_x \) is

\[ B_2 = \frac{\mu_0 \times (5)}{2\pi (\sqrt{1 + k^2})} \left[ (-a_x) \times \left( \frac{ka_x + a_\phi}{\sqrt{1 + k^2}} \right) \right] \]

\[ = \frac{5\mu_0}{2\pi (1 + k^2)} (-ka_x + a_\phi) \]

Therefore from equation (1), we get the net magnetic flux density experienced by the third filamentary current of 10 A as

\[ B = \frac{\mu_0}{2\pi (1 + k^2)} \left[ ka_x + a_\phi - ka_x + a_\phi \right] \]

\[ = \frac{5\mu_0}{2\pi (1 + k^2)} (2a_\phi) = \frac{5\mu_0}{\pi (1 + k^2)} a_\phi \]

As the force experienced by a current element \( ldI \) in the presence of magnetic flux density \( B \) is defined as

\[ dF = ldI \times B \]

where \( I \) is the current flowing in the element and \( dl \) is the differential vector length of the current element in the direction of flow of current.

Force per unit meter length experienced by the third filament is

\[ F = \int_{-0}^{1} (10a_x \cdot dx) \times \frac{5\mu_0}{\pi (1 + k^2)} a_\phi \]

\[ = 10 \times 5 \times 4\pi \times 10^{-7} a_z \]

\[ = \frac{20a_z}{(1 + k^2)} \mu N \]

or,

\[ F = \frac{12}{(1 + k^2)} \mu N \]

Thus, the graph between \( F \) and \( k \) will be as shown in the figure below:
SOL 5.2.6 Option (B) is correct.
Consider the strip is formed of many adjacent strips of width $dz$ each carrying current $Kdz$.

Since the magnetic flux density produced at a distance $\rho$ from a straight wire carrying current $I$ is defined as

$$B = \frac{\mu_0 I}{2\pi\rho}$$

So the magnetic flux density produced by each differential strip is

$$dB = \frac{\mu_0 (Kdz)}{2\pi z} a_y$$

(Using right hand rule we get the direction of the magnetic flux density along $a_y$)

Therefore the net magnetic flux density produced by the strip on the current filament is

$$B = \int_{z=0.5}^{1.5} \frac{3\mu_0 a_y}{2\pi z} dz = \frac{3\mu_0}{2\pi} \ln\left(\frac{1.5}{0.5}\right) a_y$$

($K = 3 \text{ A/m}$)

As the force experienced by a current element $Idl$ in the presence of magnetic flux density $B$ is defined as

$$dF = Idl \times B$$

where $I$ is the current flowing in the element and $dl$ is the differential vector length.
of the current element in the direction of flow of current. So the force exerted on the filament per unit length is

\[
F = \int ldl \times B = \int_{-0}^{1} (10dx a_x) \times (6.6 \times 10^{-7} a_y)
\]

\[= 2.4a_z \mu N/m\]

**SOL 5.2.7**

Option (C) is correct.

Consider the strip as made up of many adjacent strips of width \(dy\), each carrying current \(Kdy\)

\[
\begin{align*}
I &= 10 \text{ A} \\
K &= 6a_z \text{ A/m}
\end{align*}
\]

Since the magnetic flux density produced at a distance \(\rho\) from a straight wire carrying current \(I\) is defined as

\[
B = \frac{l_0 I}{2\pi \rho}
\]

So the magnetic flux density produced at distance \(y\) from the current filament located along \(z\)-axis as shown in the figure will be

\[
B = \frac{l_0 I}{2\pi y} (a_x) \quad \text{(Direction is determined using right hand rule)}
\]

As the force experienced by a current element \(ldl\) in the presence of magnetic flux density \(B\) is defined as

\[
dF = ldl \times B
\]

and since the length of strip is \(l = 2 \text{ m}\) so, the force exerted on the width \(dy\) of strip is given by

\[
dF = l(Kdy) \times B
\]

Therefore the net force exerted on the strip is

\[
F = \int_{y=1}^{3} (2)(6a_z) \times \left(-\frac{10l_0}{2\pi y}a_y\right) dy
\]

\[= -\frac{60l_0}{\pi} a_y \ln y\]

\[= -13.4a_z \mu N\]
Consider a rectangular Amperian loop of dimension $(l) \times (2y)$ inside the slab as shown in the figure below.

As from the Amperes circuital law, we have
\[ \int H \cdot dl = I_{enc} \]
So for the Amperian loop inside the slab we get
\[ H(2l) = (2y \times l) (J_0) \quad \text{for} \quad -a \leq y \leq a \]
(Net magnetic field intensity along the edge $2y$ will be cancelled due to symmetry)
Therefore the magnetic field intensity (magnetizing factor) at any point inside the slab is
\[ H = J_0 y a \]
or
\[ H = J_0 |y| \quad \text{(for} \quad |y| \leq a) \]
and the magnetic field intensity (magnetizing factor) at any point outside the slab is
\[ H = J_0 a \quad \text{(for} \quad |y| > a) \]
Thus, the plot of $H$ versus $y$ will be as shown below.

Option (A) is correct.

Force on any dipole having moment $m$ due to a magnetic flux density $B$ is defined as
\[ F = \nabla (m \cdot B) \]
Since the magnetic moment of the dipole is given as
\( m = m_0 a_z \) \hspace{1cm} (1)

and as calculated in previous question the magnetic field intensity produced due to the slab is

\[ H = J_0 y a_z \]

So we get the magnetic flux density produced due to the slab as

\[ B = \mu_0 H = \mu_0 J_0 y a_z \] \hspace{1cm} (2)

Therefore from equation (1) and (2) we get

\[ m \cdot B = 0 \]

Thus the force acting on the dipole is

\[ F = 0 \]

**SOL 5.2.10**

Option (A) is correct.

Volume current density inside a magnetic material is equal to the curl of its magnetization \( M \)

i.e.

\[ J = \nabla \times M \]

So volume current density due inside the circular cylinder is

\[ J = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho (5\rho^2) a_z = 15\rho a_z \]

or

\[ J \propto \rho \]

**SOL 5.2.11**

Option (B) is correct.

As calculated above the volume current density inside the cylinder is

\[ J = 15\rho a_z \]

So, we can get the flux density by Ampere’s circuital law as

\[ \oint B \cdot dl = \mu_0 I_{enc} \]

\[ (B) (2\pi \rho) = \mu_0 I_{enc} \]

\[ B = \frac{\mu_0}{2\pi \rho} I_{enc} \]

Now the enclosed current in the loop is

\[ I_{enc} = \int J \cdot dS = \int \int (15\rho)(\rho d\rho d\phi) = 2\pi \times 15 \left[ \frac{\rho^3}{3} \right]_0^I \]

\[ = 10\pi R^3 \]

So, the magnetic flux density inside the cylinder is

\[ B = \frac{\mu_0}{2\pi \rho} I_{enc} = 5\mu_0 \rho^2 \] \hspace{1cm} (I = 10\pi R^3)

Thus the plot of magnetic flux density \( B \) versus \( \rho \) is as shown below
SOL 5.2.12 Option (B) is correct.
The surface current density of a material in terms of its magnetization is defined as
\[ K = M \times a_n \] where \( a_n \) is unit vector normal to the surface.
So, the surface current density of the cylinder is
\[ K = (Ma_z) \times (a_r) = Ma_a \]
Therefore the surface current density is directed along \( a_\phi \) as shown in option (B).

SOL 5.2.13 Option (A) is correct.
Magnetic flux density inside a magnetic material is defined as
\[ B = \mu_0 (H + M) \]
So, \( B \) and \( M \) will be in same direction inside the cylinder.
Now as the magnetic field lines are circular so outside the cylinder it will make a loop. Thus, the magnetic field lines will be as shown below.

---

SOL 5.2.14 Option (B) is correct.
Let the circular ring being placed such that magnetization \( M \) is in \( a_0 \) direction and the ring is centered at origin.
So, we have \( M = 4a_0 \)
As the surface current density of a material in terms of its magnetization is defined as
\[
K = M \times a_n \quad \text{where } a_n \text{ is unit vector normal to the surface.}
\]
So the surface current density of the ring is
\[
K = 4a_0 \times (-a_p) = 4a_z \quad \text{and since the volume current density inside a material is equal to the curl of magnetization } M
\]
\[
J = \nabla \times M
\]
So the volume current density inside the ring is
\[
J = \nabla \times (4a_0) = 0 \quad \text{\((M = 4a_0)\)}
\]
Now from Ampere's circuital law we have
\[
\oint _C B \cdot dl = \mu_0 I_{enc}
\]
and for determining the field inside the circular ring, the current present on the inner surface of ring will be considered only. So we get
\[
(B)(2\pi \rho) = \mu_0 (K)(2\pi \rho)
\]
Therefore the magnetic flux density inside the circular ring is
\[
B = (\mu_0)(4) = 4\mu_0 \text{ wb/m}^2 \quad \text{\((K = 4 \text{ A/m})\)}
\]
Alternate Method:
Magnetic flux density inside a magnetic material is defined as
\[
B = \mu_0 M
\]
and since the magnetization of the rod is \( M = 4 \text{ A/m} \) so, we can have directly the magnetic flux density inside the ring as
\[
B = 5\mu_0 \text{ wb/m}^2
\]
SOL 5.2.15 Option (A) is correct.
As calculated above for the complete circular ring, magnetic flux density inside the ring is
\[
B = 4\mu_0 a_0 \text{ wb/m}^2
\]
(magnetic flux density will be directed along the assumed direction of magnetization)
Now we calculate the flux density contributed by the gap at its centre when it was the complete ring. The gap has its cross section in form of a square loop as shown in figure below

As calculated in previous question the surface current density of the ring is

\[ K = 4 \text{ A/m} \]

and since the width of the gap(square loop) is \( w \) so, net current in the loop is

\[ I = Kw = 4w \]

Now the magnetic flux density at any point \( P \) due to a filamentary current \( I \) is defined as

\[ H = \frac{I}{4\pi \rho} \left[ \cos \alpha_2 - \cos \alpha_1 \right] a_0 \]

where

\[ \rho \rightarrow \text{distance of point } P \text{ from the current filament.} \]
\[ \alpha_1 \rightarrow \text{angle subtended by the lower end of the filament at } P. \]
\[ \alpha_2 \rightarrow \text{angle subtended by the upper end of the filament at } P. \]

So the flux density at center of the square loop produced due to one side of the loop is

\[ B_{sq} = \frac{\mu_0 I}{4\pi \times (10^{-2})} \times \left( \frac{2}{\sqrt{2}} \right) \quad (\rho = 1 \text{ cm}, \alpha_1 = 135^\circ, \alpha_2 = 45^\circ) \]

Summing the flux density produced due to all the four sides of loop, we get total magnetic flux density produced by the square loop as

\[ B_{sq} = 4 \times \left( \frac{\mu_0 I \sqrt{2}}{4\pi (10^{-2})} \right) = \frac{\sqrt{2} \mu_0 (4w)}{\pi} \times 10^2 \quad (I = Kw) \]
\[ = \frac{\sqrt{2} \mu_0 (4 \times (4) \times (0.1 \times 10^{-3}) \times 10^2}{\pi} \quad (w = 0.1 \text{ mm}) \]

Therefore at the centre of the gap the net magnetic flux density will reduce by this amount of the flux density. Thus at the centre of the gap the net magnetic flux density at the centre of the loop will be

\[ B_{net} = B - B_{sq} \]
\[ = 4\mu_0 - \frac{4\sqrt{2} \times 10^{-2}}{\pi} \mu_0 \]
\[ = \mu_0 \left( 4 - \frac{4\sqrt{2} \times 10^{-2}}{\pi} \right) \]
\[ = 50.04 \times 10^{-7} \text{ wb/m}^2 \]
Let the magnetized sphere be of radius \( r \), centered at origin and the magnetization be \( \mathbf{M} \) in \( \mathbf{a}_z \) direction as shown in figure.

Volume current density inside a material is equal to the curl of magnetization \( \mathbf{M} \)
i.e.
\[
\mathbf{J} = \nabla \times \mathbf{M}
\]
So the volume current density inside the cylinder is
\[
\mathbf{J} = \nabla \times (\mathbf{M} \mathbf{a}_z) = 0
\]
and since the surface current density in terms of magnetization is defined as
\[
\mathbf{K} = \mathbf{M} \times \mathbf{a}_n
\]
where \( \mathbf{a}_n \) is unit vector normal to the surface.

So the surface current density on the sphere is
\[
\mathbf{K} = (\mathbf{Ma}_z) \times (\mathbf{a}_z) = M \sin \theta \mathbf{a}_z
\]
\[\text{(i)}\]

Now, consider a rotating spherical shell of uniform surface charge density \( \sigma \), that corresponds to a surface current density at any point \((r, \theta, \phi)\). So we have
\[
\mathbf{K} = \sigma \omega R \sin \theta \mathbf{a}_z
\]
\[\text{(ii)}\]
where
- \( \omega \to \) angular velocity of spherical shell across \( z \)-axis
- \( R \to \) radius of the sphere.

and the magnetic flux density produced inside the rotating spherical shell is defined as
\[
\mathbf{B} = \frac{2}{3} \mu_0 \sigma \omega R
\]
\[\text{(iii)}\]
Comparing the eq.(i) and eq.(ii) we get
\[
\mathbf{M} = \sigma \omega R
\]
Putting this value in eq.(iii) we get the magnetic flux density for the magnetized sphere as
\[
\mathbf{B} = \frac{2}{3} \mu_0 \mathbf{M}
\]
\[\text{(} \mathbf{M} = \sigma \omega R \text{)}\]
Option (B) is correct.

Since the magnetic flux density inside a magnetic material is defined as
\[ B = \mu_0 (H + M) \]

So, we have the magnetic field intensity inside the material as
\[ H = \frac{1}{\mu_0} B - M \]

and outside the material the magnetic field intensity is
\[ H = \frac{1}{\mu_0} B \]

So the field lines outside the material will be same as for the case of magnetic flux density shown in the MCQ. 1.31. Whereas inside the material the direction of magnetic field intensity will be opposite to the direction of magnetization. Thus the sketch of the field intensity will be same as shown in the option (B).

Option (B) is correct.

The magnetic flux density produced at any point \( P \) due to an infinite filamentary current \( I \) is defined as
\[ B = \frac{\mu_0 I}{2\pi \rho} \]

where \( \rho \) is the distance of point \( P \) from the infinite current filament.

Now consider a small area \( dS \) of the coil located at a distance \( x \) from the current filament. The magnetic flux density produced on it due to the current filament along \( y \)-axis is
\[ B = \frac{\mu_0 I}{2\pi x} \quad (\rho = x) \]

Since the flux density will be normal to the surface of the coil as determined by right hand rule therefore, the total magnetic flux passing through the coil is
\[ \psi_m = \int B \cdot dS = \int_{y=0}^{y=2} \int_{x=0}^{x=\rho} \left( \frac{\mu_0 I}{2\pi x} \right) (dx dy) = \frac{\mu_0 I}{2\pi} \ln 3 \]

As the mutual inductance in terms of total magnetic flux \( \psi_m \) is defined as
\[ M = \frac{N\psi_m}{I} \]

where \( I \to \) current flowing in the element that produces the magnetic flux.
\( N \to \) Total no. of turns of the coil that experiences the magnetic flux.

Thus the mutual inductance between the current filament and the loop is
\[ M = \frac{1500}{I} \left( \frac{\mu_0 I}{2\pi} \ln 3 \right) = 0.33 \text{ mH} \]

Option (A) is correct.

Consider the wire is lying along \( z \)-axis. So at any point inside the wire (at a distance \( \rho < a \) from its axis) magnetic field intensity will be determined as
\[ \int H \cdot dl = I_{enc} \quad \text{(Ampere’s circuital law)} \]

\[ H(2\pi \rho) = I \left( \frac{\pi \rho^2}{\pi a^2} \right) \quad \text{(for Amperian loop of radius} \ \rho) \]
or,
\[ H = \frac{I \rho}{2\pi a^2} a_\phi \]

The direction of the magnetic field intensity is determined using right hand rule.

Now the stored energy in the magnetic field \( H \) is defined as
\[ W_m = \int_2^1 \frac{1}{2} \mu_0 H^2 dv \]

So the stored energy in the internal magnetic filed per unit length (over the unit length in \( z \)-direction) will be
\[ W_m = \int_{y=0}^{2\pi} \int_{\rho=0}^{\infty} \frac{\mu_0 I^2 \rho^2}{2(2\pi a^2)} d\rho d\phi dz = \frac{\mu_0 I^2}{16\pi} \]

Therefore, the energy per unit length depends only on \( I \) and is uniform for the uniform current.

**SOL 5.2.20**

Option (B) is correct.

Since the two conducting plates of width \( w = 2 \text{ m} \) carry a uniform current of \( I = 4 \text{ A} \) each so, the surface current density of each plate is
\[ K = \frac{I}{w} = \frac{4}{2} = 2 \text{ A/m} \]

Now consider the first plate carrying current in \(+ a_z\) direction is located at \( y = 0 \) and the second plate carrying current in \(- a_z\) direction is located at \( y = d \), where \( d \) is a very small separation between the plates.

Since the magnetic field intensity produced at any point \( P \) due to an infinite sheet carrying uniform current density \( K \) is defined as
\[ H = \frac{1}{2} (K \times a_\phi) \]

where \( a_\phi \) is the unit vector normal to the sheet directed toward the point \( P \). So, the magnetic field intensity produced at the second plate due to the first plate is
\[ H_{12} = \frac{1}{2} (2a_\phi \times a_z) = -a_z \quad (K_1 = 2a_z, a_n = a_y) \]

Now the force per meter, exerted on the 2nd plate due to the 1st plate will be
\[ F_{12} = \int_0^1 \int_0^{2\pi} (K_2 \times B_{12}) dS \]

where \( K_2 \rightarrow \) current density of the 2nd plate
\( B_{12} \rightarrow \) magnetic flux density produced at the 2nd plate due to 1st plate

So,
\[ F_{12} = \int_0^1 \int_0^{2\pi} (-2a_z) \times (\mu_0 H_{12}) d\gamma dz \quad (K_2 = -2a_z, B_{12} = \mu_0 H_{12}) \]
\[ = \int_0^1 \int_0^{2\pi} (-2a_z) \times (-\mu_0 a_z) d\gamma dz = 4\mu_0 a_y \]

As the force applied by first plate on the 2nd plate is in \( a_y \) direction so it is a repulsive force. Therefore the repulsive force between the plates is \( 4\mu_0 \).

**SOL 5.2.21**

Option (D) is correct.

No, of turns, \( n = 20,000 \text{ turns/meter} \)

Relative permeability, \( \mu_r = 100 \)
cross sectional area, \[ S = 0.04 \, m^2 \]

Current in the solenoid, \[ I = 100 \times 10^{-3} \, A \]

So, its self inductance will be,
\[ L' = \mu_0 \mu_r n^2 S = (4\pi \times 10^{-7}) \times (100) \times (20,000)^2 \times (0.04) \]
\[ = 2.011 \times 10^3 \]

Therefore the energy stored per unit length in the field is
\[ W_m = \frac{1}{2} L' I^2 = \frac{1}{2} \times 2.011 \times 10^3 \times 10^{-2} = 10.05 \, J/m \]

**SOL 5.2.22** Option (A) is correct.

Consider the path followed by the two particles are the curvatures having radii \( r_1 \), and \( r_2 \) as shown in figure. So at balanced condition centrifugal force will be equal to magnetic force.

Therefore for the first charged particles
\[ \frac{mv^2}{r_1} = Bqv \Rightarrow r_1 = \frac{mv}{Bq} \]

and
\[ \frac{(2m)v^2}{r_2} = Bqv \Rightarrow r_2 = \frac{mv}{Bq} \]

So the distance between the two particles at releasing end is
\[ d = 2r_1 - 2r_2 = 2 \left( \frac{2mv}{Bq} \right) - 2 \left( \frac{mv}{Bq} \right) = \frac{2mv}{Bq} \]

**SOL 5.2.23** Option (A) is correct.

The wire is oriented in east-west direction and magnetic field is directed northward as shown in the figure.

Since the direction of gravitational force will be into the paper(toward the earth) so for counteracting the gravitational force, applied force must be outward.

Now the force experienced by a current element \( Idl \) in a magnetic field \( B \) is
As the magnetic field $B$ is directed toward north therefore, using right hand rule for cross vector we conclude that for producing the outward force current must flow from west to east as shown in the figure below.

Since

**SOL 5.2.24** Option (A) is correct.

Consider the current flowing in the wire is $I$. So the magnetic force applied by the field $B_0$ on the wire is

$$F_m = ILB_0$$

where $L$ is length of the wire

At balanced condition the magnetic force will be equal to the gravitational force:

$$F_m = mg$$

where $m$ is the mass of the wire and $g$ is acceleration due to gravity.

So comparing the two results we get the current flowing in the wire as

$$I = \frac{mg}{LB_0}$$

Since

$$B_0 = 0.6 \times 10^{-4} \text{ Wb/m}^2, \quad m = 0.3 \text{ kg} \quad \text{and} \quad L = 1 \text{ m}$$

Therefore

$$I = \frac{(0.3) \times 9.8}{1 \times (0.6 \times 10^{-4})} = 49 \text{ kA} \quad (g = 9.8 \text{ m/s})$$

**SOL 5.2.25** Option (B) is correct.

Consider the square loop has side $a$. Now, when the loop is situated in the field $B = 1.96 \text{ Wb/m}^2$. Suppose it swings with an angle $\alpha$. So in the new position the torque must be zero. Gravitational forces acting on all the sides of loop will be down wards and the force due to magnetic field will be in horizontal direction as shown in the figure.
So, in balanced condition, from the shown figure we have

\[ a \, m \sin \alpha (a) + 2 \, a \, m \sin \alpha \left( \frac{d}{2} \right) = IBa \]

\[ \tan \alpha = \frac{IB}{2mg} = \frac{(2)(1.96)}{(2)(0.2)(9.8)} \]

\[ \alpha = \tan^{-1}(1) = \pi/4 \]

**SOL 5.2.26** Option (B) is correct.

As discussed in Que 51 the path of electron will be parallel to the input beam but in opposite direction. So the ejected electrons will be flowing in the \(-a_y\) direction.

**SOL 5.2.27** Option (D) is correct.

At any point in between the two parallel slits the net magnetic flux density produced by the two sheets is given as

\[ B = B_1 + B_2 \]

where \(B_1\) is the flux density produced by the lower sheet and \(B_2\) is the flux density produced by upper sheet.

Now the magnetic flux density produced at point \(P\) due to a plane sheet having current density \(K\) is defined as

\[ B = \frac{\mu}{2} K \times a_n \]

where \(a_n\) is the unit vector normal to the sheet and directed toward point \(P\). So, the flux density produced by lower sheet is

\[ B_1 = \frac{\mu}{2} (4a_y) \times a_z \quad (K = 4a_z, \ a_n = a_z) \]

and the flux density produced by the lower sheet is

\[ B_2 = \frac{\mu}{2} (-4a_y) \times (-a_z) \quad (K = 4a_z, \ a_n = -a_z) \]

So the net magnetic flux density produced in the region between the two sheets is

\[ B = \frac{\mu}{2} (4a_y) \times a_z + \frac{\mu}{2} (-4a_y) \times (-a_z) \]

\[ = -4\mu a_y \]

where \(\mu\) is the permeability of the medium.

Therefore the flux density in region 1 is

\[ B_{\text{region 1}} = -4\mu_1 a_y = -8\mu_0 a_y \quad (\mu_1 = 2\mu_0) \]

and the flux density in region 2 is

\[ B_{\text{region 2}} = -4\mu_2 a_y = -16\mu_0 a_y \quad (\mu_2 = 4\mu_0) \]

So the net flux per unit length in the region between the two sheets is

\[ \frac{\phi}{T} = (B_{\text{region 1}} \text{ (width of region 1)}) + (B_{\text{region 2}} \text{ (width of region 2)}) \]

\[ = (-8\mu_0 a_y)(1) + (-16\mu_0 a_y)(2) \]

\[ = -50\mu_0 a_y \text{ Wb/m} \]

**SOL 5.2.28** Option (A) is correct.

As the permeability of the medium varies from \(\mu_1\) to \(\mu_2\) linearly. So at any distance
z from one of the plate near to which permittivity is $\mu_1$, the permeability is given as

$$\mu = \mu_1 + \frac{(\mu_2 - \mu_1)}{d} z$$  \hspace{1cm} (1)

The magnetic flux density between the two parallel sheets carrying equal and opposite current densities is defined as

$$\vec{B} = \mu\vec{K}$$

where $\vec{K}$ is the magnitude of the current density of the sheets.

Therefore the flux per unit length between the two sheets is

$$\frac{\phi}{l} = \int_0^d \vec{B} \, dz \quad \text{where } d \text{ is the separation between the two sheets.}$$

$$= \int_0^d \mu K dz = K \int_0^d \left[ \mu_1 + \frac{(\mu_2 - \mu_1)}{d} z \right] dz \quad \text{(from equation (1))}$$

$$= K \left[ \mu_1 z + \frac{\mu_2 - \mu_1}{d} \left( \frac{z^2}{2} \right) \right]_0^d = K \left( \mu_1 + \frac{\mu_2}{2} \right) d$$

**SOL 5.2.29** Option (B) is correct.

Given the field intensity inside the slab is

$$\vec{H} = 4\vec{a}_x + 2\vec{a}_y$$

So the magnetic flux density inside the slab is given as

$$\vec{B} = \mu \vec{H} \quad \text{where } \mu \text{ is the permeability of the material.}$$

$$= 2\mu_0 (4\vec{a}_x + 2\vec{a}_y) \quad (\mu = 2\mu_0)$$

Therefore the magnetization of the material is

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$$

$$= 8\vec{a}_x + 4\vec{a}_y - (4\vec{a}_x + 2\vec{a}_y) = 4\vec{a}_x + 2\vec{a}_y$$

Now the magnetization surface current density at the surfaces of a magnetic material is defined as

$$\vec{K}_m = \vec{M} \times \vec{n}$$

where $\vec{n}$ is the unit vector normal to the surface directed outward of the material

So, at $z = 0$ magnetization surface current density is

$$[\vec{K}_m]_{z=0} = \vec{M} \times (-\vec{a}_z) = 4\vec{a}_y - 2\vec{a}_x$$

and at $z = d$, the magnetization surface current density is

So,

$$[\vec{K}_m]_{z=d} = \vec{M} \times (\vec{a}_z) = (4\vec{a}_x + 2\vec{a}_y) \times (\vec{a}_z) = -4\vec{a}_y + 2\vec{a}_x$$

**SOL 5.2.30** Option (A) is correct.

As calculated in the previous question the magnetization vector of the material is

$$\vec{M} = 4\vec{a}_x + 2\vec{a}_y$$

The magnetization volume current density inside a magnetic material is equal to the curl of magnetization,

i.e.

$$\vec{J}_m = \nabla \times \vec{M}$$

Therefore the magnetization volume current density inside the slab is

$$\nabla \times \vec{M} = \vec{J}_m$$
\[ \mathbf{J}_m = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = 0 \]

**SOL 5.2.31** Option (B) is correct.

Given the \( \mathbf{B} \cdot \mathbf{H} \) curve for the material,
\[ \mathbf{B} = 2\mu_0 H \mathbf{H} \]

The work done per unit volume in magnetizing a material from 0 to \( B_0 \) that has non-uniform permeability is defined as
\[ w_m = \int_0^{B_0} \mathbf{H} \cdot d\mathbf{B} \]

Now for determining \( dB \), we can express
\[ \frac{dB}{dH} = 4\mu_0 H a_H \]

where \( a_H \) is the unit vector in direction of \( \mathbf{H} \).

So,
\[ \frac{dB}{dH} = 4\mu_0 H \]

and
\[ w_m = \int_0^{B_0} \mathbf{H} \cdot (4\mu_0 H) = 4\mu_0 \frac{H^2}{3} \bigg|_{B_0} = \frac{3\mu_0 H_0^2}{4} \]

**SOL 5.2.32** Option (A) is correct.

As \( \otimes \) shows the direction into the paper while \( \ominus \) shows the direction out of the paper. So the wire of length \( l \) carries current \( 2I \) that flows out of the paper.

The magnetic field intensity produced at a distance \( \rho \) from an infinite straight wire carrying current \( I \) is defined as
\[ H = \frac{I}{2\pi \rho} \]

So the magnetic field intensity produced at the top wire due to the infinite wire carrying current inward is
\[ H_{\text{in}} = \frac{I}{2\pi (\sqrt{2}I)} \]

\( (\rho = \sqrt{2}I) \)

and the magnetic field intensity at top wire due to the infinite wire carrying current outward is
\[ H_{\text{out}} = \frac{I}{2\pi (\sqrt{2}I)} \]

\( (\rho = \sqrt{2}I) \)
Therefore the resultant field intensity at the wire of length $l$ is

$$\mathbf{H}_r = (\mathbf{H}_1 + \mathbf{H}_2)\cos \theta$$

$$= \frac{2I}{2\pi(\sqrt{2})} \times \frac{1}{\sqrt{2}} = \frac{I}{2\pi l}$$

Since the force exerted on a current element $Idl$ by a magnetic field $\mathbf{H}$ is defined as

$$d\mathbf{F} = (\mu \mathbf{H}) (Idl)$$

So the force experienced by the wire of length $l$ is

$$\mathbf{F} = (\mu \mathbf{H}_r)(2I)l = \mu \left(\frac{I}{2\pi l}\right)(2I)l = \frac{\mu I^2}{2\pi}$$

**SOL 5.2.33** Option (C) is correct.

Consider the sheets as shown in figure that having the surface current densities $+20 \text{ mA/m} a_y$ and $-20 \text{ mA/m} a_y$

So the field intensity between the plates will be given as

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2$$

where $\mathbf{H}_1$ is the field intensity produced by the sheet located at $x = 0$ and $\mathbf{H}_2$ is the field intensity produced by the sheet located at $x = 5 \text{ cm}$.

Now the magnetic field intensity produced at point $P$ due to a plane sheet having current density $\mathbf{K}$ is defined as

$$\mathbf{H} = \frac{\mu_0}{2\pi} \mathbf{K} \times \mathbf{a}_n$$

where $\mathbf{a}_n$ is the unit vector normal to the sheet directed toward point $P$. So, the magnetic field intensity in the region between the plates is

$$\mathbf{H} = \left. \frac{\mu_0}{2\pi} \mathbf{K} \times \mathbf{a}_n \right|_{\text{at } x = 0} + \left. \frac{\mu_0}{2\pi} \mathbf{K} \times \mathbf{a}_n \right|_{\text{at } x = 5 \text{ cm}}$$

$$= \frac{1}{2}(20 \times 10^{-3} \mathbf{a}_y) \times (\mathbf{a}_z) + \frac{1}{2}(-20 \times 10^{-3} \mathbf{a}_y) \times (-\mathbf{a}_z)$$

$$= -20 \times 10^{-3} \mathbf{a}_z$$

and magnetic flux density in the region between the sheets is

$$\mathbf{B} = \mu \mathbf{H} = -40\mu_0 \times 10^{-3} \mathbf{a}_z$$

($\mu = 2\mu_0$)

Therefore the stored magnetic energy per unit volume in the region is
\[ w_m = \frac{1}{2} \mathbf{H} \cdot \mathbf{B} = \frac{1}{2}(800\mu_0 \times 10^{-6}) \]
\[ = 400 \times 4\pi \times 10^{-7} \times 10^{-6} = 160\pi J/m^3 \]

Since the separation between plates is \( d = 5 \text{ cm} \). So, stored energy per unit area between the plates is
\[ W_m/A = w_m \times d = (160\pi) \times (0.05) = 4\pi J/m^2 \]

**SOL 5.2.34** Option (C) is correct.

Since the boundary surface of the two medium is \( z = 0 \), so the normal component \( B_{1n} \) and tangential component \( B_{1t} \) of magnetic flux density in medium 1 are
\[ B_{1n} = a_z \]
\[ B_{1t} = 0.4a_x + 0.8a_y \]
As the normal component of magnetic flux density is uniform at the boundary of two medium So, the normal component of magnetic flux density in the medium 2 is
\[ B_{2n} = B_{1n} = a_z \] (1)

Now for determining tangential component of field in medium 2, we first calculate tangential component of magnetic field intensity in medium 1 which is given as
\[ H_{1t} = \frac{B_{1t}}{\mu_1} \]
\[ = \frac{1}{4\mu_0}(0.4a_x + 0.8a_y) = \frac{0.1a_x + 0.2a_y}{\mu_0} \] \( (\mu_1 = 4\mu_0) \)

Again from the boundary condition the tangential component of magnetic field intensity in the two mediums are related as
\[ a_n \times (H_{1t} - H_{2t}) = K \]
where \( H_{2t} \) and \( H_{1t} \) are the tangential components of magnetic field intensity in medium 2 and medium 1 respectively, \( K \) is the surface current density at the boundary interface of the two mediums and \( a_n \) is the unit vector normal to the boundary interface. So we have
\[ a_z \times \left[ \frac{0.1a_x + 0.2a_y}{\mu_0} - (H_{2t}a_x + H_{2y}a_y) \right] = \frac{1}{\mu_0}(0.2a_x - 0.4a_y) \]
\[ \left( \frac{0.1}{\mu_0} - H_{2t} \right)a_y - \left( \frac{0.2}{\mu_0} - H_{2y} \right)a_x = \frac{1}{\mu_0}(0.2a_x - 0.4a_y) \]

Comparing the \( x \) and \( y \)-components we get
\[ H_{2tx} = \frac{0.1}{\mu_0} + \frac{0.4}{\mu_0} = \frac{0.5}{\mu_0} \]
and
\[ H_{2ty} = \frac{0.2}{\mu_0} + \frac{0.2}{\mu_0} = \frac{0.4}{\mu_0} \]

Therefore the tangential component of magnetic field intensity in medium 2 is
\[ H_{2t} = \frac{0.5}{\mu_0}a_x + \frac{0.4}{\mu_0}a_y \]
and the tangential component of magnetic flux density in medium 2 is
\[ B_{2t} = \mu_2H_{2t} = a_x + 0.8a_y \]
Thus the net magnetic flux density in medium 2 is
\[ B_2 = B_{2t} + B_{2n} = a_x + 0.8a_y + a_z \]
SOL 5.2.35 Option (B) is correct.
Consider the square loop is of side \(2a\) as shown in the figure.

Since the sides \(BC\) and \(AD\) crosses the straight wire so no force will be experienced by the sides, while the flux density produced by the straight wire at sides \(AB\) and \(CD\) will be equal in magnitude.

Now the magnetic flux density produced at a distance \(\rho\) from a straight wire carrying current \(I\) is defined as

\[
B = \frac{\mu_0 I}{2\pi \rho}
\]

So the magnetic flux density produced by the straight wire at the two sides of the loop is

\[
B = \frac{\mu_0 (2)}{2\pi (a)} = \frac{\mu_0}{\pi a} \quad (I = 2\, \text{A}, \rho = a)
\]

Since the force exerted on a current element \(Idl\) by a magnetic field \(B\) is defined as

\[
dF = (Idl) \times B
\]

Therefore the force experienced by side \(AB\) of length \(2a\) is

\[
F_1 = \left[4(2a) a_y\right] \times \left[\frac{\mu_0}{\pi a} a_x\right] = \frac{8\mu_0}{\pi} (-a_y)
\]

\((I = 4\, \text{A})\)

Similarly force experienced by side \(CD\) is

\[
F_2 = \left[4(2a)(-a_x)\right] \times \left[\frac{\mu_0}{\pi a} (-a_y)\right] = \frac{8\mu_0}{\pi} (a_y)
\]

Thus the net force experienced by the loop is

\[
F = F_1 + F_2 = \frac{16\mu_0}{\pi} (a_y)
\]

\(= 16 \times 4 \times 10^{-7} a_y = 2.4a_y\, \text{\mu N}\)

SOL 5.2.36 Option (A) is correct.
According to Snell’s law the permeability of two mediums are related as

\[
\mu_0 \tan \theta_1 = \mu \tan \theta_2
\]

\[
\tan \theta_1 = \frac{15\mu_0}{\mu} \quad \tan \theta_2 = \frac{15\mu_0}{\mu} \quad \tan \theta_1 = 15 \tan \theta_2
\]

\((i)\)

Now, the given flux density in medium 2 is

\[
B_2 = 1.2a_y + 0.8a_z
\]
So the normal and tangential component of the magnetic flux density in medium 2 is

\[ B_{n2} = 0.8a_z \]

and

\[ B_{t2} = 1.2a_y \]

From the figure we have

\[ \tan \theta_2 = \frac{B_{n2}}{B_{t2}} = \frac{0.8}{1.2} = \frac{2}{3} \]

or

\[ \theta_2 = \tan^{-1}(2/3) \]

from equation (1)

\[ \tan \theta_1 = 15 \tan \theta_2 \]

\[ \tan \theta_1 = 10 \]

\[ \theta = \tan^{-1}(10) \]

Thus the angular deflection is

\[ \theta_1 - \theta_2 = \tan^{-1}(10) - \tan^{-1}(2/3) = 54.6^\circ \]

**SOL 5.2.37** Option (B) is correct.

For calculating total reluctance of the circuit, we have to draw the electrical analog of the circuit. In the given magnetic circuit, there are total six section for which six reluctance has been drawn below.

For a given cross sectional area \( S \) and length of the core \( l \) reluctance is defined as

\[ R = \frac{l}{\mu S} \]

Where \( \mu \) is permeability of the medium in core

So, we have

\[ R_1 = \frac{5 \times 10^{-2}}{(1000 \mu_0)(5 \times 10^{-4})} = \frac{1}{10 \mu_0} \]

\[ R_2 = \frac{5 \times 10^{-2}}{(1000 \mu_0)(10 \times 10^{-3})} = \frac{1}{20 \mu_0} \]

\[ R_3 = \frac{6 \times 10^{-2}}{(1000 \mu_0)(10 \times 10^{-3})} = \frac{3}{50 \mu_0} \]

\[ R_4 = \frac{14 \times 10^{-2}}{(1000 \mu_0)(10 \times 10^{-3})} = \frac{7}{50 \mu_0} \]

\[ R_5 = R_3 = \frac{3}{50 \mu_0} \]

\[ R_6 = \frac{4 \times 10^{-2}}{(1000 \mu_0)(10 \times 10^{-3})} = \frac{1}{25 \mu_0} \]
Since all the reluctance are connected in series so total reluctance of the magnetic circuit is
\[ R_T = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 \]
\[ = \frac{1}{10\mu_0} + \frac{1}{20\mu_0} + \frac{3}{50\mu_0} + \frac{7}{50\mu_0} + \frac{3}{50\mu_0} + \frac{1}{25\mu_0} \]
\[ = \frac{9}{20\mu_0} \]

**SOL 5.2.38** Option (C) is correct.

For a given reluctance \( R \) of a magnetic circuit, the self, inductance is defined as
\[ L = \frac{N^2}{R} \]

Where \( N \) is no. of turns of coil

Then,
\[ L = \frac{(100)^2}{(9/20\mu_0)} \]
\[ = 2.79 \times 10^{-2} = 27.9 \text{ mH} \]

**SOL 5.2.39** Option (B) is correct.

Give that
- no. of turns of coil, \( N = 50 \)
- length of the core, \( l = 0.6 \text{ m} \)
- relative permeability, \( \mu_r = 600 \)
- inductance of the coil, \( L = 0.2 \text{ mH} = 0.2 \times 10^{-3} \text{ H} \)

So, the cross sectional area of core is
\[ S = \frac{LL}{\mu N^2} = \frac{(0.2 \times 10^{-3})(0.6)}{(600\mu_0)(50)^2} \]
\[ = 6.366 \times 10^{-5} \text{ m}^2 = 1.64 \text{ cm}^2 \]

**SOL 5.2.40** Option (B) is correct.

Since the core is ideal so it’s reluctance will be zero and so the electrical analog for the magnetic circuit will be as shown below

The reluctance \( R_1, R_2 \) and \( R_3 \) is produced by the air gap.
\[ R_1 = \frac{l_1}{\mu_0 S_1} = \frac{4 \times 10^{-2}}{\mu_0 (100 \times 10^{-4})} = \frac{4}{\mu_0} \]
\[ R_2 = \frac{2 \times 10^{-2}}{\mu_0 (100 \times 10^{-4})} = \frac{2}{\mu_0} \]
\[ R_3 = \frac{2 \times 10^{-2}}{\mu_0 (100 \times 10^{-4})} = \frac{2}{\mu_0} \]
So, the total reluctance seen by coil \( N_1 \) is
\[
R_T = R_1 + R_2 + R_3
\]
\[
= \frac{4}{\mu_0} + \frac{1}{\mu_0} = \frac{5}{\mu_0}
\]
and the self inductance of coil will be
\[
L_1 = \frac{N_1^2}{R_T} = 62.8 \text{ mH}
\]

SOL 5.2.41 Option (B) is correct.
The total reluctance of the magnetic circuit as seen from the coil \( N_2 \) is
\[
R_T = (R_1 || R_3) + R_3
\]
\[
= \left( \frac{4}{\mu_0} || \frac{2}{\mu_0} \right) + \frac{2}{\mu_0}
\]
\[
= \frac{4}{3\mu_0} + \frac{2}{\mu_0} = \frac{10}{3\mu_0}
\]
Therefore the self inductance of the coil \( N_2 \) is
\[
L_2 = \frac{N_2^2}{R_T} = \frac{(250)^2}{(10/3\mu_0)} = 28.6 \text{ mH}
\]

SOL 5.2.42 Option (D) is correct.
Since the coil \( N_1 \) and \( N_2 \) are directly connected through ideal core so entire flux produced by \( N_2 \) will link with \( N_1 \).
The electrical analog of the magnetic circuit is shown below where the reluctance \( R_1 \) and \( R_2 \) are the reluctance due to the air gap.

So, the reluctance seen by coil \( N_2 \) is
\[
R_1 = \frac{L_1}{\mu_0 S} = \frac{4 \times 10^{-3}}{\mu_0 (2000 \times 10^{-6})} = \frac{2}{\mu_0}
\]
Consider the current flowing in coil \( N_2 \) is \( i_2 \). So, the total flux produced by \( N_2i_2 \) is
\[
\phi_2 = \frac{N_2i_2}{R_1} = \frac{500i_2}{2/\mu_0} = 250\mu_0 i_2
\]
Since the entire flux will link with \( N_1 \). So mutual induction between \( N_1 \) and \( N_2 \) is
\[
M = L_{12} = \frac{N_1\phi_2}{i_2} = \frac{(250)(250\mu_0 i_2)}{i_2} = 78.54 \text{ mH}
\]

SOL 5.2.43 Option (A) is correct.
As the coil \( N_1 \) and \( N_2 \) are directly connected through an ideal core so entire flux will produced by \( N_2 \) will link with \( N_1 \) and so flux linked with \( N_3 \) will be zero.
Therefore the mutual inductance between $N_3$ and $N_2$ is zero.

**SOL 5.2.44** Option (A) is correct.

Given, the expression for magnetization curve,

$$B = \frac{1}{3}H + H^2 \mu \text{Wb/m}^2$$

The energy stored per unit volume in a magnetic material having linear magnetic flux density is defined as

$$w_m = \int_{H=0}^{H_0} H \cdot dB$$

Since, magnetic field intensity varied from 0 to $210 \text{ A/m}$ So, we have

$$w_m = \int_{H=0}^{210} H dB$$

Since,

$$\frac{dB}{dH} = \frac{1}{3} + 2H$$

So, putting it in equation we get,

$$w_m = \int_{H=0}^{210} H\left(\frac{1}{3} + 2H\right) dH$$

$$= \left[\frac{H^2}{6} + \frac{2H^3}{3}\right]_0^{210}$$

$$= 6.18 \times 10^6 \text{ J/m}^3 = 6.2 \text{ MJ/m}^3$$

***********
SOLUTIONS 5.3

SOL 5.3.1 Option (A) is correct.
From boundary condition we have the following relation between the magnetic field intensity in the two mediums:

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$  \hspace{1cm} (1)

and

$$\mathbf{H}_1 - \mathbf{H}_2 \times \mathbf{a}_{n12} = \mathbf{K}$$  \hspace{1cm} (2)

where $\mathbf{H}_1$ and $\mathbf{H}_2$ are the magnetic field intensity in the two mediums, $\mathbf{a}_{n12}$ is the unit vector normal to the interface of the mediums directed from medium 1 to medium 2 and $\mathbf{K}$ is the surface current density at the interface of the two mediums.

Now, the magnetic field intensity in medium 1 is

$$\mathbf{H}_1 = 3\mathbf{a}_x + 30\mathbf{a}_y \text{ A/m}$$

As the interface lies in the plane $x = 0$ so, we have

$$H_{n1} = 3\mathbf{a}_x$$

From equation (1), the normal component of the field intensity in medium 2 is given as

$$H_{n2} = \frac{H_{n1}}{2} = 1.5\mathbf{a}_x$$

Therefore, the net magnetic field intensity in medium 2 can be considered as

$$\mathbf{H}_2 = 1.5\mathbf{a}_x + A\mathbf{a}_y + B\mathbf{a}_z$$  \hspace{1cm} (3)

where $A$ and $B$ are the constants. So, from equation (2) we have

$$[3\mathbf{a}_x + 30\mathbf{a}_y] \times \mathbf{a}_x = 10\mathbf{a}_y$$

$$[1.5\mathbf{a}_x + (30 - A)\mathbf{a}_y - B\mathbf{a}_z] \times \mathbf{a}_x = 10\mathbf{a}_y$$

$$0 - (30 - A)\mathbf{a}_y - B\mathbf{a}_z = 10\mathbf{a}_y$$

Comparing the components in the two sides we get

$$30 - A = 0 \Rightarrow A = 30$$

and

$$-B = 10 \Rightarrow B = -10$$

Putting these values in equation (3) we get the magnetic field intensity in medium 2 as

$$\mathbf{H}_2 = 1.5\mathbf{a}_x + 30\mathbf{a}_y - 10\mathbf{a}_z \text{ A/m}$$

SOL 5.3.2 Option (B) is correct.

Given,

the magnetic moment $m = 2.5 \text{ A-m}^2$

Mass of magnet, $m = 6.6 \times 10^{-3} \text{ kg}$

density of steel, $\text{density} = 7.9 \times 10^3 \text{ kg/m}^3$
So, the net volume of the magnet bar is

\[ \frac{v}{\text{density}} = \frac{6.6 \times 10^{-3}}{7.9 \times 10^4} = 0.835 \times 10^{-6} \text{ m}^3 \]

Now, the magnetization of the magnet is defined as the magnetic moment per unit volume so, we get magnetization of the magnet bar as

\[ M = \frac{m}{v} = \frac{2.5}{0.835 \times 10^{-6}} = 3 \times 10^6 \text{ A/m} \]

**SOL 5.3.3**  Option (A) is correct.

Given,

Magnetic field intensity, \( H = \frac{5a_x}{\mu} \text{ A/m} \)

Current element, \( Idl = 4 \times 10^{-4}a_y \text{ A-m} \)

So, the magnetic flux density is given as

\[ B = \mu H = 5a_x \text{ A/m} \]

Since, the force exerted on a current element \( Idl \) placed in a magnetic field \( B \) is defined as

\[ F = (Idl) \times B \]

So, putting all the values we get,

\[ F = (4 \times 10^{-4}a_y) \times (5a_x) = -2 \times 10^{-4}a_y \text{ N} = -2a_z \text{ mN} \]

**SOL 5.3.4**  Option (C) is correct.

The electrical analogy of the magnetic field are listed below:

<table>
<thead>
<tr>
<th>Electrical field</th>
<th>Magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMF (electromotive force)</td>
<td>MMF (magneto motive force)</td>
</tr>
<tr>
<td>Electric current</td>
<td>Magnetic flux</td>
</tr>
<tr>
<td>Resistance</td>
<td>Reluctance</td>
</tr>
<tr>
<td>Conductivity</td>
<td>Permeability</td>
</tr>
</tbody>
</table>

So, for the given match list we get, \( A \rightarrow 3, B \rightarrow 2, C \rightarrow 4, D \rightarrow 1 \).

**SOL 5.3.5**  Option (C) is correct.

At the surface of discontinuity (interface between two medium) the normal component of magnetic flux density are related as

\[ B_{in} = B_{2n} \]

i.e. normal component of magnetic flux density is uniform at the surface of discontinuity.

Statement 1 is correct

At the boundary interface between two mediums, the normal component of the electric flux density is related as

\[ D_{2n} - D_{in} = \rho_s \]

i.e. discontinuous
where \( \rho_s \) is surface charge density at the interface. If the interface is charge free (\( \rho_s = 0 \)) then, the equation changes to
\[
D_{2n} = D_{1n}
\]
i.e. continuous
So, the normal component of flux density at the surface of discontinuity may or may not be continuous.

SOL 5.3.6 Option (C) is correct.
Magnetic current is composed of both displacement and conduction components.

SOL 5.3.7 Option (C) is correct.
Torque exerted on a loop with dipole moment \( M \) in a magnetic field \( B \) is defined as
\[
T = M \times B
\]

SOL 5.3.8 Option (B) is correct.
Biot Savart’s law gives the magnetic flux density as defined below
\[
B = \frac{\mu_0}{4\pi} \int \mathbf{l} \times \mathbf{R} \frac{I d l}{R^2} \quad (b \to 3)
\]
Displacement current is determined by using Maxwell’s equation as
\[
\nabla \times \mathbf{H} = \mathbf{J}_s + \mathbf{J}_d \quad \text{where } \mathbf{J}_d \text{ is displacement current density (c \to 1)}
\]
Time average power flow in a field wave is determined by Poynting vector as
\[
\mathcal{P}_{\text{ave}} = \frac{1}{2} \mathbf{E} \times \mathbf{H} \quad (d \to 2)
\]
Using Gauss’s law line charge distribution can be determined. (a \to 4)

SOL 5.3.9 Option (B) is correct.
Magnetic energy density in a magnetic field is defined as
\[
w_m = \frac{1}{2} \mathbf{J} \cdot \mathbf{A} \quad (c \to 4)
\]

SOL 5.3.10 Option ( ) is correct.
Consider the two wires carrying current as shown below:

The force exerted due to the wire 2 at wire 1 is given as
\[
\mathbf{F} = (I d l) \times (B)
\]
where \( I d l \) is the small current element of the wire 1 and \( B \) is magnetic flux density produced by wire 2 at wire 1. As determined by right hand rule the magnetic flux

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
density produced due to wire 2 at wire 1 is out of the paper.
Which will be towards wire 2. In the similar way the force due to wire 1 at wire 2
will be toward wire 1 i.e. attractive and perpendicular to the wire.

**SOL 5.3.11** Option (B) is correct.
From the boundary condition for magnetic field, we have the following derived
condition as
\[ \mu_1 H_{n1} = \mu_2 H_{n2} \]
and
\[ H_{t1} = H_{t2} \]

**SOL 5.3.12** Option (C) is correct.
The magnetic flux density \( B \) and magnetic field intensity \( H \) in a medium with
permeability \( \mu \) are related as
\[ B = \mu H = \mu_r \mu_0 H \]
Now, for the different magnetic material relative permeability \( \mu_r \) are listed below:
Free space (vacuum) \( \mu_r = 1 \)
Diamagnetic \( \mu_r \leq 1 \)
Paramagnetic \( \mu_r \geq 1 \)
Ferromagnetic \( \mu_r >> 1 \)
So, the \( B-H \) curve for the respective material has been shown below (depending
on their slopes \( \mu \)).

**SOL 5.3.13** Option (A) is correct.
When a dielectric material is placed in an electric field then the electric dipoles
are created in it. This phenomenon is called polarization of the dielectric material.
So, we conclude that both the statement are correct and statement (II) is correct
explanation of (I).

**SOL 5.3.14** Option (B) is correct.
For an inhomogenous magnetic material, magnetic permeability is a variable and
so, it has some finite gradient. Now, from maxwell’s equation we know
\[ \nabla \cdot B = 0 \]
Since,
\[ B = \mu H \]
So,
\[ \nabla \cdot B = \nabla \cdot (\mu H) \]
\[ 0 = \nabla \cdot \mu + \nabla \cdot H \]
In the above equation \( \nabla \cdot \mu \) have some finite value therefore,
$\nabla \cdot \mathbf{H} \neq 0$ (in inhomogenous medium)

**SOL 5.3.15** Option (C) is correct.
Force on a current element $Idl$ kept in a magnetic field $\mathbf{B}$ is defined as

$$F = \oint \mathbf{l} dl \times \mathbf{B}$$

$$= [(10)(2)\mathbf{a}_x] \times [0.05\mathbf{a}_z] = 1.0a_y \text{ N}$$

**SOL 5.3.16** Option (C) is correct.
Magnetic energy density in a magnetic field is defined as

$$w_m = \frac{1}{2} \mathbf{J} \cdot \mathbf{A}$$

where $\mathbf{J}$ is the current density and $\mathbf{A}$ is the magnetic vector potential.

**SOL 5.3.17** Option (D) is correct.
The force on a moving charge $q$ with the velocity $\mathbf{v}$ in a region having magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ is defined as

$$F = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

**SOL 5.3.18** Option (B) is correct.
The currents in the hairpin shaped wire flows as shown in the figure.

![Hairpin current](image)

As the direction of current are opposite so the force acting between them is repulsive, and So it tend to a straight line.

**SOL 5.3.19** Option (C) is correct.
Given, the Lorentz force equation,

$$\mathbf{F} = e(\mathbf{v} \times \mathbf{B})$$

If the particle is at rest then $\mathbf{v} = 0$ and so there will be no any deflection in particle due to the magnetic field.

**SOL 5.3.20** Option (D) is correct.
Force acting on a small point charge $q$ moving in an $EM$ wave is defined as

$$\mathbf{F} = q\mathbf{E} + q(\mathbf{v} + \mathbf{B})$$

So, for $q = 1$

$$\mathbf{F} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

**SOL 5.3.21** Option (B) is correct.
Given,
Current flowing in the conductor, $I = 5 \text{ A}$
Magnetic flux density, $B = 3\mathbf{a}_x + 4\mathbf{a}_y$
Since, the force experienced by a current carrying element \( ldl \) placed in a magnetic field \( B \) is defined as
\[
dF = (ldl) \times B
\]
As the current flowing in \( a_z \) direction so, we have
\[
dl = dl a_z
\]
and the force experienced by the conductor is
\[
dF = (5dl a_z) \times (3a_x + 4a_y)
\]
Therefore, the force per unit length experienced by the conductor is
\[
d\frac{F}{dl} = 15a_y - 20a_x
\]
\[
= -20a_x + 15a_y \text{ N/m}
\]

**SOL 5.3.22** Option (B) is correct.

From the boundary condition for magnetic field we have the following relation:
Normal component of magnetic flux density is continuous
i.e.
\[
B_n1 = B_n2
\]
Any field vector is the sum of its normal and tangential component to any surface
i.e.
\[
H_1 = H_{n1} + H_{t1}
\]
When the interface between two medium carries a uniform current \( K \) then the tangential component of magnetic field intensity is not uniform.
i.e. \( H_{t1} - H_{t2} = K \)
or, \( a_{n21} \times (H_1 - H_2) = K \)
But,
\[
B_2 \neq \sqrt{B_{n2} + B_{t2}}
\]

**SOL 5.3.23** Option (A) is correct.

Given, the magnetic flux density in medium 1 is
\[
B_1 = 1.2a_x + 0.8a_y + 0.4a_z
\]
and the interface lies in the plane \( z = 0 \).
So, the tangential and normal components of magnetic flux density in the two mediums are respectively:
\[
B_{n1} = 1.2a_x + 0.8a_y
\]
\[
B_{t1} = 0.4a_z
\]
and
\[
\frac{B_{n1}}{\mu_1} = \frac{B_{n2}}{\mu_2}
\]
Therefore, we get the field components in medium 2 as
\[
B_{2n} = B_{1n} = 0.4a_z
\]
and
\[
B_{2t} = B_{1t}\left(\frac{\mu_2}{\mu_1}\right) = \frac{1}{2}(1.2a_x + 0.8a_y) = (0.6a_x + 0.4a_y)
\]
Thus, the net magnetic flux density in region 2 is...
\[ B_2 = B_{2n} + B_{2t} \]
\[ = 0.6\mathbf{a}_x + 0.4\mathbf{a}_y + 0.4\mathbf{a}_z \]

So, the magnetic field intensity in region 2 is
\[ H_2 = \frac{B_2}{\mu_0} = \frac{1}{\mu_0}(0.6\mathbf{a}_x + 0.4\mathbf{a}_y + 0.4\mathbf{a}_z) \text{ A/m} \]

SOL 5.3.24 Option (C) is correct.

Energy stored in a magnetic field is defined as
\[ W_m = \frac{1}{2} \int \mathbf{A} \cdot \mathbf{J} \, dv \]

So, \( \int \mathbf{A} \cdot \mathbf{J} \, dv \) has the units of energy.

*******
CHAPTER 6

TIME VARYING FIELD AND MAXWELL EQUATION
EXERCISE 6.1

MCQ 6.1.1 Match List I with List II and select the correct answer using the codes given below (Notations have their usual meaning)

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a  Ampere’s circuital law</td>
<td>1. $\nabla \cdot D = \rho_v$</td>
</tr>
<tr>
<td>b  Faraday’s law</td>
<td>2. $\nabla \cdot B = 0$</td>
</tr>
<tr>
<td>c  Gauss’s law</td>
<td>3. $\nabla \times E = -\frac{\partial B}{\partial t}$</td>
</tr>
<tr>
<td>d  Non existence of isolated magneticharge</td>
<td>4. $\nabla \times H = J + \frac{\partial D}{\partial t}$</td>
</tr>
</tbody>
</table>

Codes:
(A) 4 3 2 1         (B) 4 1 3 2         (C) 2 3 1 4         (D) 4 3 1 2

MCQ 6.1.2 Magneto static fields is caused by
(A) stationary charges (B) steady currents
(C) time varying currents (D) none of these

MCQ 6.1.3 Let $\mathbf{A}$ be magnetic vector potential and $\mathbf{E}$ be electric field intensity at certain time in a time varying EM field. The correct relation between $\mathbf{E}$ and $\mathbf{A}$ is
(A) $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$ (B) $\mathbf{A} = -\frac{\partial \mathbf{E}}{\partial t}$
(C) $\mathbf{E} = \frac{\partial \mathbf{A}}{\partial t}$ (D) $\mathbf{A} = \frac{\partial \mathbf{E}}{\partial t}$

MCQ 6.1.4 A closed surface $\mathbf{S}$ defines the boundary line of magnetic medium such that the field intensity inside it is $\mathbf{B}$. Total outward magnetic flux through the closed surface will be
(A) $\mathbf{B} \cdot \mathbf{S}$ (B) 0
(C) $\mathbf{B} \times \mathbf{S}$ (D) none of these
A perfect conducting sphere of radius $r$ is such that it’s net charge resides on the surface. At any time $t$, magnetic field $B(r,t)$ inside the sphere will be

(A) 0
(B) uniform, independent of $r$
(C) uniform, independent of $t$
(D) uniform, independent of both $r$ and $t$

The total magnetic flux through a conducting loop having electric field $E = 0$ inside it will be

(A) 0
(B) constant
(C) varying with time only
(D) varying with time and area of the surface both

A cylindrical wire of a large cross section made of superconductor carries a current $I$. The current in the superconductor will be confined.

(A) inside the wire  
(B) to the axis of cylindrical wire
(C) to the surface of the wire  
(D) none of these

If $B_i$ denotes the magnetic flux density increasing with time and $B_d$ denotes the magnetic flux density decreasing with time then which of the configuration is correct for the induced current $I$ in the stationary loop?

(A)  
(B)  
(C)  
(D)  

A circular loop is rotating about $z$-axis in a magnetic field $B = B_0 \cos \omega t a_z$. The total induced voltage in the loop is caused by

(A) Transformer emf 
(B) motion emf.
(C) Combination of (A) and (B)  
(D) none of these
MCQ 6.1.10  A small conducting loop is released from rest with in a vertical evacuated cylinder voltage induced in the falling loop is
(Assume earth magnetic field = 2 × 10⁻⁶ T at a constant angle of 10° below the horizontal)
(A) zero  (B) 1 mV
(C) 17.34 mV  (D) 9.8 mV

MCQ 6.1.11  A square loop of side 2 m is located in the plane x = 0 as shown in figure. A non-uniform magnetic flux density through it is given as
\[ B = 4z^2 t^2 \mathbf{a}_z, \]
The emf induced in the loop at time \( t = 2 \) sec will be
(A) 16 volt  (B) -4 volt
(C) 4 volt  (D) -2 volt

MCQ 6.1.12  A very long straight wire carrying a current \( I = 3 \) A is placed at a distance of 4 m from a square loop as shown in figure. If the side of the square loop is 4 m then the total flux passing through the square loop will be
\[ \Phi = \frac{\mu_0 I}{4 \pi} a^2 \]
\( a = 1 \) m
\( d = 2 \) m

(A) \( 0.81 \times 10^{-7} \) wb  (B) \( 10^{-6} \) wb
(C) \( 4.05 \times 10^{-7} \) wb  (D) \( 2.0 \times 10^{-7} \) wb
MCQ 6.1.13 A straight conductor $ab$ of length $l$ lying in the $xy$ plane is rotating about the centre $a$ at an angular velocity $\omega$ as shown in the figure.

If a magnetic field $B$ is present in the space directed along $a_z$ then which of the following statement is correct ?

(A) $V_{ab}$ is positive  
(B) $V_{ab}$ is negative  
(C) $V_{ba}$ is positive  
(D) $V_{ba}$ is zero

MCQ 6.1.14 In a certain region magnetic flux density is given as $B = 0.2t\mathbf{a}$, Wb/m². An electric loop with resistance $2\Omega$ and $4\Omega$ is lying in $x$-$y$ plane as shown in the figure.

If the area of the loop is $2\text{m}^2$ than, the voltage drop $V_1$ and $V_2$ across the two resistances is respectively

(A) $66.7 \text{ mV}$ and $33.3 \text{ mV}$  
(B) $33.3 \text{ mV}$ and $66.7 \text{ mV}$  
(C) $50 \text{ mV}$ and $100 \text{ mV}$  
(D) $100 \text{ mV}$ and $50 \text{ mV}$

MCQ 6.1.15 Assertion (A) : A small piece of bar magnet takes several seconds to emerge at bottom when it is dropped down a vertical aluminum pipe where as an identical unmagnetized piece takes a fraction of second to reach the bottom.

Reason (R) : When the bar magnet is dropped inside a conducting pipe, force exerted on the magnet by induced eddy current is in upward direction.

(A) Both A and R are true and R is correct explanation of A.  
(B) Both A and R are true but R is not the correct explanation of A. 
(C) A is true but R is false.  
(D) A is false but R is true.
MCQ 6.1.16  A magnetic core of uniform cross section having two coils (Primary and secondary) wound on it as shown in figure. The no. of turns of primary coil is 5000 and no. of turns of secondary coil is 3000. If a voltage source of 4 Volts is connected across the primary coil then what will be the voltage across the secondary coil?

![Diagram of magnetic core with primary and secondary coils](image)

\( V_1 = 12 \text{ Volt} \)

\( N_1 = 5000 \)

\( N_2 = 3000 \)

(A) 72 volt  
(B) 7.2 volt  
(C) 20 volt  
(D) \(-7.2 \text{ volt}\)

MCQ 6.1.17  Self inductance of a long solenoid having \( n \) turns per unit length will be proportional to

(A) \( n \)  
(B) \( \frac{1}{n} \)  
(C) \( n^2 \)  
(D) \( \frac{1}{n^2} \)

MCQ 6.1.18  A wire with resistance \( R \) is looped on a solenoid as shown in figure.

![Diagram of wire looped on solenoid](image)

If a constant current is flowing in the solenoid then the induced current flowing in the loop with resistance \( R \) will be

(A) non uniform  
(B) constant  
(C) zero  
(D) none of these

MCQ 6.1.19  A long straight wire carries a current \( I = I_0 \cos(\omega t) \). If the current returns along a coaxial conducting tube of radius \( r \) as shown in figure then magnetic field and electric field inside the tube will be respectively.

![Diagram of long straight wire and coaxial conducting tube](image)
For View Only

<table>
<thead>
<tr>
<th>For View Only</th>
<th>Shop Online at <a href="http://www.nodia.co.in">www.nodia.co.in</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) radial, longitudinal</td>
<td>(B) circumferential, longitudinal</td>
</tr>
<tr>
<td>(C) circumferential, radial</td>
<td>(D) longitudinal, circumferential</td>
</tr>
</tbody>
</table>

**MCQ 6.1.20**

**Assertion (A):** Two coils are wound around a cylindrical core such that the primary coil has \( N_1 \) turns and the secondary coils has \( N_2 \) turns as shown in figure. If the same flux passes through every turn of both coils then the ratio of emf induced in the two coils is

\[
\frac{V_{emf2}}{V_{emf1}} = \frac{N_2}{N_1}
\]

**Reason (R):** In a primitive transformer, by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

**MCQ 6.1.21**

In a non magnetic medium electric field \( E = E_0 \cos \omega t \) is applied. If the permittivity of medium is \( \varepsilon \) and the conductivity is \( \sigma \) then the ratio of the amplitudes of the conduction current density and displacement current density will be

(A) \( \mu_0/\varepsilon \omega \)  
(B) \( \sigma/\varepsilon \omega \)  
(C) \( \sigma \mu_0/\varepsilon \omega \)  
(D) \( \omega \varepsilon/\sigma \)

**MCQ 6.1.22**

In a medium where no D.C. field is present, the conduction current density at any point is given as \( J = 5 \cos(1.5 \times 10^8 t) a_y \text{A/m}^2 \). Electric flux density in the medium will be

(A) \( 133.3 \sin(1.5 \times 10^8 t) a_y \text{nC/m}^2 \)  
(B) \( 13.3 \sin(1.5 \times 10^8 t) a_y \text{nC/m}^2 \)  
(C) \( 1.33 \sin(1.5 \times 10^8 t) a_y \text{nC/m}^2 \)  
(D) \( -1.33 \sin(1.5 \times 10^8 t) a_y \text{nC/m}^2 \)

**MCQ 6.1.23**

In a medium, the permittivity is a function of position such that \( \nabla \varepsilon \approx 0 \). If the volume charge density inside the medium is zero then \( \nabla \cdot E \) is roughly equal to

(A) \( \varepsilon E \)  
(B) \( -\varepsilon E \)  
(C) 0  
(D) \( -\nabla \varepsilon \cdot E \)
MCQ 6.1.24  A conducting medium has permittivity, \( \varepsilon = 4\varepsilon_0 \) and conductivity, \( \sigma = 1.14 \times 10^8 \text{s/m} \). What will be the ratio of magnitude of displacement current and conduction current in the medium at 50 GHz ?

(A) \( 1.10 \times 10^4 \)  
(B) \( 1.025 \times 10^7 \)  
(C) \( 9.75 \times 10^{-17} \)  
(D) \( 9.75 \times 10^{-8} \)

MCQ 6.1.25  In free space, the electric field intensity at any point \((r, \theta, \phi)\) in spherical coordinate system is given by

\[
E = \frac{\sin \theta \cos(\omega t - kr)}{r} a_\phi
\]

The phasor form of magnetic field intensity in the free space will be

(A) \( \frac{k \sin \theta}{\omega \mu_0 r} e^{-jkr} a_\phi \)  
(B) \( -\frac{k \sin \theta}{\omega \mu_0 r} e^{jkr} a_\phi \)  
(C) \( \frac{k \omega \mu_0}{r} e^{-jkr} a_\phi \)  
(D) \( \frac{k \sin \theta}{r} e^{-jkr} a_\phi \)

MCQ 6.1.26  Magnetic field intensity in free space is given as

\[
H = 0.2 \cos(15\pi y) \sin(6\pi \times 10^8 t - bx) a_z, \text{ A/m}
\]

It satisfies Maxwell’s equation when \( b \) equals to

(A) \( \pm 46.5 \text{ rad/m} \)  
(B) \( \pm 41.6 \text{ rad/m} \)  
(C) \( \pm 77.5 \text{ rad/m} \)  
(D) \( \pm 60.28 \text{ rad/m} \)

***********
MCQ 6.2.1 Two parallel conducting rails are being placed at a separation of 3 m as shown in figure. One end of the rail is being connected through a resistor \( R = 10 \Omega \) and the other end is kept open. A metal bar slides frictionlessly on the rails at a speed of 5 m/s away from the resistor. If the magnetic flux density \( B = 0.2 \text{ Wb/m}^2 \) pointing out of the page fills entire region then the current \( I \) flowing in the resistor will be

(A) 0.01 A  
(B) -0.01 A  
(C) 1 A  
(D) -0.1 A

Common Data for Question 2 - 3:

A conducting wire is formed into a square loop of side 4 m. A very long straight wire carrying a current \( I = 30 \text{ A} \) is located at a distance 2 m from the square loop as shown in figure.

MCQ 6.2.2 If the loop is pulled away from the straight wire at a velocity of 5 m/s then the induced e.m.f. in the loop after 0.6 sec will be

(A) 5 \( \mu \text{volt} \)  
(B) 2.5 \( \mu \text{volt} \)  
(C) 25 \( \mu \text{volt} \)  
(D) 5 mV
MCQ 6.2.3
If the loop is pulled downward in the parallel direction to the straight wire, such that distance between the loop and wire is always 3 m then the induced e.m.f. in the loop at any time $t$ will be
(A) linearly increasing with $t$  
(B) always 0  
(C) linearly decreasing with $t$  
(D) always constant but not zero.

MCQ 6.2.4
An infinitely long straight wire with a closed switch $S$ carries a uniform current $I = 4 \text{ A}$ as shown in figure. A square loop of side $a = 2 \text{ m}$ and resistance $R = 4 \Omega$ is located at a distance 4 m from the wire. Now at any time $t = t_0$ the switch is open so the current $I$ drops to zero. What will be the total charge that passes through a corner of the square loop after $t = t_0$?
(A) 277 nC  
(B) 693 nC  
(C) −237 nC  
(D) 139 nC

MCQ 6.2.5
A circular loop of radius 5 m carries a current $I = 2 \text{ A}$. If another small circular loop of radius 1 mm lies a distance 8 m above the large circular loop such that the planes of the two loops are parallel and perpendicular to the common axis as shown in figure then total flux through the small loop will be
(A) 1.62 fWb  
(B) 25.3 nWb  
(C) 44.9 fWb  
(D) 45.4 pWb
MCQ 6.2.6  A non magnetic medium at frequency \( f = 1.6 \times 10^8 \text{ Hz} \) has permittivity \( \varepsilon = 54\varepsilon_0 \) and resistivity \( \rho = 0.77 \Omega \cdot \text{m} \). What will be the ratio of amplitudes of conduction current to the displacement current?

(A) 0.43  
(B) 0.37  
(C) 1.16  
(D) 2.70

MCQ 6.2.7  Two voltmeters \( A \) and \( B \) with internal resistances \( R_A \) and \( R_B \) respectively is connected to the diametrically opposite points of a long solenoid as shown in figure. Current in the solenoid is increasing linearly with time. The correct relation between the voltmeter’s reading \( V_A \) and \( V_B \) will be

\[
\frac{V_A}{V_B} = \frac{R_A}{R_B}
\]

(A) \( V_A = V_B \)  
(B) \( V_A = -V_B \)  
(C) \( V_A = \frac{R_A}{R_B} \)  
(D) \( V_A = -\frac{R_A}{R_B} \)

Statement for Linked Question 8 - 9 :

Two parallel conducting rails are being placed at a separation of 6 m with a resistance \( R = 10 \Omega \) connected across it’s one end. A conducting bar slides frictionlessly on the rails with a velocity of 4 m/s away from the resistance as shown in the figure.

MCQ 6.2.8  If a uniform magnetic field \( B = 4 \text{ Tesla} \) pointing out of the page fills entire region then the current \( I \) flowing in the bar will be

(A) 0 A  
(B) -40 A  
(C) 4 A  
(D) -4 A
MCQ 6.2.9 The force exerted by magnetic field on the sliding bar will be
(A) 4 N, opposes its motion
(B) 40 N, opposes its motion
(C) 40 N, in the direction of its motion
(D) 0

MCQ 6.2.10 Two small resistor of 225 Ω each is connected through a perfectly conducting filament such that it forms a square loop lying in x-y plane as shown in the figure. Magnetic flux density passing through the loop is given as

$$B = -7.5 \cos(120\pi t - 30^\circ) a_z$$

The induced current $$I(t)$$ in the loop will be

- (A) $$0.02 \sin(120\pi t - 30^\circ)$$
- (B) $$2.8 \times 10^3 \sin(120\pi t - 30^\circ)$$
- (C) $$-5.7 \sin(120\pi t - 30^\circ)$$
- (D) $$5.7 \sin(120\pi t - 30^\circ)$$

Common Data for Question 11 - 12:
In a non uniform magnetic field $$B = 8x^2 a_z$$ Tesla, two parallel rails with a separation of 10 m and connected with a voltmeter at its one end is located in x-y plane as shown in figure. The position of the bar which is sliding on the rails is given as

$$x = t(1 + 0.4t^2)$$

MCQ 6.2.11 Voltmeter reading at $$t = 0.4$$ sec will be

MCQ 6.2.12 Voltmeter reading at $x = 12$ cm will be
(A) 12.27 mvolt  
(B) −14.64 mvolt  
(C) 23.4 mvolt  
(D) −23.4 mvolt

MCQ 6.2.13 A rectangular loop of self inductance $L$ is placed near a very long wire carrying current $i_1$ as shown in figure (a). If $i_1$ be the rectangular pulse of current as shown in figure (b) then the plot of the induced current $i_2$ in the loop versus time $t$ will be (assume the time constant of the loop, $\tau \gg L/R$)

MCQ 6.2.14 Two parallel conducting rails is placed in a varying magnetic field $B = 0.2 \cos \omega t a_x$. A conducting bar oscillates on the rails such that it’s position is given by $y = 0.25(1 - \cos \omega t)$ m
If one end of the rails are terminated in a resistance $R = 5 \Omega$, then the current $i$ flowing in the rails will be

$$i = 0.01\omega\sin\omega t(1 + 2\cos\omega t)$$

(A) $0.01\omega\sin\omega t(1 + 2\cos\omega t)$
(B) $-0.01\omega\sin\omega t(1 + 2\cos\omega t)$
(C) $0.01\omega\cos\omega t(1 + 2\sin\omega t)$
(D) $0.05\omega\sin\omega t(1 + 2\sin\omega t)$

MCQ 6.2.15 Electric flux density in a medium ($\varepsilon_r = 10$, $\mu_r = 2$) is given as

$$D = 2.33\sin(3 \times 10^8 t - 0.2x)\mathbf{a}_y \mu C/m^2$$

Magnetic field intensity in the medium will be

(A) $10^{-5}\sin(3 \times 10^8 t - 0.2x)\mathbf{a}_y$ A/m
(B) $2\sin(3 \times 10^8 t - 0.2x)\mathbf{a}_y$ A/m
(C) $-4\sin(3 \times 10^8 t - 0.2x)\mathbf{a}_y$ A/m
(D) $4\sin(3 \times 10^8 t - 0.2x)\mathbf{a}_y$ A/m

MCQ 6.2.16 In a non conducting medium ($\sigma = 0$) magnetic field intensity at any point is given by $H = \cos(10^8 t - bx)\mathbf{a}_y$ A/m. The permittivity of the medium is $\varepsilon = 0.12$ nF/m and permeability of the medium is $\mu = 3 \times 10^{-5}$ H/m. If no D.C. field is present in medium, then value of $b$ for which the field satisfies Maxwell’s equation is

(A) $-600$ rad/s
(B) $600$ rad/m
(C) $3.6 \times 10^5$ rad/m
(D) (A) and (B) both

MCQ 6.2.17 A current filament located on the $x$-axis in free space with in the interval $-0.1 < x < 0.1$ m carries current $I(t) = 8t A$ in $\mathbf{a}_x$ direction. If the retarded vector potential at point $P(0,0,2)$ be $\mathbf{A}(t)$ then the plot of $\mathbf{A}(t)$ versus time will be
MCQ 6.2.18 In a non-conducting medium \((\sigma = 0, \mu = \epsilon = 2)\), the retarded potentials are given as \(V = y(x - ct)\) volt and \(A = y(x - ct)\) Wb/m where \(c\) is velocity of waves in free space. The field (electric and magnetic) inside the medium satisfies Maxwell’s equation if
(A) \(J = 0\) only
(B) \(\rho = 0\) only
(C) \(J = \rho = 0\)
(D) Can’t be possible

MCQ 6.2.19 Electric field in free space in given as
\[ E = 5\sin(10\pi y)\cos(6\pi \times 10^9 - bx)a_z \]
It satisfies Maxwell’s equation for \(b = \) ?
(A) \(\pm 20\pi\) rad/m
(B) \(\pm 300\pi\) rad/m
(C) 10\pi rad/m
(D) 30\pi rad/m

Statement for Linked Question 20 - 21:
In a region of electric and magnetic fields \(E\) and \(B\), respectively, the force experienced by a test charge \(qC\) are given as follows for three different velocities.

<table>
<thead>
<tr>
<th>Velocity m/sec</th>
<th>Force, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_x)</td>
<td>(2q(a_x + a_y))</td>
</tr>
<tr>
<td>(a_y)</td>
<td>(q a_y)</td>
</tr>
<tr>
<td>(a_z)</td>
<td>(q(2a_y + a_z))</td>
</tr>
</tbody>
</table>

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 6.2.20 What will be the magnetic field $B$ in the region?
(A) $a_x$  (B) $a_x - a_y$
(C) $a_z$  (D) $a_y - a_z$

MCQ 6.2.21 What will be electric field $E$ in the region?
(A) $a_x - a_z$  (B) $a_y - a_z$
(C) $a_y + a_z$  (D) $a_y + a_z - a_x$

MCQ 6.2.22 In Cartesian coordinates magnetic field is given by $B = \frac{2}{x} a_z$. A square loop of side 2 m is lying in $xy$ plane and parallel to the $y$-axis. Now, the loop is moving in that plane with a velocity $v = 4a_x$ as shown in the figure.

What will be the circulation of the induced electric field around the loop?
(A) $\frac{16}{x(x+2)}$  (B) $\frac{8}{x}$
(C) $\frac{8}{x(x+2)}$  (D) $\frac{x(x+2)}{16}$

Common Data for Question 23 - 24:
In a cylindrical coordinate system, magnetic field is given by
\[ B = \begin{cases} \\ 2\sin\omega t a_z & \text{for } 5 < \rho < 6 \text{ m} \\ 0 & \text{for } \rho > 6 \text{ m} \\ \end{cases} \]

MCQ 6.2.23 The induced electric field in the region $\rho < 4 \text{ m}$ will be
(A) 0  (B) $\frac{2\omega \cos \omega t}{\rho} a_\phi$
(C) $-2\cos \omega t a_\phi$  (D) $\frac{1}{2\sin \omega t} a_\phi$

MCQ 6.2.24 The induced electric field at $\rho = 4.5 \text{ m}$ is
(A) 0  (B) $-\frac{17\omega \cos \omega t}{18}$
(C) $\frac{4\omega \cos \omega t}{9}$  (D) $-\frac{17\omega \cos \omega t}{4}$
MCQ 6.2.25 The induced electric field in the region $\rho > 5 \, \text{m}$ is
(A) $-\frac{18}{\rho} \omega \cos \omega t \mathbf{a}_\phi$
(B) $-\frac{9}{\rho} \omega \cos \omega t \mathbf{a}_\phi$
(C) $-9 \rho \cos \omega t \mathbf{a}_\phi$
(D) $\frac{9}{\rho} \omega \cos \omega t \mathbf{a}_\phi$

MCQ 6.2.26 In a certain region a test charge is moving with an angular velocity $2 \, \text{rad/ sec}$ along a circular path of radius $4 \, \text{m}$ centred at origin in the $x$-$y$ plane. If the magnetic flux density in the region is $\mathbf{B} = 2 \mathbf{a}_r \, \text{Wb/m}^2$ then the electric field viewed by an observer moving with the test charge is
(A) $8 \mathbf{a}_r \, \text{V/m}$
(B) $4 \mathbf{a}_r \, \text{V/m}$
(C) $0$
(D) $-8 \mathbf{a}_r \, \text{V/m}$

MCQ 6.2.27 A $8 \, \text{A}$ current is flowing along a straight wire from a point charge situated at the origin to infinity and passing through the point $(1, 1, 1)$. The circulation of the magnetic field intensity around the closed path formed by the triangle having the vertices $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$ is equal to
(A) $\frac{7}{8} \, \text{A}$
(B) $3 \, \text{A}$
(C) $7 \, \text{A}$
(D) $1 \, \text{A}$

MCQ 6.2.28 Magnetic flux density, $\mathbf{B} = 0.1 \, t \, \mathbf{a}_z \, \text{Tesla}$ threads only the loop $abcd$ lying in the plane $xy$ as shown in the figure.

Consider the three voltmeters $V_1$, $V_2$ and $V_3$, connected across the resistance in the same $xy$ plane. If the area of the loop $abcd$ is $1 \, \text{m}^2$ then the voltmeter readings are

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>66.7 mV</td>
<td>33.3 mV</td>
<td>66.7 mV</td>
</tr>
<tr>
<td>(B)</td>
<td>33.3 mV</td>
<td>66.7 mV</td>
<td>33.3 mV</td>
</tr>
<tr>
<td>(C)</td>
<td>66.7 mV</td>
<td>66.7 mV</td>
<td>33.3 mV</td>
</tr>
<tr>
<td>(D)</td>
<td>33.3 mV</td>
<td>66.7 mV</td>
<td>66.7 mV</td>
</tr>
</tbody>
</table>
Statement for Linked Question 29 - 30:
A square wire loop of resistance $R$ rotated at an angular velocity $\omega$ in the uniform magnetic field $B = 2a_y$ mWb/m$^2$ as shown in the figure.

**MCQ 6.2.29** If the angular velocity, $\omega = 2$ rad/sec then the induced e.m.f. in the loop will be
(A) $2 \sin \theta \mu$V/m
(B) $2 \cos \theta \mu$V/m
(C) $4 \cos \theta \mu$V/m
(D) $4 \sin \theta \mu$V/m

**MCQ 6.2.30** If resistance, $R = 40$ m$\Omega$ then the current flowing in the square loop will be
(A) $0.2 \sin \theta$ mA
(B) $0.1 \sin \theta$ mA
(C) $0.1 \cos \theta$ mA
(D) $0.5 \sin \theta$ mA

**MCQ 6.2.31** In a certain region magnetic flux density is given as $B = B_0 \sin \omega t a_y$. A rectangular loop of wire is defined in the region with it’s one corner at origin and one side along $z$-axis as shown in the figure.

If the loop rotates at an angular velocity $\omega$ (same as the angular frequency of magnetic field) then the maximum value of induced e.m.f in the loop will be
(A) $\frac{1}{2} B_0 S \omega$
(B) $2 B_0 S \omega$
(C) $B_0 S \omega$
(D) $4 B_0 S \omega$
**MCQ 6.2.32**

A 50 turn rectangular loop of area 64 cm$^2$ rotates at 60 revolution per seconds in a magnetic field $B = 0.25 \sin 377t$ Wb/m$^2$ directed normal to the axis of rotation. The rms value of the induced voltage is

(A) 2.13 volt  
(B) 21.33 volt  
(C) 4.26 volt  
(D) 42.66 volt

**Statement for Linked Question 33 - 34 :**

Consider the figure shown below. Let $B = 5 \cos 120\pi t$ Wb/m$^2$ and assume that the magnetic field produced by $i(t)$ is negligible

**MCQ 6.2.33**

The value of $v_{ab}$ is

(A) $-118.43 \cos 120\pi t$ V  
(B) $118.43 \cos 120\pi t$ V  
(C) $-118.43 \sin 120\pi t$ V  
(D) $118.43 \sin 120\pi t$ V

**MCQ 6.2.34**

The value of $i(t)$ is

(A) $-0.47 \cos 120\pi t$ A  
(B) $0.47 \cos 120\pi t$ A  
(C) $-0.47 \sin 120\pi t$ A  
(D) $0.47 \sin 120\pi t$ A

***********
EXERCISE 6.3

MCQ 6.3.1
A magnetic field in air is measured to be \( \mathbf{B} = B_0 \left( \frac{x}{x^2 + y^2} \mathbf{a}_x - \frac{y}{x^2 + y^2} \mathbf{a}_y \right) \) What current distribution leads to this field?

[Hint: The algebra is trivial in cylindrical coordinates.]

(A) \( \mathbf{J} = \frac{B_0 z}{\mu_0} \left( \frac{1}{x^2 + y^2} \right) r \neq 0 \)

(B) \( \mathbf{J} = \frac{B_0 z}{\mu_0} \left( \frac{2}{x^2 + y^2} \right) r \neq 0 \)

(C) \( \mathbf{J} = 0, r \neq 0 \)

(D) \( \mathbf{J} = \frac{B_0 z}{\mu_0} \left( \frac{1}{x^2 + y^2} \right) r \neq 0 \)

MCQ 6.3.2
For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of Maxwell’s equations?

(A) \( \nabla \cdot \mathbf{E} = 0, \nabla \times \mathbf{B} = 0 \)

(B) \( \nabla \cdot \mathbf{E} = 0, \nabla \cdot \mathbf{B} = 0 \)

(C) \( \nabla \times \mathbf{E} = 0, \nabla \times \mathbf{B} = 0 \)

(D) \( \nabla \times \mathbf{E} = 0, \nabla \cdot \mathbf{B} = 0 \)

MCQ 6.3.3
If \( C \) is closed curve enclosing a surface \( S \), then magnetic field intensity \( \mathbf{H} \), the current density \( \mathbf{J} \) and the electric flux density \( \mathbf{D} \) are related by

(A) \( \int_S \mathbf{H} \cdot d\mathbf{S} = \int_C \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{l} \)

(B) \( \int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S} \)

(C) \( \int_S \mathbf{H} \cdot d\mathbf{S} = \int_C \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{l} \)

(D) \( \int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S} \)

MCQ 6.3.4
The unit of \( \nabla \times \mathbf{H} \) is

(A) Ampere

(B) Ampere/meter

(C) Ampere/meter^2

(D) Ampere-meter

MCQ 6.3.5
The Maxwell equation \( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \) is based on

(A) Ampere’s law

(B) Gauss’ law

(C) Faraday’s law

(D) Coulomb’s law

MCQ 6.3.6
A loop is rotating about the \( y \)-axis in a magnetic field \( \mathbf{B} = B_0 \cos(\omega t + \phi) \mathbf{a}_x \), T. The voltage in the loop is

(A) zero

(B) due to rotation only

(C) due to transformer action only

(D) due to both rotation and transformer action
MCQ 6.3.7
IES EC 2012
The credit of defining the following current is due to Maxwell
(A) Conduction current (B) Drift current
(C) Displacement current (D) Diffusion current

MCQ 6.3.8
IES EC 2011
A varying magnetic flux linking a coil is given by \( \Phi = \frac{2}{3} \lambda t^3 \). If at time \( t = 3 \) s, the emf induced is 9 V, then the value of \( \lambda \) is.
(A) zero (B) 1 Wb/s² (C) -1 Wb/s² (D) 9 Wb/s²

MCQ 6.3.9
IES EC 2011
Assuming that each loop is stationary and time varying magnetic field \( B \), induces current \( I \), which of the configurations in the figures are correct?

1. Increasing \( B \)
2. Decreasing \( B \)
3. Decreasing \( B \)
4. Increasing \( B \)

(A) 1, 2, 3 and 4 (B) 1 and 3 only
(C) 2 and 4 only (D) 3 and 4 only

MCQ 6.3.10
IES EC 2011
Assertion (A) : For time varying field the relation \( E = -\nabla V \) is inadequate.
Reason (R) : Faraday’s law states that for time varying field \( \nabla \times E = 0 \)
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true
MCQ 6.3.11  Who developed the concept of time varying electric field producing a magnetic field?
(A) Gauss  (B) Faraday  
(C) Hertz  (D) Maxwell

MCQ 6.3.12  A single turn loop is situated in air, with a uniform magnetic field normal to its plane. The area of the loop is 5 m² and the rate of change of flux density is 2 Wb/m²/s. What is the emf appearing at the terminals of the loop?
(A) -5 V  (B) -2 V  
(C) -0.4 V  (D) -10 V

MCQ 6.3.13  Which of the following equations results from the circuital form of Ampere’s law?
(A) \( \nabla \times E = -\frac{\partial B}{\partial t} \)  (B) \( \nabla \cdot B = 0 \)  
(C) \( \nabla \cdot D = \rho \)  (D) \( \nabla \times H = J + \frac{\partial D}{\partial t} \)

MCQ 6.3.14  Assertion (A) : Capacitance of a solid conducting spherical body of radius \( a \) is given by \( 4\pi \varepsilon_0 a \) in free space. 
Reason (R) : \( \nabla \times H = j\omega E + J \)  
(A) Both A and R are individually true and R is the correct explanation of A.  
(B) Both A and R are individually true but R is not the correct explanation of A.  
(C) A is true but R is false  
(D) A is false but R is true

MCQ 6.3.15  Two conducting thin coils \( X \) and \( Y \) (identical except for a thin cut in coil \( Y \) ) are placed in a uniform magnetic field which is decreasing at a constant rate. If the plane of the coils is perpendicular to the field lines, which of the following statement is correct? As a result, emf is induced in
(A) both the coils  
(B) coil \( Y \) only  
(C) coil \( X \) only  
(D) none of the two coils

MCQ 6.3.16  Assertion (A) : Time varying electric field produces magnetic fields. 
Reason (R) : Time varying magnetic field produces electric fields.  
(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true but R is NOT the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true
MCQ 6.3.17
Match List I (Electromagnetic Law) with List II (Different Form) and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Ampere’s law</td>
<td>1. ( \nabla \cdot D = \rho_e )</td>
</tr>
<tr>
<td>b. Faraday’s law</td>
<td>2. ( \nabla \cdot J = -\frac{\partial H}{\partial t} )</td>
</tr>
<tr>
<td>c. Gauss law</td>
<td>3. ( \nabla \times H = J + \frac{\partial D}{\partial t} )</td>
</tr>
<tr>
<td>d. Current</td>
<td>4. ( \nabla \times E = -\frac{\partial B}{\partial t} )</td>
</tr>
</tbody>
</table>

Codes:

- (A) 1 2 3 4
- (B) 3 4 1 2
- (C) 1 4 3 2
- (D) 3 2 1 4

MCQ 6.3.18
Two metal rings 1 and 2 are placed in a uniform magnetic field which is decreasing with time with their planes perpendicular to the field. If the rings are identical except that ring 2 has a thin air gap in it, which one of the following statements is correct?

- (A) No e.m.f is induced in ring 1
- (B) An e.m.f is induced in both the rings
- (C) Equal Joule heating occurs in both the rings
- (D) Joule heating does not occur in either ring.

MCQ 6.3.19
Which one of the following Maxwell’s equations gives the basic idea of radiation?

- (A) \( \nabla \times H = \frac{\partial D}{\partial t} \)
- (B) \( \nabla \cdot D = -\frac{\partial B}{\partial t} \)
- (C) \( \nabla \cdot D = \rho \)
- (D) \( \nabla \times E = -\frac{\partial B}{\partial t} \)

MCQ 6.3.20
Which one of the following is NOT a correct Maxwell equation?

- (A) \( \nabla \times H = \frac{\partial D}{\partial t} + J \)
- (B) \( \nabla \times E = \frac{\partial H}{\partial t} \)
- (C) \( \nabla \cdot D = \rho \)
- (D) \( \nabla \cdot B = 0 \)
Match List I (Maxwell equation) with List II (Description) and select the correct answer:

List I

a. \( \oint B \cdot dS = 0 \)
b. \( \oint D \cdot dS = \int \rho, dv \)
c. \( \oint E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot dS \)
d. \( \oint H \cdot dl = \int \frac{\partial (D + J)}{\partial t} \cdot dS \)

List II

1. The mmf around a closed path is equal to the conduction current plus the time derivative of the electric displacement current through any surface bounded by the path.
2. The emf around a closed path is equal to the time derivative is equal to the time derivative of the magnetic displacement through any surface bounded by the path.
3. The total electric displacement through the surface enclosing a volume is equal to total charge within the volume.
4. The net magnetic flux emerging through any closed surface is zero.

Codes:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(B)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(C)</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>(D)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

The equation of continuity defines the relation between

(A) electric field and magnetic field
(B) electric field and charge density
(C) flux density and charge density
(D) current density and charge density

What is the generalized Maxwell’s equation \( \nabla \times H = J_e + \frac{\partial D}{\partial t} \) for the free space?

(A) \( \nabla \times H = 0 \)
(B) \( \nabla \times H = J_e \)
(C) \( \nabla \times H = \frac{\partial D}{\partial t} \)
(D) \( \nabla \times H = D \)
MCQ 6.3.24
IES EE 2009
Magnetic field intensity is \( \mathbf{H} = 3 \mathbf{a}_x + 6y \mathbf{a}_y + 2z \mathbf{a}_z \) A/m. What is the current density \( \mathbf{J} \)?
(A) \(-2 \mathbf{a}_y\)  
(B) \(-7 \mathbf{a}_z\)  
(C) \(3 \mathbf{a}_x\)  
(D) \(12 \mathbf{a}_y\)

MCQ 6.3.25
IES EE 2009
A circular loop placed perpendicular to a uniform sinusoidal magnetic field of frequency \( \omega_1 \) is revolved about an axis through its diameter at an angular velocity \( \omega_2 \) rad/sec (\( \omega_2 < \omega_1 \)) as shown in the figure below. What are the frequencies for the e.m.f induced in the loop?

(A) \(\omega_1 \) and \(\omega_2\)  
(B) \(\omega_1, \omega_2 + \omega_2\) and \(\omega_2\)  
(C) \(\omega_2, \omega_1 - \omega_2\) and \(\omega_2\)  
(D) \(\omega_1 - \omega_2\) and \(\omega_1 + \omega_2\)

MCQ 6.3.26
IES EE 2009
Which one of the following is not a Maxwell’s equation?
(A) \(\nabla \times \mathbf{H} = (\sigma + j\omega \epsilon) \mathbf{E}\)  
(B) \(\mathbf{F} = Q(\mathbf{E} + v \times \mathbf{B})\)  
(C) \(\oint \mathbf{H} \cdot d\mathbf{l} = \oint \mathbf{J} \cdot d\mathbf{S} + \oint \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}\)  
(D) \(\oint \mathbf{B} \cdot d\mathbf{S} = 0\)

MCQ 6.3.27
IES EE 2008
Consider the following three equations:
1. \(\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\)
2. \(\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\)
3. \(\nabla \cdot \mathbf{B} = 0\)
Which of the above appear in Maxwell’s equations?
(A) 1, 2 and 3  
(B) 1 and 2  
(C) 2 and 3  
(D) 1 and 3
MCQ 6.3.28
IES EE 2007
A straight current carrying conductor and two conducting loops A and B are shown in the figure given below. What are the induced current in the two loops?

(A) Anticlockwise in A and clockwise in B
(B) Clockwise in A and anticlockwise in B
(C) Clockwise both in A and B
(D) Anticlockwise both in A and B

MCQ 6.3.29
IES EE 2007
Which one of the following equations is not Maxwell’s equation for a static electromagnetic field in a linear homogeneous medium?

(A) $\nabla \cdot B = 0$
(B) $\nabla \times D = 0$
(C) $\oint B \cdot dl = \mu_0 I$
(D) $\nabla^2 A = \mu_0 J$

MCQ 6.3.30
IES EE 2006
In free space, if $\rho_v = 0$, the Poisson’s equation becomes

(A) Maxwell’s divergence equation $\nabla \cdot B = 0$
(B) Laplacian equation $\nabla^2 V = 0$
(C) Kirchhoff’s voltage equation $\Sigma V = 0$
(D) None of the above

MCQ 6.3.31
IES EE 2004
Match List I with List II and select the correct answer using the codes given below:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Continuity equation</td>
<td>1. $\nabla \times H = J + \frac{\partial D}{\partial t}$</td>
</tr>
<tr>
<td>b Ampere’s law</td>
<td>2. $J = \frac{\partial D}{\partial t}$</td>
</tr>
<tr>
<td>c Displacement current</td>
<td>3. $\nabla \times E = -\frac{\partial B}{\partial t}$</td>
</tr>
<tr>
<td>d Faraday’s law</td>
<td>4. $\nabla \times J = -\frac{\partial \rho_v}{\partial t}$</td>
</tr>
</tbody>
</table>
MCQ 6.3.32

IES EE 2004

Match List I (Type of field denoted by \( A \)) with List II (Behaviour) and select the correct answer using the codes given below:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. A static electric field in a charge free region</td>
<td>1. ( \nabla \cdot A = 0 )</td>
</tr>
<tr>
<td>b. A static electric field in a charged region</td>
<td>2. ( \nabla \times A \neq 0 )</td>
</tr>
<tr>
<td>c. A steady magnetic field in a current carrying conductor</td>
<td>3. ( \nabla \cdot A \neq 0 )</td>
</tr>
<tr>
<td>d. A time-varying electric field in a charged medium with time-varying magnetic field</td>
<td>4. ( \nabla \times A = 0 )</td>
</tr>
</tbody>
</table>

Codes:

(A) 4 2 3 1
(B) 4 2 1 3
(C) 2 4 3 1
(D) 2 4 1 3

MCQ 6.3.33

IES EE 2003

Which one of the following pairs is not correctly matched?

(A) Gauss Theorem: \( \int \mathbf{D} \cdot ds = \oint \nabla \cdot \mathbf{D} dv \)

(B) Gauss’s Law: \( \int \mathbf{D} \cdot ds = \int \rho dv \)

(C) Coulomb’s Law: \( V = -\frac{d\phi_m}{dt} \)

(D) Stoke’s Theorem: \( \oint \mathbf{\xi} \cdot dl = \int (\nabla \times \mathbf{\xi}) \cdot ds \)

MCQ 6.3.34

IES EE 2003

Maxwell equation \( \nabla \times \mathbf{E} = -\left( \frac{\partial \mathbf{B}}{\partial t} \right) \) is represented in integral form as

(A) \( \oint \mathbf{E} \cdot dl = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot dl \)

(B) \( \oint \mathbf{E} \cdot dl = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot ds \)

(C) \( \oint \mathbf{E} \times dl = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot dl \)

(D) \( \oint \mathbf{E} \times dl = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot dl \)
MCQ 6.3.35

The magnetic flux through each turn of a 100 turn coil is \((t^4 - 4t)\) milli-Webers where \(t\) is in seconds. The induced e.m.f at \(t = 2\) s is

(A) 1 V
(B) \(-1\) V
(C) 0.4 V
(D) \(-0.4\) V

MCQ 6.3.36

Two conducting coils 1 and 2 (identical except that 2 is split) are placed in a uniform magnetic field which decreases at a constant rate as in the figure. If the planes of the coils are perpendicular to the field lines, the following statements are made:

1. an e.m.f is induced in the split coil 2
2. e.m.fs are induced in both coils
3. equal Joule heating occurs in both coils
4. Joule heating does not occur in any coil

Which of the above statements is/are true?

(A) 1 and 4
(B) 2 and 4
(C) 3 only
(D) 2 only

MCQ 6.3.37

For linear isotropic materials, both \(E\) and \(H\) have the time dependence \(e^{\omega t}\) and regions of interest are free of charge. The value of \(\nabla \times H\) is given by

(A) \(\sigma E\)
(B) \(j\omega E\)
(C) \(\sigma E + j\omega E\)
(D) \(\sigma E - j\omega E\)

MCQ 6.3.38

Which of the following equations is/are not Maxwell’s equations(s) ?

(A) \(\nabla \cdot J = \frac{\partial \rho}{\partial t}\)
(B) \(\nabla \cdot D = \rho\)
(C) \(\nabla \cdot E = \frac{\partial B}{\partial t}\)
(D) \(\oint H \cdot dl = \int (\sigma E + \varepsilon \frac{\partial E}{\partial t}) \cdot ds\)

Select the correct answer using the codes given below:

(A) 2 and 4
(B) 1 alone
(C) 1 and 3
(D) 1 and 4

MCQ 6.3.39

Assertion (A) : The relationship between Magnetic Vector potential \(A\) and the current density \(J\) in free space is

\[ \nabla \times (\nabla \times A) = \mu_0 J \]
For a magnetic field in free space due to a \( dc \) or slowly varying current is \( \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \)

**Reason (R):** For magnetic field due to \( dc \) or slowly varying current \( \nabla \cdot \mathbf{A} = 0 \).

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

**MCQ 6.3.40**
Given that \( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \)

**Assertion (A):** In the equation, the additional term \( \frac{\partial \mathbf{D}}{\partial t} \) is necessary.

**Reason (R):** The equation will be consistent with the principle of conservation of charge.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

**MCQ 6.3.41**
Consider coils \( C_1, C_2, C_3 \) and \( C_4 \) (shown in the given figures) which are placed in the time-varying electric field \( \mathbf{E}(t) \) and electric field produced by the coils \( C'_2, C'_3 \) and \( C'_4 \) carrying time varying current \( I(t) \) respectively:

1. Time varying electric field \( \mathbf{E}(t) \) parallel to the plane of coil \( C_1 \)
2. Coil planes are orthogonal
3. Co-planer coils
4. Coil planes are orthogonal

The electric field will induce an emf in the coils
(A) \( C_1 \) and \( C_2 \)  
(B) \( C_2 \) and \( C_3 \)
MCQ 6.3.42
A circular loop is rotating about the $y$-axis as a diameter in a magnetic field $B = B_0 \sin \omega t \ 	ext{Wb/m}^2$. The induced emf in the loop is

(A) due to transformer emf only  
(B) due to motional emf only  
(C) due to a combination of transformer and motional emf  
(D) zero

MCQ 6.3.43
Match List I (Law/quantity) with List II (Mathematical expression) and select the correct answer:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Gauss’s law</td>
<td>1. $\nabla \cdot D = \rho$</td>
</tr>
<tr>
<td>b. Ampere’s law</td>
<td>2. $\nabla \times E = -\frac{\partial B}{\partial t}$</td>
</tr>
<tr>
<td>c. Faraday’s law</td>
<td>3. $\mathcal{P} = E \times H$</td>
</tr>
<tr>
<td>d. Poynting vector</td>
<td>4. $F = q(E + v \times B)$</td>
</tr>
<tr>
<td></td>
<td>5. $\nabla \times H = J_e + \frac{\partial D}{\partial t}$</td>
</tr>
</tbody>
</table>

Codes:

- (A) 1 2 4 3
- (B) 3 5 2 1
- (C) 1 5 2 3
- (D) 3 2 4 1

***********
SOLUTIONS 6.1

SOL 6.1.1 Option (C) is correct.

SOL 6.1.2 Option (B) is correct.
The line integral of magnetic field intensity along a closed loop is equal to the current enclosed by it.
i.e. \[ \int \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} \]
So, for the constant current, magnetic field intensity will be constant i.e. magnetostatic field is caused by steady currents.

SOL 6.1.3 Option (D) is correct.
From Faraday’s law the electric field intensity in a time varying field is defined as
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
where \( \mathbf{B} \) is magnetic flux density in the EM field.
and since the magnetic flux density is equal to the curl of magnetic vector potential i.e.
\[ \mathbf{B} = \nabla \times \mathbf{A} \]
So, putting it in equation (1), we get
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \]
or
\[ \nabla \times \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{A}}{\partial t}\right) \]
Therefore,
\[ \mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \]

SOL 6.1.4 Option (B) is correct.
Since total magnetic flux through a surface \( S \) is defined as
\[ \Phi = \int_S \mathbf{B} \cdot d\mathbf{S} \]
From Maxwell’s equation it is known that curl of magnetic flux density is zero
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \int_S \mathbf{B} \cdot d\mathbf{S} = \int_v (\nabla \cdot \mathbf{B}) \, dv = 0 \quad \text{(Stokes Theorem)} \]
Thus, net outwards flux will be zero for a closed surface.

SOL 6.1.5 Option (A) is correct.
From Faraday’s law, the relation between electric field and magnetic field is
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
Since the electric field inside a conducting sphere is zero.
i.e. \( \mathbf{E} = 0 \)
So the rate of change in magnetic flux density will be
\[
\frac{\partial \mathbf{B}}{\partial t} = - (\nabla \times \mathbf{E}) = 0
\]
Therefore \( \mathbf{B}(r,t) \) will be uniform inside the sphere and independent of time.

**SOL 6.1.6**
Option (B) is correct.
From the integral form of Faraday’s law we have the relation between the electric field intensity and net magnetic flux through a closed loop as
\[
\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}
\]
Since electric field intensity is zero (\( \mathbf{E} = 0 \)) inside the conducting loop. So, the rate of change in net magnetic flux through the closed loop is
\[
\frac{d\Phi}{dt} = 0
\]
i.e. \( \Phi \) is constant and doesn’t vary with time.

**SOL 6.1.7**
Option (A) is correct.
A superconductor material carries zero magnetic field and zero electric field inside it.
i.e. \( \mathbf{B} = 0 \) and \( \mathbf{E} = 0 \)
Now from Ampere-Maxwell equation we have the relation between the magnetic flux density and electric field intensity as
\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\]
So,
\[
\mathbf{J} = 0 \quad \quad (\mathbf{B} = 0, \mathbf{E} = 0)
\]
Since the net current density inside the superconductor is zero so all the current must be confined at the surface of the wire.

**SOL 6.1.8**
Option (A) is correct.
According to Lenz’s law the induced current \( \mathbf{I} \) in a loop flows such as to produce a magnetic field that opposes the change in \( \mathbf{B}(t) \).
Now the configuration shown in option (A) and (B) for increasing magnetic flux \( \mathbf{B}_i \), the change in flux is in same direction to \( \mathbf{B}_i \) as well as the current \( \mathbf{I} \) flowing in the loop produces magnetic field in the same direction so it does not follow the Lenz’s law.
For the configuration shown in option (D), as the flux \( \mathbf{B}_d \) is decreasing with time so the change in flux is in opposite direction to \( \mathbf{B}_d \) as well as the current \( \mathbf{I} \) flowing in the loop produces the magnetic field in opposite direction so it also does not follow the Lenz’s law.
For the configuration shown in option (C), the flux density \( \mathbf{B}_d \) is decreasing with time so the change in flux is in opposite direction to \( \mathbf{B}_d \) but the current \( \mathbf{I} \) flowing in the loop produces magnetic field in the same direction to \( \mathbf{B}_d \) (opposite to the direction of change in flux density). Therefore this is the correct configuration.
SOL 6.1.9 Option (A) is correct.

Induced emf in a conducting loop is given by

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is total magnetic flux passing through the loop.

Since, the magnetic field is non-uniform so the change in flux will be caused by it and the induced emf due to it is called transformer emf.

Again the field is in $a_y$ direction and the loop is rotating about z-axis so flux through the loop will also vary due to the motion of the loop. This causes the emf which is called motion emf. Thus, total induced voltage in the rotating loop is caused by the combination of both the transformer and motion emf.

SOL 6.1.10 Option (D) is correct.

As the conducting loop is falling freely So, the flux through loop will remain constant. Therefore, the voltage induced in the loop will be zero.

SOL 6.1.11 Option (B) is correct.

The magnetic flux density passing through the loop is given as

$$B = 4\pi^2 a_t$$

Since the flux density is directed normal to the plane $x = 0$ so the total magnetic flux passing through the square loop located in the plane $x = 0$ is

$$\Phi = \int B \cdot dS = \int_{y = 0}^{1} \int_{z = 0}^{1} (4\pi^2 t^2) dydz = t^2 \quad (dS = (dydz) a_x)$$

Induced emf in a loop placed in magnetic field is defined as

$$V_{emf} = -\frac{d\Phi}{dt}$$

where $\Phi$ is the total magnetic flux passing through the loop. So the induced emf in the square loop is

$$V_{emf} = -\frac{d(t^2)}{dt} = -2t \quad (\Phi = t^2)$$

Therefore at time $t = 2$ sec the induced emf is

$$V_{emf} = -4 \text{ volt}$$

SOL 6.1.12 Option (A) is correct.

Magnetic flux density produced at a distance $\rho$ from a long straight wire carrying current $I$ is defined as

$$B = \frac{\mu_0 I}{2\pi \rho} a_o$$

where $a_o$ is the direction of flux density as determined by right hand rule. So the flux density produced by straight wire at a distance $\rho$ from it is

$$B = \frac{\mu_0 I}{2\pi \rho} a_o \quad (a_o \text{ is unit vector normal to the loop})$$

Therefore the total magnet flux passing through the loop is

$$\Phi = \int B \cdot dS = \int_{d}^{d+\alpha} \frac{\mu_0 I}{2\pi \rho} d\rho \quad (dS = d\rho a_o)$$

where $d\rho$ is width of the strip of loop at a distance $\rho$ from the straight wire. Thus,
For View Only Shop Online at www.nodia.co.in

\[ \Phi = \int_2^3 \left( \frac{\mu_0 I}{2\pi} \right) \frac{d\rho}{\rho} = \frac{\mu_0 I}{2\pi} \ln\left( \frac{3}{2} \right) = \frac{\mu_0 (5)}{2\pi} \ln(1.5) \]
\[ = (2 \times 10^{-7}) (5) \ln(1.5) = 4.05 \times 10^{-7} \text{ Wb} \]

**SOL 6.1.13** Option (D) is correct.

Electric field intensity experienced by the moving conductor \( ab \) in the presence of magnetic field \( B \) is given as
\[ E = v \times B \]
where \( v \) is the velocity of the conductor.

So, electric field will be directed from \( b \) to \( a \) as determined by right hand rule for the cross vector. Therefore, the voltage difference between the two ends of the conductor is given as
\[ V_{ab} = -\int_a^b E \cdot dl \]

Thus, the positive terminal of voltage will be \( a \) and \( V_{ab} \) will be positive.

**SOL 6.1.14** Option (B) is correct.

Given magnetic flux density through the square loop is
\[ B = 0.1t a, \text{ Wb/m}^2 \]
So, total magnetic flux passing through the loop is
\[ \Phi = B \cdot dS = (0.1t)(1) = 0.1t \]
The induced emf (voltage) in the loop is given as
\[ V_{\text{emf}} = -\frac{d\Phi}{dt} = -0.2 \text{ Volt} \]

As determined by Lenz’s law the polarity of induced emf will be such that
\[ V_1 + V_2 = -V_{\text{emf}} \]
Therefore, the voltage drop in the two resistances are respectively,
\[ V_1 = \left( \frac{2}{2+4} \right)(-V_{\text{emf}}) = 0.1 \times \frac{1}{3} = 33.3 \text{ mV} \]
and
\[ V_2 = \left( \frac{4}{2+4} \right)(-V_{\text{emf}}) = 66.7 \text{ mV} \]

**SOL 6.1.15** Option (D) is correct.

Consider a magnet bar being dropped inside a pipe as shown in figure.
Suppose the current \( I \) in the magnet flows counter clockwise (viewed from above) as shown in figure. So near the ends of pipe, it’s field points upward. A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches and hence by Lenz’s law a current will be induced in it such as to produce downward flux.

Thus, \( I_{\text{ind}} \) must flow clockwise which is opposite to the current in the magnet. Since opposite currents repel each other so, the force exerted on the magnet due to the induced current is directed upward. Meanwhile a ring above the magnet experiences a decreasing upward flux; so it’s induced current parallel to \( I \) and it attracts magnet upward. And flux through the rings next to the magnet bar is constant. So no current is induced in them.
Thus, for all we can say that the force exerted by the eddy current (induced current according to Lenz’s law) on the magnet is in upward direction which causes the delay to reach the bottom. Whereas in the cases of unmagnetized bar no induced current is formed. So it reaches in fraction of time. Thus, A and R both true and R is correct explanation of A.

**SOL 6.1.16**

Option (B) is correct.

Voltage, \[ V_1 = -N_1 \frac{d\Phi}{dt} \]

where \( \Phi \) is total magnetic flux passing through it.

Again \[ V_2 = -N_2 \frac{d\Phi}{dt} \]

Since both the coil are in same magnetic field so, change in flux will be same for both the coil.

Comparing the equations (1) and (2) we get

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} \]

\[ V_2 = V_1 \frac{N_2}{N_1} = (12) \frac{3000}{5000} = 7.2 \text{ volt} \]

**SOL 6.1.17**

Option (A) is correct.

The magnetic flux density inside a solenoid of \( n \) turns per unit length carrying current \( I \) is defined as

\[ B = \mu_0 n I \]

Let the length of solenoid be \( l \) and its cross sectional radius be \( r \). So, the total magnetic flux through the solenoid is

\[ \Phi = (\mu_0 n I) (\pi r^2) (nl) \]  \( (1) \)

Since the total magnetic flux through a coil having inductance \( L \) and carrying current \( I \) is given as

\[ \Phi = LI \]

So comparing it with equation (1) we get,
\[ L = \mu_0 n^2 I \pi^2 l \]

and as for a given solenoid, radius \( r \) and length \( l \) is constant therefore \( L \propto n^2 \)

**SOL 6.1.18** Option (A) is correct.

The magnetic flux density inside the solenoid is defined as

\[ B = \mu_0 n I \]

where \( n \rightarrow \) no. of turns per unit length
\( I \rightarrow \) current flowing in it.

So the total magnetic flux through the solenoid is

\[ \Phi = \int B \cdot dS = (\mu_0 n I) (\pi a^2) \]

where \( a \rightarrow \) radius of solenoid

Induced emf in a loop placed in a magnetic field is defined as

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} \]

where \( \Phi \) is the total magnetic flux passing through the loop. Since the resistance \( R \) is looped over the solenoid so total flux through the loop will be equal to the total flux through the solenoid and therefore the induced emf in the loop of resistance will be

\[ V_{\text{emf}} = -\pi a^2 \mu_0 n \frac{dI}{dt} \]

Since current \( I \) flowing in the solenoid is constant so, the induced emf is

\[ V_{\text{emf}} = 0 \]

and therefore the induced current in the loop will be zero.

**SOL 6.1.19** Option (B) is correct.

It will be similar to the current in a solenoid.
So, the magnetic field will be in circumferential while the electric field is longitudinal.

**SOL 6.1.20** Option (B) is correct.

In Assertion (A) the magnetic flux through each turn of both coils are equal So, the net magnetic flux through the two coils are respectively

\[ \Phi_1 = N_1 \Phi \]
\[ \Phi_2 = N_2 \Phi \]

where \( \Phi \) is the magnetic flux through a single loop of either coil and \( N_1, N_2 \) are the total no. of turns of the two coils respectively.

Therefore the induced emf in the two coils are

\[ V_{\text{emf1}} = -\frac{d\Phi_1}{dt} = -N_1 \frac{d\Phi}{dt} \]
\[ V_{\text{emf2}} = -\frac{d\Phi_2}{dt} = -N_2 \frac{d\Phi}{dt} \]

Thus, the ratio of the induced emf in the two loops are

\[ \frac{V_{\text{emf2}}}{V_{\text{emf1}}} = \frac{N_2}{N_1} \]
Now, in Reason (R): a primitive transformer is similar to the cylinder core carrying wound coils. It is the device in which by choosing the appropriate no. of turns, any desired secondary emf can be obtained.

So, both the statements are correct but R is not the explanation of A.

SOL 6.1.21 Option (B) is correct.

Electric flux density in the medium is given as

\[ D = \varepsilon E = \varepsilon E_0 \cos \omega t \quad (E = E_0 \cos \omega t) \]

Therefore the displacement current density in the medium is

\[ J_d = \frac{\partial D}{\partial t} = -\omega \varepsilon E_0 \sin \omega t \]

and the conduction current density in the medium is

\[ J_c = \sigma E = \sigma E_0 \cos \omega t \]

So, the ratio of amplitudes of conduction current density and displacement current density is

\[ \frac{|J_c|}{|J_d|} = \frac{\sigma}{\omega \varepsilon} \]

SOL 6.1.22 Option (D) is correct.

The displacement current density in a medium is equal to the rate of change in electric flux density in the medium.

\[ J_d = \frac{\partial D}{\partial t} \]

Since the displacement current density in the medium is given as

\[ J_d = 20 \cos(1.5 \times 10^8 t) \text{A/m}^2 \]

So, the electric flux density in the medium is

\[ D = \int J_d \, dt + C \quad (C \rightarrow \text{constant}) \]

\[ = \int 20 \cos(1.5 \times 10^8 t) \text{A} \, dt + C \]

As there is no D.C. field present in the medium so, we get \( C = 0 \) and thus,

\[ D = \frac{20 \sin(1.5 \times 10^8 t)}{1.5 \times 10^8} a_y = 1.33 \times 10^{-7} \sin(1.5 \times 10^8 t) a_y \]

\[ = 153.3 \sin(1.5 \times 10^8 t) a_y \text{nC/m}^2 \]

SOL 6.1.23 Option (A) is correct.

Given the volume charge density, \( \rho_v = 0 \)

So, from Maxwell’s equation we have

\[ \nabla \cdot D = \rho_v \]

\[ \nabla \cdot D = 0 \quad (1) \]

Now, the electric flux density in a medium is defined as

\[ D = \varepsilon E \quad (\text{where } \varepsilon \text{ is the permittivity of the medium}) \]

So, putting it in equation (1) we get,

\[ \nabla \cdot (\varepsilon E) = 0 \]

or, \( E \cdot (\nabla \varepsilon) + \varepsilon (\nabla \cdot E) = 0 \)
and since \( \nabla \varepsilon = 0 \Rightarrow \nabla \varepsilon \approx 0 \) \( \text{(given)} \)

Therefore,
\[ \nabla \cdot \mathbf{E} \approx 0 \]

**SOL 6.1.24** Option (C) is correct.
The ratio of magnitudes of displacement current to conduction current in any medium having permittivity \( \varepsilon \) and conductivity \( \sigma \) is given as
\[ \left| \frac{\text{Displacement current}}{\text{Conduction current}} \right| = \frac{\omega \varepsilon}{\sigma} \]
where \( \omega \) is the angular frequency of the current in the medium.

Given frequency, \( f = 50 \text{ GHz} \)
Permittivity, \( \varepsilon = 4\varepsilon_0 = 4 \times 8.85 \times 10^{-12} \)
Conductivity, \( \sigma = 1.14 \times 10^8 \text{ S/m} \)

So,
\[ \omega = 2\pi f = 2\pi \times 50 \times 10^9 = 100\pi \times 10^9 \]

Therefore, the ratio of magnitudes of displacement current to the conduction current is
\[ \left| \frac{I_d}{I_c} \right| = \frac{100\pi \times 10^9 \times 4 \times 8.85 \times 10^{-12}}{1.14 \times 10^8} = 9.75 \times 10^{-8} \]

**SOL 6.1.25** Option (D) is correct.
Given the electric field intensity in time domain as
\[ \mathbf{E} = \frac{\sin \theta \cos(\omega t - kr)}{r} \mathbf{a}_\theta \]
So, the electric field intensity in phasor form is given as
\[ \mathbf{E}_s = \frac{\sin \theta}{r} e^{-jk} \mathbf{a}_\theta \]
and
\[ \nabla \times \mathbf{E}_s = \frac{1}{r} \frac{\partial}{\partial r}(rE_{\theta 0}) \mathbf{a}_\phi = (-jk) \frac{\sin \theta}{r} e^{-jk} \mathbf{a}_\phi \]
Therefore, from Maxwell’s equation we get the magnetic field intensity as
\[ \mathbf{H}_s = -\frac{\nabla \times \mathbf{E}_s}{jk \omega \varepsilon_0} = \frac{k}{\omega \varepsilon_0} \frac{\sin \theta}{r} e^{-jk} \mathbf{a}_\phi \]

**SOL 6.1.26** Option (B) is correct.
In phasor form the magnetic field intensity can be written as
\[ \mathbf{H}_s = 0.1 \cos(15\pi y) e^{-jk} \mathbf{a}_z \text{ A/m} \]
Similar as determined in MCQ 42 using Maxwell’s equation we get the relation
\[ (15\pi)^2 + b^2 = \omega^2 \varepsilon_0 \]
Here \( \omega = 6\pi \times 10^9 \)
So,
\[ (15\pi)^2 + b^2 = \left( \frac{6\pi \times 10^9}{3 \times 10^8} \right)^2 \]
\[ (15\pi)^2 + b^2 = 400\pi^2 \]
\[ b^2 = 175\pi^2 \Rightarrow b = \pm 23.6 \text{ rad/m} \]

**********
SOL 6.2.1  Option (D) is correct.

Induced emf. in the conducting loop formed by rail, bar and the resistor is given by

\[ V_{emf} = -\frac{d\Phi}{dt} \]

where \( \Phi \) is total magnetic flux passing through the loop.

Consider the bar be located at a distance \( x \) from the resistor at time \( t \). So the total magnetic flux passing through the loop at time \( t \) is

\[ \Phi = \int B \cdot dS = Blx \quad \text{(area of the loop is } S = lx) \]

Now the induced emf in a loop placed in magnetic field is defined as

\[ V_{emf} = -\frac{d\Phi}{dt} \]

where \( \Phi \) is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

\[ V_{emf} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \quad (\Phi = Blx) \]

Since from the given figure, we have

\[ l = 2 \text{ m} \quad \text{and} \quad B = 0.1 \text{ Wb/m}^2 \]

and

\[ dx/dt = \text{velocity of bar} = 5 \text{ m/s} \]

So, induced emf is

\[ V_{emf} = -(0.1)(2)(5) = -1 \text{ volt} \]

According to Lenz’s law the induced current \( I \) in a loop flows such as to produce magnetic field that opposes the change in \( B(t) \). As the bar moves away from the resistor the change in magnetic field will be out of the page so the induced current will be in the same direction of \( I \) shown in figure.

Thus, the current in the loop is

\[ I = -\frac{V_{emf}}{R} = -\frac{-1}{10} = 0.23 \text{ A} \quad (R = 10 \Omega) \]

SOL 6.2.2  Option (B) is correct.

Magnetic flux density produced at a distance \( \rho \) from a long straight wire carrying current \( I \) is defined as

\[ B = \frac{\mu_0 I}{2\pi \rho} a_\phi \]

where \( a_\phi \) is the direction of flux density as determined by right hand rule. So, the magnetic flux density produced by the straight conducting wire linking through the
loop is normal to the surface of the loop.
Now consider a strip of width \( dp \) of the square loop at distance \( \rho \) from the wire for which the total magnetic flux linking through the square loop is given as

\[
\Phi = \int_S B \cdot dS = \frac{\mu_0 I}{2\pi} \int_{\rho}^{\rho + a} \frac{1}{\rho} (adp)
\]

(area of the square loop is \( dS = adp \))

The induced emf due to the change in flux (when pulled away) is given as

\[
V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I a}{2\pi} \frac{d}{dt} \ln \left( \frac{\rho + a}{\rho} \right)
\]

Therefore,

\[
V_{\text{emf}} = -\frac{\mu_0 I a}{2\pi} \left( \frac{1}{\rho + a} \frac{dp}{dt} - \frac{1}{\rho} \frac{dp}{dt} \right)
\]

Given \( \frac{dp}{dt} = \text{velocity of loop} = 5 \text{ m/s} \)

and since the loop is currently located at 3 m distance from the straight wire, so after 0.6 sec it will be at

\[
\rho = 3 + (0.6) \times v \\
v = 3 + 0.6 \times 5 = 6 \text{ m}
\]

So,

\[
V_{\text{emf}} = -\frac{\mu_0 \times (300) \times 2}{2\pi} \left[ \frac{1}{8} (5) - \frac{1}{6} (5) \right]
\]

\[
= 25 \times 10^{-7} \text{ volt} = 2.5 \mu\text{volt}
\]

SOL 6.2.3 Option (B) is correct.
Since total magnetic flux through the loop depends on the distance from the straight wire and the distance is constant. So the flux linking through the loop will be constant, if it is pulled parallel to the straight wire. Therefore the induced emf in the loop is

\[
V_{\text{emf}} = -\frac{d\Phi}{dt} = 0
\]

(\( \Phi \) is constant)

SOL 6.2.4 Option (D) is correct.
Magnetic flux density produced at a distance \( \rho \) from a long straight wire carrying current \( I \) is defined as

\[
B = \frac{\mu_0 I}{2\pi \rho} a_x
\]

where \( a_x \) is the direction of flux density as determined by right hand rule.
Since the direction of magnetic flux density produced at the loop is normal to the surface of the loop So, total flux passing through the loop is given by

\[
\Phi = \int_S B \cdot dS = \frac{\mu_0 I}{2\pi} \int_{\rho = 2}^{\rho + 2} \frac{dp}{\rho} = \frac{\mu_0 I_2}{2\pi} \ln 2 = \frac{\mu_0 I}{\pi} \ln (2)
\]

The current flowing in the loop is \( I_{\text{loop}} \) and induced e.m.f. is \( V_{\text{emf}} \).
So, 

\[ V_{\text{emf}} = \int_{\text{loop}} R dt = -\frac{d\Phi}{dt} \]

\[ \frac{dQ}{dt}(R) = -\frac{\mu_0}{4\pi} \ln(2) \frac{dI}{dt} \]

where \( Q \) is the total charge passing through a corner of square loop.

\[ \frac{dQ}{dt} = -\frac{\mu_0}{4\pi} \ln(2) \frac{dI}{dt} \quad (R = 4 \, \Omega) \]

\[ dQ = -\frac{\mu_0}{4\pi} \ln(2) \, dI \]

Therefore the total charge passing through a corner of square loop is

\[ Q = -\frac{\mu_0}{4\pi} \ln(2) \int_0^1 dI = -\frac{\mu_0}{4\pi} \ln(2) (0 - 4) \]

\[ = 4 \times 4\pi \times 10^{-7} \ln(2) = 2.77 \times 10^{-7} \, C = 277 \, \text{nC} \]

**SOL 6.2.5** Option (A) is correct.

Since the radius of small circular loop is negligible in comparison to the radius of the large loop. So, the flux density through the small loop will be constant and equal to the flux on the axis of the loops.

So,

\[ B = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} a_z \]

where \( R \rightarrow \) radius of large loop = 5 m

\( z \rightarrow \) distance between the loops = 12 m

\[ B = \frac{\mu_0 \times 2}{2} \times \frac{(5)^2}{[(12)^2 + (5)^2]^{3/2}} a_z = \frac{25\mu_0}{(13)^3} a_z \]

Therefore, the total flux passing through the small loop is

\[ \Phi = \int B \cdot dS = \frac{25\mu_0}{(13)^3} \times \pi r^2 \quad \text{where} \, r \text{ is radius of small circular loop.} \]

\[ = \frac{25 \times 4\pi \times 10^{-7}}{(13)^3} \times \pi (10^{-3})^2 = 65.9 \, \text{fWb} \]

**SOL 6.2.6** Option (C) is correct.

Electric field in any medium is equal to the voltage drop per unit length.

\[ E = \frac{V}{d} \]

where \( V \rightarrow \) potential difference between two points.

\( d \rightarrow \) distance between the two points.

The voltage difference between any two points in the medium is

\[ V = V_0 \cos 2\pi ft \]

So the conduction current density in the medium is given as

\[ J_c = \sigma E \quad (\sigma \rightarrow \text{conductivity of the medium}) \]

\[ = \frac{E}{\rho} \quad (\rho \rightarrow \text{resistivity of the medium}) \]

\[ = \frac{V}{\rho d} = \frac{V_0 \cos 2\pi ft}{\rho d} \quad (V = V_0 \cos 2\pi ft) \]
or, \[ |J_d| = \frac{V_0}{\rho d} \]

and displacement current density in the medium is given as
\[
J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{\partial}{\partial t} \left[ \frac{V_0 \cos(2\pi ft)}{d} \right] = \varepsilon \frac{V_0}{d} [-2\pi ft \sin(2\pi ft)]
\]

or, \[ |J_d| = \frac{2\pi \varepsilon V_0}{d} \]

Therefore, the ratio of amplitudes of conduction current and displacement current in the medium is
\[
\left| \frac{I_c}{I_d} \right| = \left| \frac{J_c}{J_d} \right| = \left( \frac{V_0}{\rho d} \right) / \left( \frac{2\pi \varepsilon V_0}{d} \right) = \frac{1}{2\pi \varepsilon \rho}
\]
\[
= \frac{2\pi \times (1.6 \times 10^{-19}) \times (54 \times 8.85 \times 10^{-12}) \times 0.77}{2.7} = 2.7
\]

SOL 6.2.7 Option (C) is correct.

Total magnetic flux through the solenoid is given as
\[ \Phi = \mu_0 n I \]

where \( n \) is the no. of turns per unit length of solenoid and \( I \) is the current flowing in the solenoid.

Since the solenoid carries current that is increasing linearly with time i.e.
\[ I \propto t \]

So the net magnetic flux through the solenoid will be
\[ \Phi \propto t \]

or, \[ \Phi = kt \]

where \( k \) is a constant.

Therefore the emf induced in the loop consisting resistances \( R_A, R_B \) is
\[ V_{\text{emf}} = -\frac{d\Phi}{dt} \]
\[ V_{\text{emf}} = -k \]

and the current through \( R_1 \) and \( R_2 \) will be
\[ I_{\text{ind}} = -\frac{k}{R_1 + R_2} \]

Now according to Lenz’s law the induced current \( I \) in a loop flows such as to produce a magnetic field that opposes the change in \( B(t) \).

i.e. the induced current in the loop will be opposite to the direction of current in solenoid (in anticlockwise direction).

So,
\[ V_A = I_{\text{ind}} R_A = -\frac{kR_A}{R_A + R_B} \]

and
\[ V_B = -I_{\text{ind}} R_B = \left( \frac{kR_B}{R_A + R_B} \right) \]

Thus, the ratio of voltmeter readings is
\[ \frac{V_A}{V_B} = -\frac{R_A}{R_B} \]
For View Only

SOL 6.2.8
Option (C) is correct.

Induced emf in the conducting loop formed by rail, bar and the resistor is given by

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} \]

where \( \Phi \) is total magnetic flux passing through the loop.

The bar is located at a distance \( x \) from the resistor at time \( t \). So the total magnetic flux passing through the loop at time \( t \) is

\[ \Phi = \int B \cdot dS = Blx \]

where \( l \) is separation between the rails

Now the induced emf in a loop placed in magnetic field is defined as

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} \]

where \( \Phi \) is the total magnetic flux passing through the loop. Therefore the induced emf in the square loop is

\[ V_{\text{emf}} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} \]

Since from the given figure, we have

\[ l = 5 \text{ m} \]
\[ B = 2 \text{ T} \]

and \( \frac{dx}{dt} \to \) velocity of bar = 4 m/s

So, induced emf is

\[ V_{\text{emf}} = -(2)(5)(4) = 40 \text{ volt} \]

Therefore the current in the bar loop will be

\[ I = \frac{V_{\text{emf}}}{R} = \frac{40}{10} = 5 \text{ A} \]

SOL 6.2.9
Option (B) is correct.

As obtained in the previous question the current flowing in the sliding bar is

\[ I = -4 \text{ A} \]

Now we consider magnetic field acts in \( a_z \) direction and current in the sliding bar is flowing in \( +a_z \) direction as shown in the figure.

Therefore, the force exerted on the bar is

\[ F = \int ldl \times B = \int_0^l (-4dz \cdot a_z) \times (2a_z) \]

\[ = -16a_z[z] = -40a_z \text{ N} \]
i.e. The force exerted on the sliding bar is in opposite direction to the motion of the sliding bar.

**SOL 6.2.10** Option (A) is correct.

Given the magnetic flux density through the square loop is

\[ B = 7.5 \cos(120\pi t - 30^\circ) \mathbf{a}_z \]

So the total magnetic flux passing through the loop will be

\[ \Phi = \oint \mathbf{B} \cdot d\mathbf{S} = \left[ -7.5 \cos(120\pi t - 30^\circ) \mathbf{a}_z \right] (1 \times 1) (-\mathbf{a}_z) = 7.5 \cos(120\pi t - 30^\circ) \]

Now, the induced emf in the square loop is given by

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} = 7.5 \times 120\pi \sin(120\pi t - 30^\circ) \]

The polarity of induced emf (according to Lenz’s law) will be such that induced current in the loop will be in opposite direction to the current \( I(t) \) shown in the figure. So we have

\[ I(t) = -\frac{V_{\text{emf}}}{R} = -\frac{7.5 \times 120\pi}{500} \sin(120\pi t - 30^\circ) \quad (R = 250 + 250 = 500 \, \Omega) \]
\[ = -4.7 \sin(120\pi t - 30^\circ) \]

**SOL 6.2.11** Option (D) is correct.

As shown in figure the bar is sliding away from origin.

Now when the bar is located at a distance \( dx \) from the voltmeter, then, the vector area of the loop formed by rail and the bar is

\[ d\mathbf{S} = (20 \times 10^{-2}) (dx) \mathbf{a}_z \]

So, the total magnetic flux passing through the loop is

\[ \Phi = \int B \cdot d\mathbf{S} = \int_0^x (8x^2 \mathbf{a}_z) (20 \times 10^{-2} dx \mathbf{a}_z) = \frac{1.6 t(1 + 0.4t^2)}{3} \]

Therefore, the induced e.m.f. in the loop is given as

\[ V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{1.6}{3} \times 3(t + 0.4t^3) \times (1 + 1.2t^2) \]
\[ V_{\text{emf}} = -1.6[(0.4) + (0.4)^3] \times [1 + (1.2)(0.4)^2] \quad (t = 0.4 \, \text{sec}) \]
\[ = -0.35 \, \text{volt} \]

Since the voltmeter is connected in same manner as the direction of induced emf (determined by Lenz’s law).

So the voltmeter reading will be

\[ V = V_{\text{emf}} = -0.35 \, \text{volt} \]

**SOL 6.2.12** Option (C) is correct.

Since the position of bar is give as

\[ x = t(1 + 0.4t^2) \]

So for the position \( x = 12 \, \text{cm} \) we have
\[ 0.12 = t(1 + 0.4t^2) \]

or,
\[ t = 0.1193 \text{ sec} \]

As calculated in previous question, the induced emf in the loop at a particular time \( t \) is
\[ V_{\text{emf}} = -(1.6)[t + 0.4t^2](1 + 1.2t^2) \]

So, at \( t = 0.1193 \) sec,
\[ V_{\text{emf}} = -1.6[(0.1193) + 0.4(0.1193)^2][1 + (1.2)(0.1193)^2] \]
\[ = -0.02344 = -23.4 \text{ mV} \]

Since the voltmeter is connected in same manner as the direction of induced emf as determined by Lenz’s law. Therefore, the voltmeter reading at \( x = 12 \text{ cm} \) will be
\[ V = V_{\text{emf}} = -23.4 \text{ mV} \]

**SOL 6.2.13**

Option (D) is correct.

Consider the mutual inductance between the rectangular loop and straight wire be \( M \). So applying KVL in the rectangular loop we get,
\[ M\frac{di_2}{dt} = I_1 \frac{di_2}{dt} + Ri_2 \quad \ldots (1) \]

Now from the shown figure (b), the current flowing in the straight wire is given as
\[ i_1 = I_1 u(t) - I_1 u(t - T) \quad (I_1 \text{ is amplitude of the current}) \]

or,
\[ \frac{di_1}{dt} = I_1 \delta(t) - I_1 \delta(t - T) \quad \ldots (2) \]

So, at \( t = 0 \)
\[ \frac{di_1}{dt} = I_1 \]

and
\[ MI_1 = I_1 \frac{di_2}{dt} + Ri_2 \quad \text{(from equation (1))} \]

Solving it we get
\[ i_2 = \frac{M}{L} I_1 e^{(R/L)t} \quad \text{for } 0 < t < T \]

Again in equation (2) at \( t = T \) we have
\[ \frac{di_1}{dt} = -I_1 \]

and
\[ -MI_1 = I_1 \frac{di_2}{dt} + Ri_2 \quad \text{(from equation (1))} \]

Solving it we get
\[ i_2 = -\frac{M}{L} I_1 e^{-(R/L)(t - T)} \quad \text{for } t > T \]

Thus, the current in the rectangular loop is
\[ i_2 = \begin{cases} \frac{M}{L} I_1 e^{(R/L)t} & 0 < t < T \\ -\frac{M}{L} I_1 e^{-(R/L)(t - T)} & t > T \end{cases} \]

Plotting \( i_2 \) versus \( t \) we get
SOL 6.2.14 Option (D) is correct.

Total magnetic flux passing through the loop formed by the resistance, bar and the rails is given as:

\[
\Phi = \int_S B \cdot dS
\]

\[
= B \cdot S = [0.2 \cos \omega t a_x] \cdot [0.5(1 - y) a_z]
\]

\[
= 0.1[1 - 0.5(1 - \cos \omega t)] \cos \omega t \quad (y = 0.5(1 - \cos \omega t) \text{ m})
\]

\[
= 0.05 \cos \omega t + \cos^2 \omega t
\]

So, the induced emf in the loop is

\[
V_{\text{emf}} = -\frac{d\Phi}{dt}
\]

and as determined by Lenz’s law, the induced current will be flowing in opposite direction to the current \(i\). So the current \(i\) in the loop will be

\[
i = -\frac{V_{\text{emf}}}{R} = -\frac{1}{R} \left( \frac{d\Phi}{dt} \right)
\]

\[
i = 0.05 [-\omega \sin \omega t - 2\omega \cos \omega t \sin \omega t]
\]

\[
i = -0.23\omega \sin \omega t(1 + 2 \cos \omega t)
\]

SOL 6.2.15 Option (C) is correct.

Given the electric flux density in the medium is

\[
D = 1.33 \sin(3 \times 10^8 t - 0.2x) \text{ } \mu \text{C/m}^2
\]

So, the electric field intensity in the medium is given as

\[
E = \frac{D}{\varepsilon} = \frac{1.33 \times 10^{-6} \sin(3 \times 10^8 t - 0.2x)}{10 \times 8.85 \times 10^{-12}} a_y
\]

\[
(\varepsilon_r = 10)
\]

or,

\[
E = \frac{D}{\varepsilon \varepsilon_0} = \frac{1.33 \times 10^{-6} \sin(3 \times 10^8 t - 0.2x)}{10 \times 8.85 \times 10^{-12}} a_y
\]

\[
= 1.5 \times 10^4 \sin(3 \times 10^8 t - 0.2x) a_y
\]

Now, from maxwell’s equation we have

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]

or,

\[
\frac{\partial B}{\partial t} = -\nabla \times E
\]

\[
= -\frac{\partial E_y}{\partial x} a_z
\]
Integrating both sides, we get the magnetic flux density in the medium as

$$B = \int 3 \times 10^8 \cos(3 \times 10^8 t - 0.2x) \, dx$$

$$= 3 \times 10^8 \sin(3 \times 10^8 t - 0.2x) \, a_y$$

Therefore the magnetic field intensity in the medium is

$$H = \frac{B}{\mu} = \frac{B}{\mu_0}$$

$$= 10^{-5} \sin(3 \times 10^8 t - 0.2x)$$

$$\frac{2 \times 4\pi \times 10^{-7}}{\mu_r = 2}$$

Thus

$$H = 2 \sin(3 \times 10^8 t - 0.2x) \, a_y \text{ A/m}$$

SOL 6.2.16

Option (D) is correct.

Given the magnetic field intensity in the medium is

$$H = \cos(10^{10} t - bx) \, a_z \text{ A/m}$$

Now from the Maxwell’s equation, we have

$$\nabla \times H = \frac{\partial D}{\partial t}$$

or,

$$\frac{\partial D}{\partial t} = - \frac{\partial H}{\partial x} \, a_y = - b \sin(10^{10} t - bx) \, a_y$$

$$D = \int - b \sin(10^{10} t - bx) \, dt + C$$

where $C$ is a constant.

Since no D.C. field is present in the medium so, we get $C = 0$ and therefore,

$$D = \frac{b}{10^{10}} \cos(10^{10} t - bx) \, a_y \text{ C/m}^2$$

and the electric field intensity in the medium is given as

$$E = \frac{D}{\varepsilon} = \frac{b}{0.12 \times 10^{-9} \times 10^{10} \cos(10^{10} t - bx) \, a_y} \quad (\varepsilon = 0.12 \text{ nF/m})$$

Again From the Maxwell’s equation

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

or,

$$\frac{\partial B}{\partial t} = - \nabla \times \left[ \frac{b}{1.2} \cos(10^{10} t - bx) \, a_y \right]$$

$$= - \frac{b^2}{1.2} \sin(10^{10} t - bx) \, a_z$$

So, the magnetic flux density in the medium is

$$B = - \int \frac{b^2}{1.2} \sin(10^{10} t - bx) \, a_z \, dt$$

$$= \frac{b^2}{(1.2) \times 10^{10} \cos(10^{10} t - bx) \, a_z}$$

We can also determine the value of magnetic flux density as :

$$B = \mu H$$
Comparing the results of equation (1) and (2) we get,

\[
\frac{b^2}{(1.2) \times 10^{10}} = 3 \times 10^{-5}
\]

\[
b^2 = 5.6 \times 10^4
\]

\[
b = \pm 344 \text{ rad/m}
\]

SOL 6.2.17 Option (B) is correct.

The magnetic vector potential for a direct current flowing in a filament is given as

\[
A = \int \frac{\mu_0 I}{4\pi R} a_x dx
\]

Here current \(I(t)\) flowing in the filament shown in figure is varying with time as

\(I(t) = 8t\ A\)

So, the retarded vector potential at the point \(P\) will be given as

\[
A = \int \frac{\mu_0 I(t - R/c)}{4\pi R} a_x dx
\]

where \(R\) is the distance of any point on the filamentary current from \(P\) as shown in the figure and \(c\) is the velocity of waves in free space. So, we have

\(R = \sqrt{x^2 + 4}\) and \(c = 3 \times 10^8\ m/s\)

Therefore,

\[
A = \int_{x=-0.1}^{x=0.1} \frac{\mu_0}{4\pi R} \left[ \frac{8(t - R/c)}{dx} - \int_{x=-0.1}^{x=0.1} \frac{1}{c} dx \right]
\]

\[
= 8 \times 10^{-7} \left[ \ln(x + \sqrt{x^2 + 4}) \right]_{-0.1}^{0.1} - \frac{8 \times 10^{-7}}{3 \times 10^7} \left[ x \right]_{-0.1}^{0.1}
\]

\[
= 8 \times 10^{-7} \ln \left( \frac{0.1 + \sqrt{4.01}}{-0.1 + \sqrt{4.01}} \right) - 0.53 \times 10^{-15}
\]

or,

\[
A = (80t - 5.3 \times 10^{-7}) a_x \text{nWb/m}
\]

So, when \(A = 0\)

\(t = 6.6 \times 10^{-9} = 6.6\ nsec\)

and when \(t = 0\)

\(A = -5.3 \times 10^{-4} \text{nWb/m}\)

From equation (1) it is clear that \(A\) will be linearly increasing with respect to time. Therefore the plot of \(A\) versus \(t\) is
Note: Time varying potential is usually called the retarded potential.

**SOL 6.2.18**

Option (A) is correct.

Given

Retarded scalar potential,

\[ V = y(x - ct) \text{ volt} \]

and retarded vector potential,

\[ A = y\left(\frac{x}{c} - t\right) a_z \text{ Wb/m} \]

Now the magnetic flux density in the medium is given as

\[ B = \nabla \times A \]

\[ = -\frac{\partial A_z}{\partial y} a_y = \left(t - \frac{x}{c}\right) a_z \text{ Tesla} \quad (1) \]

So, the magnetic field intensity in the medium is

\[ H = \frac{B}{\mu_0} \quad (\mu_0 \text{ is the permittivity of the medium}) \]

\[ = \frac{1}{\mu_0}\left(t - \frac{x}{c}\right) a_y \text{ A/m} \quad (2) \]

and the electric field intensity in the medium is given as

\[ E = -\nabla V - \frac{\partial A}{\partial t} \]

\[ = -(x - ct) a_y - y a_x + y a_x = (ct - x) a_y \quad (3) \]

So, the electric flux density in the medium is

\[ D = \varepsilon_0 E \quad (\varepsilon_0 \text{ is the permittivity of the medium}) \]

\[ = \varepsilon_0 (ct - x) a_y \text{ C/m}^2 \quad (4) \]

Now we determine the condition for the field to satisfy all the four Maxwell’s equations.

(i) \( \nabla \cdot D = \rho_e \)

or,

\[ \rho_e = \nabla \cdot [\varepsilon_0 (ct - x) a_y] \quad (\text{from equation (4)}) \]

It means the field satisfies Maxwell’s equation if \( \rho_e = 0 \).

(ii) \( \nabla \cdot B = 0 \)

Now,

\[ \nabla \cdot B = \nabla \cdot \left[\left(t - \frac{x}{c}\right) a_z\right] = 0 \quad (\text{from equation (1)}) \]

So, it already satisfies Maxwell’s equation.

(iii) \( \nabla \times H = J + \frac{\partial D}{\partial t} \)
Now, \( \nabla \times \mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t} \mathbf{a}_y = \frac{1}{\mu_0 c} \mathbf{a}_y = \frac{\varepsilon_0}{\mu_0} \mathbf{a}_y \) (from equation (2))

and from equation (4) we have

\[
\frac{\partial \mathbf{D}}{\partial t} = \varepsilon_0 c \mathbf{a}_y = \sqrt{\frac{\varepsilon_0}{\mu_0}} \mathbf{a}_y
\]

(Since in free space \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \))

Putting the two results in Maxwell’s equation, we get the condition

\[
\mathbf{J} = 0
\]

(iv) \[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

Now \[ \nabla \times \mathbf{E} = \frac{\partial E_y}{\partial x} \mathbf{a}_z = -\mathbf{a}_z \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \mathbf{a}_z \]

So, it already satisfies Maxwell’s equation. Thus, by combining all the results we get the two required conditions as \( \mathbf{J} = 0 \) and \( \mathbf{\rho}_s = 0 \) for the field to satisfy Maxwell’s equation.

**SOL 6.2.19** Option (B) is correct.

Given the electric field in time domain as

\[ \mathbf{E} = 5 \sin(10\pi y) \cos(6\pi \times 10^6 \text{ } \text{ } \beta x) \mathbf{a}_z \]

Comparing it with the general equation for electric field intensity given as

\[ \mathbf{E} = E_0 \cos(\omega t - \beta x) \mathbf{a}_z \]

We get, \( \omega = 6\pi \times 10^6 \)

Now in phasor form, the electric field intensity is

\[ \mathbf{E}_c = 5 \sin(10\pi y) e^{-j\beta x} \mathbf{a}_z \] (1)

From Maxwell’s equation we get the magnetic field intensity as

\[ \mathbf{H}_c = -\frac{1}{j\omega\mu_0} (\nabla \times \mathbf{E}_c) = \frac{j}{\omega\mu_0} \left[ \frac{\partial E_y}{\partial x} \mathbf{a}_z - \frac{\partial E_z}{\partial y} \mathbf{a}_z \right] \]

\[ = \frac{j}{\omega\mu_0} \left[ 50\pi \cos(10\pi y) e^{-j\beta x} \mathbf{a}_z + j5b \sin(10\pi y) \mathbf{a}_y e^{-j\beta x} \right] \]

Again from Maxwell’s equation we have the electric field intensity as

\[ \mathbf{E}_c = \frac{1}{j\omega\varepsilon_0} (\nabla \times \mathbf{H}_c) = \frac{1}{j\omega\varepsilon_0} \left[ \frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} \right] \mathbf{a}_z \]

\[ = \frac{1}{j\omega^2 \mu_0 \varepsilon_0} \left[ (\overline{f}b) (\overline{-jb}) \sin(10\pi y) e^{-j\beta x} + (50\pi) (10\pi) \sin(10\pi y) e^{-j\beta x} \right] \mathbf{a}_z \]

Comparing this result with equation (1) we get

\[ \frac{1}{\omega^2 \mu_0 \varepsilon_0} (5b^2 + 500\pi^2) = 5 \]

or,

\[ b^2 + 100\pi^2 = \omega^2 \mu_0 \varepsilon_0 \]

\[ b^2 + 100\pi^2 = (6\pi \times 10^6)^2 \times \frac{1}{(3 \times 10^8)} \quad (\omega = 6\pi \times 10^6, \sqrt{\mu_0 \varepsilon_0} = \frac{1}{c}) \]

\[ b^2 + 100\pi^2 = 400\pi^2 \]

**GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia**
SOL 6.2.20 Option (D) is correct.

The force experienced by a test charge \( q \) in presence of both electric field \( E \) and magnetic field \( B \) in the region will be evaluated by using Lorentz force equation as
\[
F = q(E + v \times B)
\]
So, putting the given three forces and their corresponding velocities in above equation we get the following relations
\[
q(a_x + a_z) = q(E + a_x \times B)
\]
\[
q(a_y) = q(E + a_y \times B)
\]
\[
q(2a_y + a_z) = q(E + a_z \times B)
\]
Subtracting equation (2) from (1) we get
\[
a_z = (a_x - a_y) \times B
\]
and subtracting equation (1) from (3) we get
\[
a_y = (a_y - a_z) \times B
\]
Now we substitute \( B = B_x a_x + B_y a_y + B_z a_z \) in eq (4) to get
\[
a_x = B_x a_x - B_y a_y + B_z a_z
\]
So, comparing the \( x, y \) and \( z \) components of the two sides we get
\[
B_x + B_y = 1
\]
and
\[
B_z = 0
\]
Again by substituting \( B = B_x a_x + B_y a_y + B_z a_z \) in eq (5), we get
\[
a_y = B_x a_x - B_y a_y + B_z a_z
\]
So, comparing the \( x, y \) and \( z \) components of the two sides we get
\[
B_x + B_z = 1
\]
and
\[
B_y = 0
\]
as calculated above \( B_z = 0 \), therefore \( B_z = 1 \)
Thus, the magnetic flux density in the region is
\[
B = a_x Wb/m^2
\]
\[
B = 1, B_y = 0, B_z = 0
\]

SOL 6.2.21 Option (A) is correct.

As calculated in previous question the magnetic flux density in the region is
\[
B = a_x Wb/m^2
\]
So, putting it in Lorentz force equation we get
\[
F = q(E + v \times B)
\]
or,
\[
q(a_x + a_z) = q(E + a_x \times a_x)
\]
Therefore, the electric field intensity in the medium is
\[
E = a_y + a_z V/m
\]

SOL 6.2.22 Option (D) is correct.

Given the magnetic flux density through the loop is
\[
B = -4/x a_z
\]
So the total magnetic flux passing through the loop is given as
\[ \Phi = \int B \cdot dS = \int_x^{x+2} \int_y^{y+2} \left( -\frac{2}{x} a_x \right) \cdot (-dxdy a_x) \]

\[ = (2 \ln \frac{x+2}{x})(2) = 4 \ln \left( \frac{x+2}{x} \right) \]

Therefore, the circulation of induced electric field in the loop is

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \left[ 4 \ln \left( \frac{x+2}{x} \right) \right] \]

\[ = -\frac{4x}{x+2} \left( \frac{2}{x^2} \right) \]

\[ = \frac{8}{x(x+2)} \left( \frac{x}{x+4} \right) \]

\[ \left( \frac{dx}{dt} = v = 2a_x \right) \]

**SOL 6.2.23** Option (D) is correct.

As the magnetic flux density for \( \rho < 4 \) is \( B = 0 \) so, the total flux passing through the closed loop defined by \( \rho = 4 \) m is

\[ \Phi = \int B \cdot dS = 0 \]

So, the induced electric field circulation for the region \( \rho < 4 \) m is given as

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} = 0 \]

or,

\[ E = 0 \]

for \( \rho < 4 \) m

**SOL 6.2.24** Option (B) is correct.

As the magnetic field for the region \( \rho < 4 \) m and \( \rho > 5 \) m is zero so we get the distribution of magnetic flux density as shown in figure below.

At any distance \( \rho \) from origin in the region \( 4 < \rho < 5 \) m, the circulation of induced electric field is given as

\[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \left( \int B \cdot dS \right) \]

\[ = -\frac{d}{dt} \left[ 2 \sin \omega t \left( \pi \rho^2 - \pi 4^2 \right) \right] \]
or, \[ E(2\pi\rho) = -2\omega \cos \omega t(\pi \rho^2 - 16\pi) \]

\[ E = -\frac{2(\rho^2 - 16)\omega \cos \omega t}{2\rho} \]

So, the induced electric field intensity at \( \rho = 4.5 \) m is

\[ E = -\frac{2}{4.5}((4.5)^2 - 16)\omega \cos \omega t \]

\[ = -\frac{17}{13}\omega \cos \omega t \]

**SOL 6.2.25** Option (B) is correct.

For the region \( \rho > 5 \) m the magnetic flux density is 0 and so the total magnetic flux passing through the closed loop defined by \( \rho = 5 \) m is

\[ \Phi = \int_0^\rho B \cdot dS = \int_0^5 B \cdot dS + \int_5^\rho B \cdot dS = 0 + \int_5^\rho (2\sin \omega t) a_z \cdot dS \]

\[ = (2\sin \omega t)[\pi(5)^2 - \pi(4)^2] = 18\pi \sin \omega t \]

So, the circulation of magnetic flux density for any loop in the region \( \rho > 5 \) m is

\[ \oint E \cdot dl = -\frac{d\psi}{dt} \]

\[ E(2\pi\rho) = -\frac{d}{dt}(18\pi \sin \omega t) \]

\[ = -18\pi \omega \cos \omega t \]

So, the induced electric field intensity in the region \( \rho > 5 \) m is

\[ E = -\frac{18\pi \omega \cos \omega t}{2\pi \rho} a_\phi \]

\[ = -\frac{18}{\rho} \omega \cos \omega t a_\phi \]

**SOL 6.2.26** Option (D) is correct.

Let the test charge be \( q \) coulomb So the force presence of experienced by the test charge in the presence of magnetic field is

\[ F = q(v \times B) \]

and the force experienced can be written in terms of the electric field intensity as

\[ F = qE \]

Where \( E \) is field viewed by observer moving with test charge.

Putting it in Eq. (i)

\[ qE = q(v \times B) \]

\[ E = (\omega p a_\phi) \times (2a_z) \]

where \( \omega \) is angular velocity and \( \rho \) is radius of circular loop.

\[ = (2)(2)(2)a_{\rho} = 8a_{\rho} \text{ V/m} \]

**SOL 6.2.27** Option (A) is correct.

Let the point charge located at origin be \( Q \) and the current \( I \) is flowing out of the
As the current \( I \) flows away from the point charge along the wire, the net charge at origin will change with increasing time and given as

\[
\frac{dQ}{dt} = -I
\]

So the electric field intensity will also vary through the surface and for the varying field circulation of magnetic field intensity around the triangular loop is defined as

\[
\oint C \mathbf{H} \cdot d\mathbf{l} = [I_c]_{\text{enc}} + [I_d]_{\text{enc}}
\]

where \([I_c]_{\text{enc}}\) is the actual flow of charge called enclosed conduction current and \([I_d]_{\text{enc}}\) is the current due to the varying field called enclosed displacement current which is given as

\[
[I_d]_{\text{enc}} = \frac{d}{dt} \oint_S (\varepsilon_0 \mathbf{E}) \cdot d\mathbf{S} = \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S}
\]

From symmetry the total electric flux passing through the triangular surface is

\[
\int_S \mathbf{D} \cdot d\mathbf{S} = \frac{Q}{8}
\]

So,

\[
[I_c]_{\text{enc}} = \frac{d}{dt} \left( \frac{Q}{8} \right) = \frac{1}{8} \frac{dQ}{dt} = -\frac{I}{8}
\]

(from equation (1))

Where as \([I_c]_{\text{enc}} = I\)

So, the net circulation of the magnetic field intensity around the closed triangular loop is

\[
\oint C \mathbf{H} \cdot d\mathbf{l} = [I_c]_{\text{enc}} + [I_d]_{\text{enc}}
\]

\[
= -\frac{I}{8} + I = \frac{7}{8}(8) = 14 \text{ A}
\]

\(I = 8 \text{ A}\)

**SOL 6.2.28** Option (C) is correct.

The distribution of magnetic flux density and the resistance in the circuit are same as given in section A (Q. 31) so, as calculated in the question, the two voltage drops in the loop due to magnetic flux density \(B = 0.1 t \mathbf{a}_z\) are

\[V_1 = 33.3 \text{ mV}\]

and

\[V_2 = 66.67 \text{ mV} = 66.7 \text{ mV}\]

Now \(V_3\) (voltmeter) which is directly connected to terminal \(cd\) is in parallel to
both $V_2$ and $V_1$. It must be kept in mind that the loop formed by voltmeter $V_3$ and resistance $2 \Omega$ also carries the magnetic flux density crossing through it. So, in this loop the induced emf will be produced which will be same as the field produced in loop $abcd$ at the enclosed fluxes will be same.

Therefore as calculated above induced emf in the loop of $V_3$ is

$$V_{\text{emf}} = 100 \text{ mV}$$

According to lenz’s law it’s polarity will be opposite to $V_3$ and so

$$-V_{\text{emf}} = V_1 + V_3$$

or,

$$V_3 = 100 - 33.3 = 66.7 \text{ mV}$$

**SOL 6.2.29** Option (C) is correct.

The induced emf in a closed loop is defined as

$$V_{\text{emf}} = -\frac{\partial \Phi}{\partial t}$$

where $\Phi$ is the total magnetic flux passing through the square loop

At any time $t$, angle between $B$ and $dS$ is $\theta$ since $B$ is in $a_y$ direction so the total magnetic flux passing through the square loop is

$$\Phi = \int B \cdot dS = (B)(S) \cos \theta = (5 \times 10^{-3})(20 \times 10^{-3} \times 20 \times 10^{-3}) \cos \theta = 2 \times 10^{-6} \cos \theta$$

Therefore the induced emf in the loop is

$$V_{\text{emf}} = -\frac{\partial \Phi}{\partial t} = -2 \times 10^{-6} \frac{d}{dt}(\cos \theta) = 2 \times 10^{-6} \sin \theta \frac{d\theta}{dt}$$

and as

$$\frac{d\theta}{dt} = \text{angular velocity} = 2 \text{ rad/sec}$$

So,

$$V_{\text{emf}} = (2 \times 10^{-6}) \sin \theta (2) = 4 \times 10^{-6} \sin \theta \text{ V/m} = 24 \sin \theta \mu\text{V/m}$$

**SOL 6.2.30** Option (B) is correct.

As calculated in previous question the induced emf in the closed square loop is

$$V_{\text{emf}} = 4 \sin \theta \mu\text{V/m}$$

So the induced current in the loop is

$$I = \frac{V_{\text{emf}}}{R} = \frac{4 \sin \theta \times 10^{-6}}{40 \times 10^{-3}} = 0.1 \sin \theta \text{ mA}$$

**SOL 6.2.31** Option (A) is correct.

The total magnetic flux through the square loop is given as

$$\Phi = \int B \cdot dS = (B_t \sin \omega t)(S) \cos \theta$$

So, the induced emf in the loop is

$$V_{\text{emf}} = -\frac{\partial \Phi}{\partial t} = -\frac{d}{dt}(B_t \sin \omega t) (S) \cos \theta$$
\[ V_{\text{emf}} = -B_0 S \frac{d}{dt}[\sin \omega t \cos \omega t] = -B_0 S \cos 2\omega t \]

Thus, the maximum value of induced emf is

\[ |V_{\text{emf}}| = B_0 S \omega \]

**SOL 6.2.32** Option (B) is correct.

As calculated in previous question the maximum induced voltage in the rotating loop is given as

\[ |V_{\text{emf}}| = B_0 S \omega \]

From the given data, we have

\[ B_0 = 0.25 \text{ Wb/m}^2 \]
\[ S = 64 \text{ cm}^2 = 64 \times 10^{-4} \text{ m}^2 \]

and \[ \omega = 60 \times 2\pi = 377 \text{ rad/sec} \] (In one revolution \(2\pi\) radian is covered)

So, the r.m.s. value of the induced voltage is

\[ [V_{\text{emf}}]_{\text{rms}} = \frac{1}{\sqrt{2}} |V_{\text{emf}}| = \frac{1}{\sqrt{2}} B_0 S \omega = \frac{1}{\sqrt{2}} (0.25 \times 64 \times 10^{-4} \times 377) = 0.4265 \]

Since the loop has 50 turns so net induced voltage will be 50 times the calculated value.

i.e.

\[ [V_{\text{emf}}]_{\text{rms}} = 50 \times (0.4265) = 21.33 \text{ volt} \]

**SOL 6.2.33** Option (A) is correct.

e.m.f. induced in the loop due to the magnetic flux density is given as

\[ V_{\text{emf}} = -\frac{\delta \Phi}{\delta t} = -\frac{\partial}{\partial t}(10 \cos 120\pi t)(10^2) \]

\[ = -\pi (10 \times 10^{-2})^2 \times (120\pi)(-10 \sin 120\pi t) = 12\pi^2 \sin 120\pi t \]

As determined by Lenz’s law the polarity of induced e.m.f will be such that \( b \) is at positive terminal with respect to \( a \).

i.e.

\[ V_{ba} = V_{\text{emf}} = 12\pi^2 \sin 120\pi t \]

or

\[ V_{ab} = -12\pi^2 \sin 120\pi t = -118.43 \sin 120\pi t \text{ Volt} \]

**SOL 6.2.34** Option (C) is correct.

As calculated in previous question, the voltage induced in the loop is

\[ V_{ab} = -12\pi^2 \sin 120\pi t \]

Therefore, the current flowing in the loop is given as

\[ I(t) = \frac{V_{ab}}{250} = \frac{12\pi^2 \sin 120\pi t}{250} = 2.47 \sin 120\pi t \]

************
SOLUTIONS 6.3

SOL 6.3.1
Option (A) is correct.

Given, the magnetic flux density in air as
\[ B = B_0 \left( \frac{x}{x^2 + y^2} a_x - \frac{y}{x^2 + y^2} a_y \right) \]  
\[ \text{...(1)} \]

Now, we transform the expression in cylindrical system, substituting
\[ x = r \cos \phi \quad \text{and} \quad y = r \sin \phi \]
\[ a_x = \cos \phi a_r - \sin \phi a_\phi \]
and
\[ a_y = \sin \phi a_r + \cos \phi a_\phi \]
So, we get
\[ B = B_0 a_\phi \]

Therefore, the magnetic field intensity in air is given as
\[ H = \frac{B}{\mu_0} = \frac{B_0 a_\phi}{\mu_0}, \text{ which is constant} \]

So, the current density of the field is
\[ J = \nabla \times H = 0 \quad \text{(since } H \text{ is constant)} \]

SOL 6.3.2
Option (C) is correct.

Maxwell equations for an EM wave is given as
\[ \nabla \cdot B = 0 \]
\[ \nabla \cdot E = \frac{\rho_c}{\varepsilon} \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times H = \frac{\partial D}{\partial t} + J \]

So, for static electric magnetic fields
\[ \nabla \cdot B = 0 \]
\[ \nabla \cdot E = \rho_c / \varepsilon \]
\[ \nabla \times E = 0 \]
\[ \nabla \times H = J \quad \text{(since } H \text{ is constant)} \]
\[ \left( \frac{\partial B}{\partial t} = 0 \right) \]
\[ \left( \frac{\partial D}{\partial t} = 0 \right) \]

SOL 6.3.3
Option (C) is correct.

Maxwell Equations
\[ \nabla \times H = J + \frac{\partial D}{\partial t} \]
Integral form
\[ \oint_S (\nabla \times H) \cdot dS = \int_S (J + \frac{\partial D}{\partial t}) \cdot dS \]
\[ \oint H \cdot dl = \iint_S (J + \frac{\partial D}{\partial t}) \cdot dS \]  
Stokes Theorem

**SOL 6.3.4**  
Option (A) is correct.  
From Maxwells equations we have  
\[ \nabla \times H = \frac{\partial D}{\partial t} + J \]  
Thus, \( \nabla \times H \) has unit of current density \( J \) (i.e., \( \text{A/m}^2 \))

**SOL 6.3.5**  
Option (D) is correct.  
This equation is based on Ampere’s law as from Ampere’s circuital law we have  
\[ \oint H \cdot dl = I_{\text{enclosed}} \]  
or,  
\[ \oint H \cdot dl = \int_S J \cdot dS \]  
Applying Stoke’s theorem we get  
\[ \int_S (\nabla \times H) \cdot dS = \int_S J \cdot dS \]  
\[ \nabla \times H = J \]  
Then, it is modified using continuity equation as  
\[ \nabla \times H = J + \frac{\partial D}{\partial t} \]

**SOL 6.3.6**  
Option (C) is correct.  
When a moving circuit is put in a time varying magnetic field induced emf have two components. One due to time variation of magnetic flux density \( B \) and other due to the motion of circuit in the field.

**SOL 6.3.7**  
Option (A) is correct.  
From maxwell equation we have  
\[ \nabla \times H = J + \frac{\partial D}{\partial t} \]  
The term \( \frac{\partial D}{\partial t} \) defines displacement current.

**SOL 6.3.8**  
Option (A) is correct.  
Emf induced in a loop carrying a time varying magnetic flux \( \phi \) is defined as  
\[ V_{\text{ind}} = -\frac{d\phi}{dt} \]  
\[ 9 = -\frac{d}{dt}(\frac{1}{3} \lambda t^3) \]  
\[ 9 = -\lambda t^2 \]  
at time, \( t = 3 \text{s} \), we have  
\[ 9 = -\lambda (3)^2 \]  
\[ \lambda = -2 \text{ Wb/s}^2 \]

**SOL 6.3.9**  
Option (B) is correct.  
According to Lenz’s law the induced emf (or induced current) in a loop flows such as to produce a magnetic field that opposed the change in \( B \). The direction of the magnetic field produced by the current is determined by right hand rule.
Now, in figure (1), $B$ directed upward increases with time whereas the field produced by current $I$ is downward so, it obeys the Lenz’s law.

In figure (2), $B$ directed upward is decreasing with time whereas the field produced by current $I$ is downwards (i.e. additive to the change in $B$) so, it doesn’t obey Lenz’s law.

In figure (3), $B$ directed upward is decreasing with time whereas current $I$ produces the field directed upwards (i.e. opposite to the change in $B$) So, it also obeys Lenz’s law.

In figure (4), $B$ directed upward is increasing with time whereas current $I$ produces field directed upward (i.e. additive to the change in $B$) So, it doesn’t obey Lenz’s law.

Thus, the configuration 1 and 3 are correct.

SOL 6.3.10 Option (A) is correct.

Faraday’s law states that for time varying field,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Since, the curl of gradient of a scalar function is always zero

i.e.

$$\nabla \times (\nabla V) = 0$$

So, the expression for the field,

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

i.e. A is true but R is false.

SOL 6.3.11 Option (B) is correct.

Faraday develops the concept of time varying electric field producing a magnetic field. The law he gave related to the theory is known as Faraday’s law.

SOL 6.3.12 Option (C) is correct.

Given, the area of loop

$$S = 5 \text{ m}^2$$

Rate of change of flux density,

$$\frac{\partial B}{\partial t} = 2 \text{ Wb/m}^2/\text{S}$$

So, the emf in the loop is

$$V_{\text{emf}} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{S} = (5)(-2) = -10 \text{ V}$$

SOL 6.3.13 Option (C) is correct.

The modified Maxwell’s differential equation.

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

This equation is derived from Ampere’s circuitual law which is given as

$$\int \mathbf{H} \cdot d\mathbf{l} = I_{\text{cur}}$$

$$\int (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int \mathbf{J} d\mathbf{S}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$
SOL 6.3.14 Option (B) is correct.

Electric potential of an isolated sphere is defined as

\[ C = 4\pi\varepsilon_0 a \quad \text{(free space)} \]

The Maxwell’s equation in phasor form is written as

\[ \nabla \times \mathbf{H} = j\omega \varepsilon \mathbf{E} + \sigma \mathbf{E} = j\omega \varepsilon \mathbf{E} + \mathbf{J} \quad \text{(J = \sigma \mathbf{E})} \]

So A and R both are true individually but R is not the correct explanation of A.

SOL 6.3.15 Option (D) is correct.

If a coil is placed in a time varying magnetic field then the e.m.f. will induce in coil. So here in both the coil e.m.f. will be induced.

SOL 6.3.16 Option (B) is correct.

Both the statements are individually correct but R is not explanation of A.

SOL 6.3.17 Option ( ) is correct.

Ampere’s law \[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{(a \rightarrow 3)} \]

Faraday’ law \[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad \text{(b \rightarrow 4)} \]

Gauss law \[ \nabla \cdot \mathbf{D} = \rho_c \quad \text{(c \rightarrow 1)} \]

Current continuity \[ \nabla \cdot \mathbf{J} = \frac{\partial \rho}{\partial t} \quad \text{(d \rightarrow 2)} \]

SOL 6.3.18 Option (B) is correct.

Since, the magnetic field perpendicular to the plane of the ring is decreasing with time so, according to Faraday’s law emf induced in both the ring is

\[ V_{\text{emf}} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{S} \]

Therefore, emf will be induced in both the rings.

SOL 6.3.19 Option (D) is correct.

The Basic idea of radiation is given by the two Maxwells equation

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

SOL 6.3.20 Option (B) is correct.

The correct maxwell’s equation are

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \cdot \mathbf{D} = \rho \]

\[ \nabla \cdot \mathbf{B} = 0 \]

SOL 6.3.21 Option (B) is correct.

In List I

a. \[ \oint \mathbf{B} \cdot d\mathbf{S} = 0 \]

The surface integral of magnetic flux density over the closed surface is zero or in other words, net outward magnetic flux through any closed surface is zero. \(a \rightarrow 4\)

b. \[ \oint \mathbf{D} \cdot d\mathbf{S} = \int \rho, dv \]
Total outward electric flux through any closed surface is equal to the charge enclosed in the region. \( \text{(b)} \quad \text{(3)} \)

c. \[ \int E \cdot dl = - \int \frac{\partial B}{\partial t} \cdot dS \]
i.e. The line integral of the electric field intensity around a closed path is equal to the surface integral of the time derivative of magnetic flux density \( \text{(c)} \quad \text{(2)} \)

d. \[ \oint H \cdot dS = \int \left( \frac{\partial D}{\partial t} + J \right) da \]
i.e. The line integral of magnetic field intensity around a closed path is equal to the surface integral of sum of the current density and time derivative of electric flux density. \( \text{(d)} \quad \text{(1)} \)

**SOL 6.3.22** Option (C) is correct.
The continuity equation is given as \[ \nabla \cdot J = -\rho \]
i.e. it relates current density \( J \) and charge density \( \rho \).

**SOL 6.3.23** Option (A) is correct.
Given Maxwell’s equation is \[ \nabla \times H = J + \frac{\partial D}{\partial t} \]
For free space, conductivity, \( \sigma = 0 \) and so,
\[ J = \sigma E = 0 \]
Therefore, we have the generalized equation \[ \nabla \times H = \frac{\partial D}{\partial t} \]

**SOL 6.3.24** Option (D) is correct.
Given the magnetic field intensity, \[ H = 3a_x + 7a_y + 2xa_z \]
So from Ampere’s circuital law we have \[ J = \nabla \times H \]
\[
\begin{vmatrix}
  a_x & a_y & a_z \\
  \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
  3 & 7y & 2x 
\end{vmatrix}
= a_x(0) - a_y(2 - 0) + a_z(0) = -2a_y
\]

**SOL 6.3.25** Option (D) is correct.
The emf in the loop will be induced due to motion of the loop as well as the variation in magnetic field given as \[ V_{\text{emf}} = -\int \frac{\partial B}{\partial t} \cdot dS + \oint (v \times B) \cdot dl \]
So, the frequencies for the induced e.m.f. in the loop is \( \omega_1 \) and \( \omega_2 \).

**SOL 6.3.26** Option (B) is correct. \[ F = Q(E + v \times B) \text{ is Lorentz force equation.} \]
SOL 6.3.27  Option (D) is correct.  
All of the given expressions are Maxwell’s equation.

SOL 6.3.28  Option (D) is correct.  
The direction of magnetic flux due to the current ‘i’ in the conductor is determined by right hand rule. So, we get the flux through A is pointing into the paper while the flux through B is pointing out of the paper.  
According to Lenz’s law the induced e.m.f. opposes the flux that causes it. So again by using right hand rule we get the direction of induced e.m.f. is anticlockwise in A and clockwise in B.

SOL 6.3.29  Option (C) is correct.  
\[ \nabla^2 A = -\mu_0 J \]
This is the wave equation for static electromagnetic field.  
i.e. It is not Maxwell’s equation.

SOL 6.3.30  Option (B) is correct.  
Poisson’s equation for an electric field is given as  
\[ \nabla^2 V = -\frac{\rho}{\varepsilon} \]
where, V is the electric potential at the point and \( \rho \varepsilon \) is the volume charge density in the region. So, for \( \rho \varepsilon = 0 \) we get,  
\[ \nabla^2 V = 0 \]
Which is Laplacian equation.

SOL 6.3.31  Option (B) is correct.  
Continuity equation  
\[ \nabla \times J = \frac{\partial \rho}{\partial t} \]  
Ampere’s law  
\[ \nabla \times H = J + \frac{\partial D}{\partial t} \]
Displacement current  
\[ J = \frac{\partial D}{\partial t} \]
Faraday’ law  
\[ \nabla \times E = \frac{\partial B}{\partial t} \]

SOL 6.3.32  Option (B) is correct.  
A static electric field in a charge free region is defined as  
\[ \nabla \cdot E = 0 \]  
and  
\[ \nabla \times E = 0 \]
A static electric field in a charged region have  
\[ \nabla \cdot E = \frac{\rho}{\varepsilon} \neq 0 \]  
and  
\[ \nabla \times E = 0 \]
A steady magnetic field in a current carrying conductor have  
\[ \nabla \cdot B = 0 \]
\[ \nabla \times B = \mu_0 J \neq 0 \]
A time varying electric field in a charged medium with time varying magnetic field
have
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0 \quad (d \to 3) \]
\[ \nabla \cdot \vec{E} = \frac{\partial \rho}{\partial t} \neq 0 \]

**SOL 6.3.33**
Option (A) is correct.
\[ V = -\frac{d\Phi}{dt} \]
It is Faraday’s law that states that the change in flux through any loop induces e.m.f. in the loop.

**SOL 6.3.34**
Option (B) is correct.
From stokes theorem, we have
\[ \int (\nabla \times \vec{E}) \cdot d\vec{S} = \int \vec{E} \cdot d\vec{l} \]
(1)

Given, the Maxwell’s equation
\[ \nabla \times \vec{E} = -(\frac{\partial \vec{B}}{\partial t}) \]
Putting this expression in equation (1) we get,
\[ \int \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S} \]

**SOL 6.3.35**
Option (B) is correct.
Induced emf in a coil of \( N \) turns is defined as
\[ V_{\text{emf}} = -N \frac{d\Phi}{dt} \]
where \( \Phi \) is flux linking the coil. So, we get
\[ V_{\text{emf}} = -100 \frac{d}{dt}(t^2 - 2t) \]
\[ = -100(3t^2 - 2) = -100(3(2)^2 - 2) = -1000 \text{ mV} \quad (\text{at } t = 2 \text{ s}) \]
\[ = -2 \text{ V} \]

**SOL 6.3.36**
Option (C) is correct.
Since, the flux linking through both the coil is varying with time so, emf are induced in both the coils.
Since, the loop 2 is split so, no current flows in it and so joule heating does not occur in coil 2 while the joule heating occurs in closed loop 1 as current flows in it. Therefore, only statement 2 is correct.

**SOL 6.3.37**
Option (A) is correct.
The electric field intensity is
\[ \vec{E} = \vec{E}_0 e^{j\omega t} \]
where \( \vec{E}_0 \) is independent of time
So, from Maxwell’s equation we have
\[ \nabla \times \vec{H} = \vec{J} + \frac{\varepsilon}{\varepsilon} \frac{\partial \vec{E}}{\partial t} \]
\[ = \sigma \vec{E} + \varepsilon (j\omega) \vec{E}_0 e^{j\omega t} = \sigma \vec{E} + j\omega \varepsilon \vec{E} \]

**SOL 6.3.38**
Option (A) is correct.
Equation (1) and (3) are not the Maxwell’s equation.
404 Time Varrying Field and Maxwell Equation Chap 6

For View Only

SOL 6.3.39  Option (D) is correct.

From the Maxwell’s equation for a static field (DC) we have
\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]
\[ \nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J} \]
\[ \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu_0 \mathbf{J} \]

For static field (DC), \( \nabla \cdot \mathbf{A} = 0 \)
therefore we have, \( \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \)
So, both A and R are true and R is correct explanation of A.

SOL 6.3.40  Option (D) is correct.

For a static field, Maxwells equation is defined as
\[ \nabla \times \mathbf{H} = \mathbf{J} \]
and since divergence of the curl is zero
i.e. \( \nabla \cdot (\nabla \times \mathbf{H}) = 0 \)
\[ \nabla \cdot \mathbf{J} = 0 \]
but in the time varying field, from continuity equation (conservation of charges)
\[ \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \neq 0 \]
So, an additional term is included in the Maxwell’s equation.

Therefore A and R both are true and R is correct explanation of A.

SOL 6.3.41  Option (D) is correct.

For any loop to have an induced e.m.f., magnetic flux lines must link with the coil. Observing all the given figures we conclude that loop \( C_1 \) and \( C_2 \) carries the flux lines through it and so both the loop will have an induced e.m.f.

SOL 6.3.42  Option (A) is correct.

Since, the circular loop is rotating about the \( y \)-axis as a diameter and the flux lines is directed in \( \alpha \) direction. So, due to rotation magnetic flux changes and as the flux density is function of time so, the magnetic flux also varies w.r.t. time and therefore the induced e.m.f. in the loop is due to a combination of transformer and motional e.m.f. both.

SOL 6.3.43  Option (A) is correct.

Gauss’s law \[ \nabla \cdot \mathbf{D} = \rho \quad (a \rightarrow 1) \]
Ampere’s law \[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (b \rightarrow 5) \]
Faraday’s law \[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad (c \rightarrow 2) \]
Poynting vector \[ \mathbf{P} = \mathbf{E} \times \mathbf{H} \quad (d \rightarrow 3) \]
CHAPTER 7

ELECTROMAGNETIC WAVES
EXERCISE 7.1

MCQ 7.1.1 What will be the direction of wave propagation in a non magnetic medium in which magnetic field intensity at any point is given by
\[ H = 4 \cos(\omega t - ky) \hat{a}_z, \text{ A/m} \]
(A) +\( \hat{a}_x \) direction (B) -\( \hat{a}_x \) direction
(C) +\( \hat{a}_x \) direction (D) +\( \hat{a}_y \) direction

MCQ 7.1.2 In a certain medium electric field intensity of a propagating wave is given by
\[ E(x, t) = 2E_0 e^{-\alpha t} \sin(\omega t - \beta z) \hat{a}_x, \text{ V/m} \]
The electric field phasor of the wave will be
(A) \( E_0 e^{-(\alpha + \beta)x} \hat{a}_y, \text{ V/m} \)
(B) \( jE_0 e^{-(\alpha + \beta)x} \hat{a}_y, \text{ V/m} \)
(C) \( -jE_0 e^{-(\alpha + \beta)x} \hat{a}_y, \text{ V/m} \)
(D) \( -jE_0 e^{(\alpha - \beta)x} \hat{a}_y, \text{ V/m} \)

MCQ 7.1.3 In air, magnetic field intensity is given by
\[ H = 10 \cos(6 \times 10^7 t - ky) \hat{a}_z, \text{ A/m} \]. Wave number \( k \) for the EM wave will be
(A) 1.8 rad/m (B) 2 rad/m
(C) 0.2 rad/m (D) 5 rad/m

MCQ 7.1.4 An electromagnetic wave is propagating in certain non magnetic material such that the magnetic field intensity at any point is given by
\[ H = 3 \cos(10^9 t - 5z) \hat{a}_z, \text{ A/m} \]
The phase velocity of the wave in the medium will be
(A) \( 1.5 \times 10^8 \) m/s (B) \( 5 \times 10^6 \) m/s
(C) \( 0.5 \times 10^8 \) m/s (D) \( 2 \times 10^8 \) m/s

MCQ 7.1.5 Magnetic field intensity in a certain non-magnetic medium is given by
\[ H = H_0 \cos(\omega t - \beta y) \hat{a}_x, \text{ A/m} \]
If the wavelength of the EM wave in the medium be 12.6 m then what will be the phase constant \( \beta \) in that medium?
(A) 0.25 rad/m (B) 0.5 rad/m
(C) 1.12 rad/m (D) 6.3 rad/m
In a nonmagnetic material electric field intensity is given by

\[ E = 16 \cos(4 \times 10^8 t - 2x) \hat{a}_y \text{ V/m} \]

The relative permittivity of the medium will be

(A) 1.5 \hspace{1cm} (B) 2.25

(C) 0.44 \hspace{1cm} (D) 225

Electric field intensity in free space is \( E = 24 \cos(5 \times 10^8 t - \beta z) \hat{a}_z \text{ V/m} \). The time period of the wave will be

(A) 7.96 ns \hspace{1cm} (B) 1.26 ns

(C) 8 \times 10^7 \text{ sec} \hspace{1cm} (D) 12.57 ns

In air, a propagating wave has electric field intensity given by

\[ E = 9 \cos(4 \times 10^8 t - \beta x) \hat{a}_x \text{ V/m} \]

The time taken by the wave to travel one-fourth of its total wave length is

(A) 61.42 ns \hspace{1cm} (B) 3.05 ns

(C) 7.23 ns \hspace{1cm} (D) 3.93 ns

What will be the intrinsic impedance of a lossless, nonmagnetic dielectric material having relative permittivity \( \varepsilon_r = 2.25 \)?

(A) 235.62 \Omega \hspace{1cm} (B) 167.56 \Omega

(C) 8.95 m\Omega \hspace{1cm} (D) 251.33 \Omega

A radio wave is propagating at a frequency of 0.5 MHz in a medium \( \sigma = 3 \times 10^7 \text{ S/m} \), \( \mu_r = \varepsilon_r \approx 1 \). The wave length of the radio wave in that medium will be

(A) 0.8 mm \hspace{1cm} (B) 0.26 mm

(C) 0.4 mm \hspace{1cm} (D) 0.13 mm

The skin depth in a poor conductor is independent of

(A) Permittivity \hspace{1cm} (B) Permeability

(C) Frequency \hspace{1cm} (D) None of these

Assertion (A) : \( E = E_0 \sin(z) \cos(ct) \hat{a}_z \) represents the electric field of a plane wave in free space.

Reason (R) : A plane wave \( f \) propagating with velocity \( v_p \) in \( + \hat{a}_z \) direction must satisfy the equation

\[ \frac{\partial^2 f}{\partial t^2} - v_p^2 \frac{\partial^2 f}{\partial z^2} = 0 \]

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true
MCQ 7.1.13  A propagating wave in free space has magnetic field intensity
\[ \mathbf{H} = 0.2 \cos(10^5 t - \beta y) \mathbf{a}_x \text{ A/m} \]
What will be the electric field intensity of the wave at \( y = 1 \text{ cm} \) at time, \( t = 0.1 \text{ ns} \)?
(A) \(-37.7 \mathbf{a}_x \) V/m  
(B) \(-36.2 \mathbf{a}_x \) V/m  
(C) \(-19.8 \mathbf{a}_x \) V/m  
(D) \(37.6 \mathbf{a}_x \) V/m

MCQ 7.1.14  Phasor form of magnetic field intensity of a uniform plane wave in free space is given as
\[ \mathbf{H}_s = (2 + \beta)(4a_y + 2ja_z)e^{-\beta z} \text{ A/m} \]
The maximum electric field of the plane wave equals to
(A) 24.1 V/m  
(B) 14.22 kV/m  
(C) 9.08 kV/m  
(D) 0

MCQ 7.1.15  Electric field intensity of linearly polarized plane wave in free space is given by
\[ \mathbf{E} = (6a_y - 5a_x)\cos(\omega t - 50z) \text{ V/m} \]
The phasor form of magnetic field intensity of the wave will be
(A) \(-\eta_0(5a_x + 6a_y)e^{-\beta_0 z} \text{ V/m} \)  
(B) \((5a_x - 6a_y)\frac{e^{-\beta_0 z}}{\eta_0} \text{ V/m} \)  
(C) \((5a_x - 6a_y)e^{-\beta_0 z} \text{ V/m} \)  
(D) \(-(5a_x + 6a_y)\frac{e^{-\beta_0 z}}{\eta_0} \text{ V/m} \)

MCQ 7.1.16  In a perfect conductor (resistivity, \( \rho \approx 0 \)) magnetic field of any EM wave
(A) lags electric field by 90°  
(B) leads electric field by 45°  
(C) lags electric field by 45°  
(D) will be in phase with electric field

Statement for Linked Question 17 - 18:
An electromagnetic wave travels in free space with the electric field component
\[ \mathbf{E}_s = (10a_x + 5a_y)e^{-\beta(4t-2z)} \text{ V/m} \]

MCQ 7.1.17  What will be the phasor form of magnetic field intensity of the wave ?
(A) \(-29.66e^{-\beta(4t-2z)} \text{ mA/m} \)  
(B) \(-5\sqrt{5}e^{-\beta(4t-2z)} \text{ mA/m} \)  
(C) \(-29.66e^{\beta(4t-2z)} \text{ mA/m} \)  
(D) \(-29.66e^{\beta(4t-2z)} \text{ A/m} \)

MCQ 7.1.18  What will be the time average power density of the electromagnetic wave ?
(A) \((665.9a_x - 331.6a_y) \text{ W/m}^2 \)  
(B) \((148.9a_x - 74.15a_y) \text{ W/m}^2 \)  
(C) \((-331.6a_x + 665.9a_y) \text{ W/m}^2 \)  
(D) \((-74.15a_x + 148.9a_y) \text{ W/m}^2 \)
A propagating wave has the phasor form of its electric field intensity defined as

\[ E_s = (-2\sqrt{3}a_x + \sqrt{3}a_y - a_z) e^{-j(3z + 3y - 2t)} \text{ V/m} \]

The wave is linearly polarized along the direction of

(A) \(-3a_x + \sqrt{3}a_y - 2a_z\)  
(B) \(-2\sqrt{3}a_x + \sqrt{3}a_y - a_z\)  
(C) \(3a_x - \sqrt{3}a_y + 2a_z\)  
(D) \(2\sqrt{3}a_x - \sqrt{3}a_y + a_z\)

**Statement for Linked Question 20 - 21:**

In free space an electric field intensity vector is given by

\[ E = 200\cos(\omega t - \beta z)a_x \]

where \(\omega\) and \(\beta\) are constants.

**MCQ 7.1.20**

If \(\beta = \frac{\omega}{\sqrt{\mu_0\varepsilon_0}}\), what will be the magnetic flux density vector \(B\) ?

(A) \(3 \times 10^8 \cos(\omega t - \beta z)a_y\)  
(B) \(3.33 \times 10^7 \cos(\omega t - \beta z)a_y\)  
(C) \(3 \times 10^8 \cos(\omega t - \beta z)a_y\)  
(D) \(3.33 \times 10^8 \cos(\omega t - \beta z)a_y\)

**MCQ 7.1.21**

The poynting vector of the \(E-M\) field will be

(A) \(10^4 \sqrt{\frac{\varepsilon_0}{\mu_0}} \cos^2(\omega t - \beta z)a_x\)  
(B) \(10^4 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos^2(\omega t - \beta z)a_x\)  
(C) \(10^4 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos^2(\omega t - \beta z)a_x\)  
(D) \(10^4 \cos^2(\omega t - \beta z)a_x\)

**MCQ 7.1.22**

The electric field associated with a sinusoidally time varying electromagnetic field is given by

\[ E = 15\sin\pi x\sin(2\pi \times 10^8 - \sqrt{3}\pi z)a_y \text{ V/m} \]

The time average stored energy density in the electric field is

(A) \(\frac{4}{25}\varepsilon_0\sin^2\pi x\)  
(B) \(\frac{25\varepsilon_0}{4}\sin^2\pi x\)  
(C) \(\frac{5\varepsilon_0}{4}\sin 2\pi x\)  
(D) \(\frac{25\varepsilon_0}{4}\sin 2\pi x\)

**MCQ 7.1.23**

Electric field associated with a sinusoidally time varying electromagnetic field is given by

\[ E = 20\sin(\pi y)\sin(6\pi \times 10^8 t - \sqrt{3}\pi x)a_z \text{ V/m} \]

What will be the time average stored energy density in the magnetic field ?

(A) \(\frac{10^{-9}}{\pi}(25 + 50\sin^2\pi x)\)  
(B) \(\frac{10^{-9}}{144\pi}(25 + 50\sin^2\pi x)\)  
(C) \(\frac{144}{\pi} \times 10^9(25 + 50\sin^2\pi x)\)  
(D) \(\frac{\pi}{144}(25 + 50\sin^2\pi x)\)
MCQ 7.1.24  An electromagnetic wave is propagating from free space to a certain medium having relative permittivity $\varepsilon_r = 5$. If wavelength of the wave in the medium be 20 cm then what would be it’s wavelength in free space?
(A) 6.67 cm  
(B) 60 cm  
(C) 180 cm  
(D) 18 cm

MCQ 7.1.25  If some free charge is being imbedded in a piece of glass, then the charge will flow out to the surface nearly after (relative permittivity of glass, $\varepsilon_r = 4.25$ conductivity of glass, $\sigma = 10^{-12}$ S/m)
(A) 2 sec  
(B) 4 sec  
(C) 35 sec  
(D) 20 sec

MCQ 7.1.26  An electromagnetic wave propagating in free space is incident on the surface of a dielectric medium ($\mu_0$, $4\varepsilon_0$). If the magnitude of the electric field of incident wave is $E_0$ then what will be the magnitude of the electric field of the reflected wave?
(A) $-2E_0/3$  
(B) $-E_0/3$  
(C) $E_0/2$  
(D) $-E_0$
MCQ 7.2.1 Magnetic field intensity of a propagating wave in free space is given by

\[ H = 0.3 \cos(\omega t - \beta y) a_z \, \text{A/m} \]

If the total time period of the wave be \( T \) then the plot of \( H \) versus \( y \) at time, \( t = \frac{T}{8} \) will be

(A)  
(B)  
(C)  
(D)  

MCQ 7.2.2 A uniform plane wave is propagating with a velocity of \( 7.5 \times 10^7 \) m/s in a lossless medium having relative permeability \( \mu_r = 2.8 \). The electric field phasor of the wave is given by

\[ E_z = 5e^{0.3x} a_z \, \text{V/m} \]

What will be the magnetic field intensity of the wave?

(A) \( 11.05 \cos(9.54 \times 10^6 t + 0.3x) a_y \, \text{mA/m} \)

(B) \( 22.13 \cos(9.54 \times 10^6 t + 0.3x) a_y \, \text{mA/m} \)

(C) \( 22.13 \cos(9.54 \times 10^6 t + 0.3x) a_y \, \text{mA/m} \)

(D) \( 11.05 \cos(2.25 \times 10^5 t + 0.3x) a_y \, \text{mA/m} \)
MCQ 7.2.3 In a certain medium \((\varepsilon_r = 4, \mu_r = 4)\). A plane wave is propagating such that the electric field intensity of the wave is \(E = E_0 e^{-\gamma z} \sin(10^6 t - \beta x)\, \boldsymbol{a}_y\) V/m. The loss tangent of the medium will be

- (A) 1.94
- (B) 0.27
- (C) 0.35
- (D) 0.52

**Statement for Linked Question 4 - 5:**

In a lossy medium \((\varepsilon_r = 8, \mu_r = 0.5, \sigma = 0.01\, \text{S/m})\) a plane wave is travelling in \(+\, \boldsymbol{a}_x\) direction that has the electric field intensity \(E = 0.5 \cos(10^6 \pi t + \pi/3)\, \boldsymbol{a}_x\) at \(z = 0\).

**MCQ 7.2.4** What will be the distance travelled by the wave to have a phase shift of \(10^\circ\) ?

- (A) 20.95 mm
- (B) 477.3 mm
- (C) 8.33 V
- (D) 3.65 m

**MCQ 7.2.5** After traveling a distance \(z\), the amplitude of the wave is reduced by 40%. So, the value of \(z\) equals to

- (A) 481.5 mm
- (B) 542 mm
- (C) 1.06 m
- (D) 2.08 m

**MCQ 7.2.6** A uniform plane wave is propagating at a velocity of \(7 \times 10^7\, \text{m/s}\) in a perfect dielectric such that the electric and magnetic fields of the wave are given by

\[
E(x, t) = 300 \cos(5 \times 10^6 \pi t - \beta x)\, \boldsymbol{a}_y\, \text{V/m} \\
H(x, t) = 1.9 \cos(5 \times 10^6 \pi t - \beta x)\, \boldsymbol{a}_z\, \text{V/m}
\]

The relative permittivity and relative permeability of the medium will be respectively

- (A) 1.70, 2.69
- (B) 3.4, 5.37
- (C) 1.70, 1.58
- (D) 2.37, 2.69

**MCQ 7.2.7** An electromagnetic wave is propagating in free space in \(-\, \boldsymbol{a}_z\) direction with a frequency \(\omega\) and phase angle zero. The EM wave is polarized in \(+\, \boldsymbol{a}_x\) direction. If the amplitude of electric field of the wave is \(E_0\) then the magnetic field of the wave will be

- (A) \(\frac{E_0}{\eta_0} \cos\left(\omega t + \frac{\omega}{c} x\right)\, \boldsymbol{a}_y\)
- (B) \(\eta_0 E_0 \cos(\omega t + \omega cx)\, \boldsymbol{a}_y\)
- (C) \(-\frac{E_0}{\eta_0} \cos\left(\omega t + \frac{\omega}{c} x\right)\, \boldsymbol{a}_y\)
- (D) \(\frac{E_0}{\eta_0} \cos\left(\omega t - \frac{\omega}{c} x\right)\, \boldsymbol{a}_y\)

**MCQ 7.2.8** What will be the electric field of a plane wave polarized parallel to the \(x-z\) plane and propagating in free space in the direction from origin to the point \((1,1,1)\), that has the amplitude \(E_0\) and frequency \(\omega\) with zero phase angle?

- (A) \(E_0 \left[ \omega t - \frac{\omega}{\sqrt{3} c} (x + y + z) \left(\frac{a_x + a_z}{\sqrt{2}}\right) \right]\)
- (B) \(E_0 \cos\left(\omega t + \frac{\omega}{\sqrt{3} c} (x + y + z) \left(\frac{a_x - a_z}{\sqrt{2}}\right) \right]\)
MCQ 7.2.9
A plane wave is propagating with frequency \( f = 50 \text{ kHz} \) in a medium \((\sigma = 2 \text{ S/m}, \varepsilon_r = 80, \mu_r = 4)\). What will be the skin depth of the medium?
(A) 1.7 m  
(B) 0.4 m  
(C) 0.8 m  
(D) 1.3 m

MCQ 7.2.10
With a thickness \( t \), silver coating is done for a microwave experiment to operate at a frequency of 10 GHz. For the successful experiment \( t \) should be (for silver, \( \mu_r = \varepsilon_r \approx 1, \sigma = 6.25 \times 10^7 \text{ S/m} \))
(A) greater than 0.64 \( \mu \text{m} \)  
(B) less than 0.64 \( \mu \text{m} \)  
(C) exactly equal to 0.64 \( \mu \text{m} \)  
(D) none of these

MCQ 7.2.11
In a nonmagnetic material of conductivity \( \sigma = 2 \times 10^7 \text{ S/m} \), electric field of a propagating plane wave is given by
\[
E = 5 \cos(10^7 t - 0.2y) \mathbf{a}_x + 2 \sin(10^7 t - 0.2y) \mathbf{a}_y \text{ V/m}
\]
What will be the value of complex permittivity of the medium?
(A) \((2 - j 36 \varepsilon_0) \text{ F/m} \)  
(B) \((36 \varepsilon_0 - j 2) \text{ F/m} \)  
(C) \((36 - j 2) \text{ F/m} \)  
(D) \((36 \varepsilon_0 + j 2) \text{ F/m} \)

MCQ 7.2.12
**Assertion (A)**: All the metals are opaque.
**Reason (R)**: Skin depth of metals are in the range of nanometers.
(A) Both A and R are true and R is the correct explanation of A.  
(B) Both A and R are true but R is not the correct explanation of A.  
(C) A is true but R is false.  
(D) A is false but R is true.

MCQ 7.2.13
In the plane \( z = 0 \), electric field of a wave propagating in \(+ \mathbf{a}_z\) direction in free space is \( E_0 \) which is varying with time \( t \) as shown in the figure.
If the magnetic field intensity of the wave at \( t = 1 \mu \text{sec} \) be \( H_1 \), then the plot of \( H_1 \) versus \( z \) will be

**MCQ 7.2.14** Three different dielectrics of permittivities \( 4\varepsilon_0 \), \( 9\varepsilon_0 \) and \( 3\varepsilon_0 \) are defined in the space as shown in figure. If the leading edge of a uniform plane wave propagating in \( \mathbf{a}_x \) direction is incident on the plane \( x = -3 \text{ m} \) then how much time it will take to strike the interface defined by the dielectric 2 and dielectric 3?

- (A) 6 nsec
- (B) 0.02 \( \mu \) sec
- (C) 3 nsec
- (D) 0.06 \( \mu \) sec

**MCQ 7.2.15** An electromagnetic wave propagating in medium 1 (\( \mu_0, \varepsilon_1 \)) is incident on medium 2 (\( \mu_0, \varepsilon_2 \)) as shown in figure such that the electric field of reflected wave is \( 1/5 \) times of the electric field of incident wave. The value of \( \varepsilon_1/\varepsilon_2 \) equals to
Statement for Linked Question 16 - 17:
An electromagnetic wave of 50 MHz frequency is incident on a dielectric medium such that its skin depth is 0.32 mm. (permittivity of dielectric = 6.28 \times 10^{-7})

MCQ 7.2.16 What will be the conductivity of the dielectric?
(A) 0.32 \times 10^2 S/m  
(B) 1.01 \times 10^{-5} S/m  
(C) 320 S/m  
(D) 0.99 \times 10^5 S/m

MCQ 7.2.17 If an electromagnetic wave of 8 GHz frequency travels a distance of 0.275 mm in the dielectric medium then its field intensity will be reduced by
(A) 20 dB  
(B) 60 dB  
(C) 0 dB  
(D) 30 dB

MCQ 7.2.18 Electric field of an electromagnetic wave propagating in a medium in \(+a_x\) direction is given by
\[ E_x = E_0(a_x - j\alpha_x) e^{-j\alpha_x} \]
The wave is
(A) left hand circularly polarized  
(B) Right hand circularly polarized  
(C) elliptically polarized  
(D) linearly polarized

MCQ 7.2.19 An electromagnetic wave has the electric field intensity in the phasor form given by
\[ E_x = 2(a_x - j\alpha_x) e^{-j\alpha y} \]
The EM wave is incident on a perfect conductor located at \(y = 0\). What will be the polarization of the reflected wave?
(A) left hand circular  
(B) Right hand circular  
(C) elliptical  
(D) linear
MCQ 7.2.20
An electromagnetic wave propagating in free space is incident on a perfectly conducting slab placed at \( x \geq 0 \). The electric field of the incident wave in the phasor form is given by
\[
E_{is} = 5a_x e^{-j(6y + 8z)} \text{ V/m}
\]
The net electric field of the total wave (incident and reflected both) in free space after reflection will be
\[
\begin{align*}
(A) & \quad 10a_x e^{-j(6y - 8z)} \text{ V/m} \\
(B) & \quad -10a_x e^{-j(6y - 8z)} \text{ V/m} \\
(C) & \quad -j20a_x e^{-j8y} \sin 8x \text{ V/m} \\
(D) & \quad j20a_x e^{-j8y} \sin 8x \text{ V/m}
\end{align*}
\]
MCQ 7.2.21
Electric field intensity of an EM wave propagating in free space is given by
\[
E_{is} = 25a_x e^{-j(6x + 8y)} \text{ V/m}
\]
If the wave is incident on a perfectly conducting plane at \( y = 0 \) then the magnetic field intensity of the reflected wave will be
\[
\begin{align*}
(A) & \quad -\left(\frac{a_x}{8\pi} + \frac{a_z}{6\pi}\right)e^{-j(6y - 8z)} \text{ A/m} \\
(B) & \quad \left(\frac{a_x}{8\pi} + \frac{a_z}{6\pi}\right)e^{-j(6y - 8z)} \text{ A/m} \\
(C) & \quad \left(\frac{a_x}{8\pi} + \frac{a_z}{6\pi}\right)e^{j(6y - 8z)} \text{ A/m} \\
(D) & \quad -\left(\frac{a_x}{8\pi} + \frac{a_z}{6\pi}\right)e^{j(6y - 8z)} \text{ A/m}
\end{align*}
\]
MCQ 7.2.22
An electromagnetic wave propagating in free space has magnetic field intensity
\[
\mathbf{H} = 0.4 \cos(\omega t - \beta y) \mathbf{a}_z \text{ A/m}
\]
What will be the total power passing through a square plate of side 20 cm located in the plane \( x + y = 2 \)?
\[
\begin{align*}
(A) & \quad 0.53 \text{ Watt} \\
(B) & \quad 1.88 \text{ Watt} \\
(C) & \quad 18.8 \text{ mW} \\
(D) & \quad 53.31 \text{ mW}
\end{align*}
\]
MCQ 7.2.23
An electromagnetic wave propagating in a lossless medium \( (\mu_1 = 4\mu_0, \varepsilon_1 = \varepsilon_0, \sigma_1 = 0) \) defined in the region \( y > 0 \) is incident on a lossy medium \( (\mu_2 = \mu_0, \varepsilon_2 = 4\varepsilon_0, \sigma_2 = 0.1 \text{ S/m}) \) defined in the region \( y \leq 0 \). The electric field intensity of the incident wave in lossless medium is given by
\[
E_{is} = 6e^{-j8y} a_y \text{ V/m}
\]
What will be the standing wave ratio ?
\[
\begin{align*}
(A) & \quad 1.22 \\
(B) & \quad 0.8186 \\
(C) & \quad 0.0997 \\
(D) & \quad 10.025
\end{align*}
\]
MCQ 7.2.24
The complex electric field vector of a uniform plane wave propagating in free space is given by
\[
\mathbf{E}_s = (\sqrt{3} \mathbf{a}_y - 2\sqrt{3} \mathbf{a}_z) e^{-j0.01xy - (3\sqrt{3}y^2)} \text{ V/m}
\]
The unit vector in the direction of propagation of the wave will be
\[
\begin{align*}
(A) & \quad -3\mathbf{a}_x + \sqrt{3} \mathbf{a}_y - 2\mathbf{a}_z \\
(B) & \quad -3\mathbf{a}_x + \sqrt{3} \mathbf{a}_y - 2\mathbf{a}_z \\
(C) & \quad -4(3\mathbf{a}_x - \sqrt{3} \mathbf{a}_y + 2\mathbf{a}_z) \\
(D) & \quad -4(3\mathbf{a}_x + \sqrt{3} \mathbf{a}_y - 2\mathbf{a}_z)
\end{align*}
\]
MCQ 7.2.25 Phasor form of electric field intensity of a uniform plane wave is given by

\[ E_s = \left(\sqrt{2} a_x - \frac{2}{\sqrt{3}} a_y \right) e^{-j0.04\pi(\sqrt{3}x - 2y + z)} \text{ V/m} \]

The wavelength along the direction of propagation is
(A) 3.16 m  (B) 0.08 m
(C) 12.5 m  (D) 15.7 m

MCQ 7.2.26 In free space the complex magnetic field vector of a uniform plane wave is given by

\[ H_s = -\left(\sqrt{3} a_x + a_y \right) e^{-j0.04\pi(\sqrt{3}x - 2y - z)} \text{ A/m} \]

Frequency of the plane wave will be
(A) 3.75 MHz  (B) $2.4 \times 10^6$ Hz
(C) 24 MHz   (D) 2.4 MHz

Statement for Linked Question 27 - 28:
In free space complex electric field vector of a uniform plane wave is given by

\[ E_s = \left(\sqrt{2} a_x + a_y \right) e^{-j0.04\pi(\sqrt{3}x - 5y - z)} \text{ V/m} \]

MCQ 7.2.27 The apparent wavelengths along the x, y and z axes are

(A) 16.7 m  25 m  28.87 m
(B) 28.87 m 16.7 m  25 m
(C) 16.7 m  28.87 m  25 m
(D) 28.87 m  25 m  16.7 m

MCQ 7.2.28 The apparent phase velocities along the x, y and z axes are

(A) $1.73 \times 10^9$ m/s  $1.5 \times 10^9$ m/s  $1 \times 10^9$ m/s
(B) $6.93 \times 10^8$ m/s  $6 \times 10^8$ m/s  $4 \times 10^8$ m/s
(C) $2.77 \times 10^7$ m/s  $2.4 \times 10^7$ m/s  $1.6 \times 10^7$ m/s
(D) $1.2 \times 10^9$ m/s  $1.2 \times 10^9$ m/s  $1.2 \times 10^9$ m/s

MCQ 7.2.29 Which of the following complex vector field represents the electric field of a uniform plane wave?

(A) $\left( -j a_x - 2a_y - j\sqrt{3} a_z \right) e^{-j0.04\pi(x + \sqrt{3}y + z)}$
(B) $\left( a_x - j2a_y - \sqrt{3} a_z \right) e^{-j0.04\pi(x + \sqrt{3}z)}$
(C) $\left( \sqrt{3} + j\frac{1}{2} \right) a_x + \left( 1 + j\frac{\sqrt{3}}{2} \right) a_y - j\sqrt{3} a_z e^{-j0.02\pi(x + 3y + 2z)}$
(D) $\left( -\sqrt{3} - j\frac{1}{2} \right) a_x + \left( 1 - j\frac{\sqrt{3}}{2} \right) a_y + j\sqrt{3} a_z e^{-j0.02\pi(x + 3y + 2z)}$
MCQ 7.2.30 Which of the following pairs of vector $E$ and $H$ field represents the complex electric and magnetic field vectors of a uniform plane wave?

(A) $(ja_x + 2a_y + j\sqrt{3} a_z)e^{-j\sqrt{3} (x + z)} \text{ V/m}$

(B) $(ja_x + \sqrt{3} a_z)e^{-j\sqrt{3} (x + z)} \text{ V/m}$

(C) $(ja_x - j\sqrt{3} a_z)e^{-j\sqrt{3} (x + z)} \text{ V/m}$

(D) $(-ja_x - 2a_y + j\sqrt{3} a_z)e^{-j\pi(x + z)} \text{ V/m}$

MCQ 7.2.31 The following fields exist in charge free regions

$P = 65\sin(\omega t + 10y) a_x$

$Q = \frac{10}{\rho} \cos(\omega t - 2\rho) a_\phi$

$R = 3\rho^2 \cot \phi a_\rho + \frac{1}{\rho} \cos \phi a_\phi$

$S = \frac{6}{\gamma} \sin \theta \sin(\omega t - 6\tau) a_\theta$

The possible electromagnetic fields are

(A) $P, Q$

(B) $R, S$

(C) $P, R$

(D) $Q, S$

MCQ 7.2.32 A uniform plane wave in region 1 is normally incident on the planner boundary separating regions 1 and 2. Both region are lossless and $\varepsilon_1 = \mu_1$, $\varepsilon_2 = \mu_2$. If the 20% of the energy in the incident wave is reflected at the boundary, the ratio $\varepsilon_2 / \varepsilon_1$ is

(A) 1.48

(B) 17.9

(C) 25.6

(D) 58.3

************
EXERCISE 7.3

MCQ 7.3.1
A plane wave propagating in air with $E = (4a_x + 6a_y + 5a_z) e^{i(\omega t + 3x - 4y)}$ V/m is incident on a perfectly conducting slab positioned at $x \leq 0$. The $E$ field of the reflected wave is

(A) $(-8a_x - 6a_y - 5a_z) e^{i(\omega t - 3x + 4y)}$ V/m
(B) $(-8a_x + 6a_y - 5a_z) e^{i(\omega t + 3x - 4y)}$ V/m
(C) $(-8a_x + 6a_y - 5a_z) e^{i(\omega t - 3x - 4y)}$ V/m
(D) $(-8a_x - 6a_y - 5a_z) e^{i(\omega t - 3x - 4y)}$ V/m

MCQ 7.3.2
The electric field of a uniform plane electromagnetic wave in free space, along the positive $x$ direction is given by $E = 10(a_x + j a_y) e^{-j2x}$ V/m. The frequency and polarization of the wave, respectively, are

(A) 1.2 GHz and left circular
(B) 4 Hz and left circular
(C) 1.2 GHz and right circular
(D) 4 Hz and right circular

MCQ 7.3.3
Consider the following statements regarding the complex Poynting vector $\mathcal{P}$ for the power radiated by a point source in an infinite homogeneous and lossless medium. $\text{Re}(\mathcal{P})$ denotes the real part of $\mathcal{P}$, $S$ denotes a spherical surface whose centre is at the point source, and $\mathbf{a}_n$ denotes the unit surface normal on $S$. Which of the following statements is TRUE?

(A) $\text{Re}(\mathcal{P})$ remains constant at any radial distance from the source
(B) $\text{Re}(\mathcal{P})$ increases with increasing radial distance from the source
(C) $\int_S \text{Re}(\mathcal{P}) \cdot (dS \mathbf{a}_n)$ remains constant at any radial distance from the source
(D) $\int_S \text{Re}(\mathcal{P}) \cdot (dS \mathbf{a}_n)$ decreases with increasing radial distance from the source

MCQ 7.3.4
The electric field component of a time harmonic plane EM wave traveling in a nonmagnetic lossless dielectric medium has an amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time-average power density vector (in W/m$^2$) is

(A) $\frac{1}{30\pi}$
(B) $\frac{1}{60\pi}$
(C) $\frac{1}{120\pi}$
(D) $\frac{1}{240\pi}$
A plane wave having the electric field components \( \mathbf{E}_x = 6 \cos(3 \times 10^8 t - \beta y) \mathbf{a}_x \) V/m and traveling in free space is incident normally on a lossless medium with \( \mu = \mu_0 \) and \( \varepsilon = 9\varepsilon_0 \), which occupies the region \( y \geq 0 \). The reflected magnetic field component is given by:

(A) \( \frac{1}{10\pi} \cos(3 \times 10^8 t + y) \mathbf{a}_x \) A/m

(B) \( \frac{1}{20\pi} \cos(3 \times 10^8 t + y) \mathbf{a}_x \) A/m

(C) \( -\frac{1}{20\pi} \cos(3 \times 10^8 t + y) \mathbf{a}_x \) A/m

(D) \( -\frac{1}{10\pi} \cos(3 \times 10^8 t + y) \mathbf{a}_x \) A/m

A uniform plane wave in the free space is normally incident on an infinitely thick dielectric slab (dielectric constant \( \varepsilon_r = 9 \)). The magnitude of the reflection coefficient is:

(A) 0

(B) 0.3

(C) 0.5

(D) 0.8

A plane wave of wavelength \( \lambda \) is traveling in a direction making an angle 30° with positive \( x \)-axis and 90° with positive \( y \)-axis. The \( \mathbf{E} \) field of the plane wave can be represented as (\( E_0 \) is constant):

(A) \( \mathbf{E} = a_y E_0 e^{(\omega t - \frac{\sqrt{3} \lambda}{\lambda} x - \frac{\pi}{3} y)} \)

(B) \( \mathbf{E} = a_y E_0 e^{(\omega t - \frac{\sqrt{3} \lambda}{\lambda} x + \frac{\pi}{3} y)} \)

(C) \( \mathbf{E} = a_y E_0 e^{(\omega t + \frac{\sqrt{3} \lambda}{\lambda} x - \frac{\pi}{3} y)} \)

(D) \( \mathbf{E} = a_y E_0 e^{(\omega t + \frac{\sqrt{3} \lambda}{\lambda} x + \frac{\pi}{3} y)} \)

The \( \mathbf{H} \) field (in A/m) of a plane wave propagating in free space is given by

\[ \mathbf{H} = a_x \frac{5\sqrt{5}}{\eta_0} \cos(\omega t - \beta z) + a_y \left( \omega t - \beta z + \frac{\pi}{2} \right). \]

The time average power flow density in Watts is:

(A) \( \frac{\eta_0}{100} \)

(B) \( \frac{100}{\eta_0} \)

(C) \( 50\eta_0^2 \)

(D) \( \frac{50}{\eta_0} \)

A right circularly polarized (RCP) plane wave is incident at an angle 60° to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant \( \varepsilon_r \) is:

(A) \( \sqrt{2} \)

(B) \( \sqrt{3} \)

(C) 2

(D) 3
MCQ 7.3.10  

The electric field of an electromagnetic wave propagation in the positive direction is given by \( E = 2a_x \sin(\omega t - \beta z) + a_y \sin(\omega t - \beta z + \pi/2) \). The wave is

(A) Linearly polarized in the \( z \)-direction

(B) Elliptically polarized

(C) Left-hand circularly polarized

(D) Right-hand circularly polarized

MCQ 7.3.11  

When a plane wave traveling in free-space is incident normally on a medium having \( \varepsilon_r = 4.0 \) then the fraction of power transmitted into the medium is given by

(A) \( \frac{8}{9} \)  

(B) \( \frac{1}{2} \)  

(C) \( \frac{1}{3} \)  

(D) \( \frac{5}{6} \)

MCQ 7.3.12  

A medium of relative permittivity \( \varepsilon_r = 2 \) forms an interface with free-space. A point source of electromagnetic energy is located in the medium at a depth of 1 meter from the interface. Due to the total internal reflection, the transmitted beam has a circular cross-section over the interface. The area of the beam cross-section at the interface is given by

(A) \( 2\pi \text{ m}^2 \)  

(B) \( \pi^2 \text{ m}^2 \)  

(C) \( \frac{\pi}{2} \text{ m}^2 \)  

(D) \( \pi \text{ m}^2 \)

MCQ 7.3.13  

A medium is divided into regions I and II about \( x = 0 \) plane, as shown in the figure below.

\[
\begin{array}{c|c}
\text{Region I} & \text{Region II} \\
\hline 
\mu_1=\mu_2 & \mu_2=\mu_2 \\
\varepsilon_\parallel=3 & \varepsilon_\parallel=4 \\
\sigma_1=0 & \sigma_2=0 \\
E_x & E_y \\
x<0 & x>0 \\
\end{array}
\]

An electromagnetic wave with electric field \( E_1 = 4a_x + 5a_y + 5a_z \) is incident normally on the interface from region I. The electric field \( E_2 \) in region II at the interface is

(A) \( E_2 = E_1 \)  

(B) \( 4a_x + 0.75a_y - 1.25a_z \)  

(C) \( 3a_x + 3a_y + 5a_z \)  

(D) \( -3a_x + 3a_y + 5a_z \)

MCQ 7.3.14  

The magnetic field intensity vector of a plane wave is given by \( \mathbf{H}(x, y, z, t) = 10 \sin(50000t + 0.004x + 30) \mathbf{a}_y \) where \( \mathbf{a}_y \) denotes the unit vector in the \( y \)-direction. The wave is propagating with a phase velocity

(A) \( 5 \times 10^4 \text{ m/s} \)  

(B) \( -3 \times 10^8 \text{ m/s} \)  

(C) \( -1.25 \times 10^7 \text{ m/s} \)  

(D) \( 3 \times 10^8 \text{ m/s} \)
MCQ 7.3.15
GATE 2005
Refractive index of glass is 1.5. Find the wavelength of a beam of light with frequency of $10^{14}$ Hz in glass. Assume velocity of light is $3 \times 10^8$ m/s in vacuum.
(A) 3 $\mu$m
(B) 3 mm
(C) 2 $\mu$m
(D) 1 mm

MCQ 7.3.16
GATE 2004
If $E = (a_x + j a_y) e^{j(kz - \omega t)}$ and $H = (k/\omega \mu)(a_y + j a_x) e^{j(kz - \omega t)}$, the time-averaged Poynting vector is
(A) null vector
(B) $(k/\omega \mu) a_z$
(C) $(2k/\omega \mu) a_z$
(D) $(k/2\omega \mu) a_z$

MCQ 7.3.17
GATE 2004
A plane electromagnetic wave propagating in free space is incident normally on a large slab of loss-less, non-magnetic, dielectric material with $\varepsilon > \varepsilon_0$. Maxima and minima are observed when the electric field is measured in front of the slab. The maximum electric field is found to be 5 times the minimum field. The intrinsic impedance of the medium should be
(A) 120$\pi$ $\Omega$
(B) 60$\pi$ $\Omega$
(C) 600$\pi$ $\Omega$
(D) 24$\pi$ $\Omega$

MCQ 7.3.18
GATE 2003
The depth of penetration of electromagnetic wave in a medium having conductivity $\sigma$ at a frequency of 1 MHz is 25 cm. The depth of penetration at a frequency of 4 MHz will be
(A) 6.25 dm
(B) 12.50 cm
(C) 50.00 cm
(D) 100.00 cm

MCQ 7.3.19
GATE 2003
A uniform plane wave traveling in air is incident on the plane boundary between air and another dielectric medium with $\varepsilon_r = 5$. The reflection coefficient for the normal incidence, is
(A) zero
(B) $0.5/180^\circ$
(C) $0.333/0^\circ$
(D) $0.333/180^\circ$

MCQ 7.3.20
GATE 2003
If the electric field intensity associated with a uniform plane electromagnetic wave traveling in a perfect dielectric medium is given by $E(z, t) = 10 \cos(2\pi 10^7 t - 0.1\pi z)$ V/m, then the velocity of the traveling wave is
(A) $3.00 \times 10^7$ m/sec
(B) $2.00 \times 10^8$ m/sec
(C) $6.28 \times 10^7$ m/sec
(D) $2.00 \times 10^7$ m/sec

MCQ 7.3.21
GATE 2002
A plane wave is characterized by $E = (0.5 a_x + a_y e^{r/2}) e^{j(\omega t - kz)}$. This wave is
(A) linearly polarized
(B) circularly polarized
(C) elliptically polarized
(D) unpolarized

MCQ 7.3.22
GATE 2002
Distilled water at $25^\circ$C is characterized by $\sigma = 1.7 \times 10^{-4}$ mho/m and $\varepsilon = 78\varepsilon_0$ at a frequency of 3 GHz. Its loss tangent $\tan \delta$ is
$$\varepsilon = \frac{10^7}{30} \text{ F/m}$$
MCQ 7.3.23 If a plane electromagnetic wave satisfies the equation $\frac{\partial^2 E_x}{\partial z^2} = c^2 \frac{\partial^2 E_z}{\partial t^2}$, the wave propagates in the
(A) $x$-direction
(B) $z$-direction
(C) $y$-direction
(D) $x$-$y$ plane at an angle of 45° between the $x$ and $z$ direction

MCQ 7.3.24 A material has conductivity of $10^{-2}$ mho/m and a relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is
(A) 45 MHz
(B) 90 MHz
(C) 450 MHz
(D) 900 MHz

MCQ 7.3.25 A uniform plane electromagnetic wave incident on a plane surface of a dielectric material is reflected with a VSWR of 3. What is the percentage of incident power that is reflected?
(A) 10%
(B) 25%
(C) 50%
(D) 75%

MCQ 7.3.26 A uniform plane wave in air impinges at 45° angle on a lossless dielectric material with dielectric constant $\varepsilon_r$. The transmitted wave propagates in a 30° direction with respect to the normal. The value of $\varepsilon_r$ is
(A) 1.5
(B) $\sqrt{1.5}$
(C) 2
(D) $\sqrt{2}$

MCQ 7.3.27 Two coaxial cable 1 and 2 are filled with different dielectric constants $\varepsilon_{r1}$ and $\varepsilon_{r2}$ respectively. The ratio of the wavelength in the cables $(\lambda_{1}/\lambda_{2})$ is
(A) $\sqrt{\varepsilon_{r1}/\varepsilon_{r2}}$
(B) $\sqrt{\varepsilon_{r2}/\varepsilon_{r1}}$
(C) $\varepsilon_{r1}/\varepsilon_{r2}$
(D) $\varepsilon_{r2}/\varepsilon_{r1}$

MCQ 7.3.28 Identify which one of the following will NOT satisfy the wave equation.
(A) $50e^{j(\omega t - 3\beta z)}$
(B) $\sin[\omega(10z + 5t)]$
(C) $\cos(y^2 + 5t)$
(D) $\sin(x)\cos(t)$

MCQ 7.3.29 A plane wave propagating through a medium [$\varepsilon_r = 8, \nu_r = 2, \text{and } \sigma = 0$] has its electric field given by $E = 0.5Xe^{-(1/3)}\sin(10^4t - \beta z)$ V/m. The wave impedance, in ohms is
(A) 377
(B) 198.5/180°
(C) 182.9/14°
(D) 133.3
MCQ 7.3.30

The intrinsic impedance of copper at high frequencies is
(A) purely resistive
(B) purely inductive
(C) complex with a capacitive component
(D) complex with an inductive component

MCQ 7.3.31

The time average poynting vector, in W/m², for a wave with \( \mathbf{E} = 12e^{j(c_0+a_0)} \mathbf{a}_x \) in free space is
(A) \( \frac{2.4}{\pi} \mathbf{a}_x \)
(B) \( \frac{4.8}{\pi} \mathbf{a}_x \)
(C) \( \frac{2.4}{\pi} \mathbf{a}_x \)
(D) \( \frac{4.8}{\pi} \mathbf{a}_x \)

MCQ 7.3.32

The wavelength of a wave with propagation constant \((0.1\pi + j0.2\pi)\) m⁻¹ is
(A) \( \frac{2}{\sqrt{0.05}} \) m
(B) 10 m
(C) 20 m
(D) 30 m

MCQ 7.3.33

The depth of penetration of wave in a lossy dielectric increases with increasing
(A) conductivity
(B) permeability
(C) wavelength
(D) permittivity

MCQ 7.3.34

The polarization of wave with electric field vector \( \mathbf{E} = E_0e^{j(c_0+a_0)}(\mathbf{a}_x + \mathbf{a}_y) \) is
(A) linear
(B) elliptical
(C) left hand circular
(D) right hand circular

MCQ 7.3.35

The skin depth at 10 MHz for a conductor is 1 cm. The phase velocity of an electromagnetic wave in the conductor at 1,000 MHz is about
(A) \( 6 \times 10^6 \) m/sec
(B) \( 6 \times 10^7 \) m/sec
(C) \( 3 \times 10^8 \) m/sec
(D) \( 6 \times 10^8 \) m/sec

MCQ 7.3.36

A uniform plane wave in air is normally incident on infinitely thick slab. If the refractive index of the glass slab is 1.5, then the percentage of incident power that is reflected from the air-glass interface is
(A) 0%
(B) 4%
(C) 20%
(D) 100%

MCQ 7.3.37

Some unknown material has a conductivity of \( 10^6 \) mho/m and a permeability of \( 4\pi \times 10^{-7} \) H/m. The skin depth for the material at 1 GHz is
(A) 15.9 \( \mu \)m
(B) 20.9 \( \mu \)m
(C) 25.9 \( \mu \)m
(D) 30.9 \( \mu \)m

MCQ 7.3.38

The plane wave travelling in a medium of \( \varepsilon_r = 1, \mu_r = 1 \) (free space) has an electric field intensity of \( 100\sqrt{\pi} \) V/m. Determine the total energy density of this field.
MCQ 7.3.39
For a plane wave propagating in an unbounded medium (say, free space), the minimum angle between electric field and magnetic field vectors is
(A) 0°
(B) 60°
(C) 90°
(D) 180°

MCQ 7.3.40
For no reflection condition, a vertically polarized wave should be incident at the interface between two dielectrics having \(\varepsilon_1 = 4\) and \(\varepsilon_2 = 7\), with an incident angle of
(A) \(\tan^{-1}\left(\frac{9}{4}\right)\)
(B) \(\tan^{-1}\left(\frac{3}{2}\right)\)
(C) \(\tan^{-1}\left(\frac{2}{3}\right)\)
(D) \(\tan^{-1}\left(\frac{4}{5}\right)\)

MCQ 7.3.41
The electric field component of a wave in free space is given by
\[
\mathbf{E} = 10 \cos(10^7 t + k z) \mathbf{a}_y \text{ V/m}
\]
Following is a list of possible inferences:
1. Wave propagates along \(\mathbf{a}_y\)
2. Wavelength \(\lambda = 188.5\) m
3. Wave amplitude is 10 V/m
4. Wave number \(= 0.33\) rad/m
5. Wave attenuates as it travels
Which of these inferences can be drawn from \(\mathbf{E}\)?
(A) 1, 2, 3, 4 and 5
(B) 2 and 3 only
(C) 3 and 4 only
(D) 4 and 5 only

MCQ 7.3.42
A plane wave is generated under water \((\varepsilon = 81\varepsilon_0\) and \(\mu = \mu_0\)). The wave is parallel polarized. At the interface between water and air, the angle \(\alpha\) for which there is no reflection is
\[
\begin{align*}
\text{Air} & \quad \theta_1 \\
\text{Water} & \quad \theta_2
\end{align*}
\]
(A) 83.88°
(B) 83.66°
(C) 84.86°
(D) 84.08°
**MCQ 7.3.43**

An elliptically polarized wave travelling in the positive $z$-direction in air has $x$ and $y$ components

$$E_x = 3\sin(\omega t - \beta z) \text{ V/m}$$

$$E_y = 3\sin(\omega t - \beta z + 75^\circ) \text{ V/m}$$

If the characteristic impedance of air is $360 \Omega$, the average power per unit area conveyed by the wave is

(A) 8 W/m$^2$

(B) 4 W/m$^2$

(C) 62.5 mW/m$^2$

(D) 125 mW/m$^2$

**MCQ 7.3.44**

The intrinsic impedance of copper at 3 GHz (with parameters: $\mu = 4\pi \times 10^{-7} \text{ H/m;}$

$\varepsilon = 10^{-79}/36\pi$; and $\sigma = 5.8 \times 10^7 \text{ mho/m}$) will be

(A) $0.02e^{\pi/4}$ ohm

(B) $0.02e^{\pi/2}$ ohm

(C) $0.2e^{\pi/2}$ ohm

(D) $0.2e^{\pi/4}$ ohm

**MCQ 7.3.45**

Consider the following statements regarding depth of penetration or skin depth in a conductor:

1. It increases as frequency increases.
2. It is inversely proportional to square root of $\mu$ and $\sigma$.
3. It is inversely proportional to square root of $f$
4. It is directly proportional to square root of $\mu$ and $\sigma$.

Which of the above statements are correct?

(A) 1 and 2 only

(B) 3 and 4 only

(C) 2 and 3 only

(D) 1, 2, 3 and 4

**MCQ 7.3.46**

Consider the following statements:

1. (Electric or magnetic) field must have two orthogonal linear components.
2. The two components must have the same magnitude.
3. The two components must have a time-phase difference of odd multiple of 90°.

Which of these are the necessary and sufficient conditions for a time-harmonic wave to be circularly polarized at a given point in space?

(A) 1 and 2 only

(B) 2 and 3 only

(C) 1, 2 and 3

(D) 1 and 3 only

**MCQ 7.3.47**

**Assertion (A):** The velocity of light in any medium is slower than that of vacuum.

**Reason (R):** The dielectric constant of the vacuum is unity and is lesser than that of any other medium.

(A) Both A and R are individually true and R is the correct explanation of A

(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 7.3.48
IES EC 2009

In which direction is the plane wave \( E = 35 \sin(10^8 t + 2z) a_y \) V/m, (where \( a_y \) is the unit vector in \( y \)-direction), travelling?
(A) along \( y \) direction
(B) along \( y \) direction
(C) along \( z \) direction
(D) along \( z \) direction

MCQ 7.3.49
IES EC 2008

According to Poynting theorem, the vector product \( \mathbf{E} \times \mathbf{H} \) is a measure of which one of the following?
(A) Stored energy density of the electric field
(B) Stored energy density of the magnetic field
(D) Power dissipated per unit volume
(E) Rate of energy flow per unit area

MCQ 7.3.50
IES EC 2007

If \( \mathbf{E} = (a_x + j a_y) e^{-j k z} \), then the wave is said to be which one of the following?
(A) Right circularly polarized
(B) Right elliptically polarized
(C) Left circularly polarized
(D) Left elliptically polarized

MCQ 7.3.51
IES EC 2007

What must be angle \( \theta \) of a corner reflector, such that an incident wave is reflected in the same direction?
(A) 30°
(B) 45°
(C) 60°
(D) 90°

MCQ 7.3.52
IES EC 2007

Poynting vector is a measure of which one of the following?
(A) Maximum power flow through a surface surrounding the source
(B) Average power flow through the surface
(C) Instantaneous power flow through the surface
(D) Power dissipated by the surface

MCQ 7.3.53
IES EC 2006

The electric field component of a wave in free space is given by
\[
E = 25 \sin(10^8 t + k z) a_y \text{ V/m}
\]
Which one of the following is the correct inference that can be drawn from this expression?
(A) The wave propagates along the $y$-axis
(B) The wavelength is 188.5 m
(C) The wave number $k = 0.33$ rad/m
(D) The wave attenuates as it travels

MCQ 7.3.54
For an electromagnetic wave incident on a conducting medium, the depth of penetration
(A) is directly proportional to the attenuation constant
(B) is inversely proportional to the attenuation constant
(C) has a logarithmic relationship with the attenuation constant
(D) is independent of the attenuation constant

MCQ 7.3.55
Which one of the following statements is correct?
A right circularly polarised wave is incident from air onto polystyrene ($\varepsilon_r = 2.7$). The reflected wave is
(A) right circularly polarised
(B) left circularly polarised
(C) right elliptically polarised
(D) left elliptically polarised

MCQ 7.3.56
The electric field of a wave propagating through a lossless medium ($\mu_0, 81\varepsilon_0$) is
$E = 50\cos(6\pi \times 10^8 t - bx) \bar{a}_x$
What is the phase constant $\beta$ of the wave?
(A) $2\pi$ rad/m
(B) $9\pi$ rad/m
(C) $18\pi$ rad/m
(D) $81\pi$ rad/m

MCQ 7.3.57
If the phase velocity of a plane wave in a perfect dielectric is 0.4 times its value in free space, then what is the relative permittivity of the dielectric?
(A) 6.25
(B) 4.25
(C) 2.5
(D) 1.25

MCQ 7.3.58
In free space $E(x,t) = 60(\omega t - 2x) \bar{a}_y$ V/m. What is the average power crossing a circular area of radius 4 m in the plane $x = \text{constant}$?
(A) 480 W
(B) 340 W
(C) 120 W
(D) 60 W

MCQ 7.3.59
What is the effect of the earth’s magnetic field in the reflected wave at frequencies in the vicinity of gyro-frequency?
(A) No attenuation in the reflected wave
(B) Decreased attenuation in the reflected wave
(C) Increased attenuation in the reflected wave
(D) Nominal attenuation in the reflected wave
A plane electromagnetic wave travelling in a perfect dielectric medium of intrinsic impedance $\eta_1$ is incident normally on its boundary with another perfect dielectric medium of characteristic impedance $\eta_2$. The electric and magnetic field strengths of the incident wave are denoted by $E_1$ and $H_1$ respectively whereas $E_r$ and $H_r$ denote these quantities for the reflected wave, and $E_t$ and $H_t$ for the transmitted wave. Which of the following relations are correct?

1. $EH_1 = \eta_1 H_1$
2. $EH_r = \eta_1 H_r$
3. $E_t = \eta_2 H_t$

Select the correct answer using the codes given below:

(A) 1, 2 and 3
(B) 1 and 2
(C) 1 and 3
(D) 2 and 3

A plane electromagnetic wave travelling in a perfect dielectric medium of dielectric constant $\varepsilon_1$ is incident on its boundary with another perfect dielectric medium of dielectric constant $\varepsilon_2$. The incident ray makes an angle of $\theta_1$ with the normal to the boundary surface. The ray transmitted into the other medium makes an angle of $\theta_2$ with the normal. If $\varepsilon_1 = 2\varepsilon_2$ and $\theta_1 = 60^\circ$, which one of the following is correct?

(A) $\theta_2 = 45^\circ$
(B) $\theta_2 = \sin^{-1}0.433$
(C) $q_2 = \sin^{-1}0.612$
(D) There will be no transmitted wave

Match List I (Nature of Polarization) with List II (Relationship Between $X$ and $Y$ Components) for a propagating wave having cross-section in the $XY$ plane and propagating along $z$-direction and select the correct answer:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Linear</td>
<td>1. $X$ and $Y$ components are in same phase</td>
</tr>
<tr>
<td>b. Left circular</td>
<td>2. $X$ and $Y$ components have arbitrary phase difference</td>
</tr>
<tr>
<td>c. Right circular</td>
<td>3. $X$ component leads $Y$ by $90^\circ$</td>
</tr>
<tr>
<td>d. Elliptical</td>
<td>4. $X$ component lags behind $Y$ by $90^\circ$</td>
</tr>
</tbody>
</table>

Codes:

(A) 1 4 2 3
(B) 4 1 2 3
(C) 1 4 3 2
(D) 4 1 3 2
Assertion (A) : For an EM wave normally incident on a conductor surface the magnetic field $H$ undergoes a $180^\circ$ phase reversal and the phase of electric field $E$ remains same.

Reason (R) : The direction of propagation of incident wave will reverse after striking a conductor surface.

(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

Match List I with List II and select the correct answer :

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Propagation constant</td>
<td>$\sqrt{\omega \mu \sigma / 2}$</td>
</tr>
<tr>
<td>b. Radiation intensity</td>
<td>$\frac{\mu}{\pi} E^2$</td>
</tr>
<tr>
<td>c. Wave impedance</td>
<td>$E / H$</td>
</tr>
<tr>
<td>d. $E \times H$</td>
<td></td>
</tr>
</tbody>
</table>

Codes :

(A) 1 2 3
(B) 4 3 2
(C) 1 3 2
(D) 4 2 3

If the $E$ field of a plane polarized EM wave travelling in the $z$-direction is : $E = a_x E_x + a_y E_y$, then its $H$ field is :

(A) $a_y \frac{E_x}{Z_0} - a_x \frac{E_y}{Z_0}$
(B) $a_x \frac{E_x}{Z_0} + a_y \frac{E_y}{Z_0}$
(C) $a_x \frac{E_x}{Z_0} - a_y \frac{E_y}{Z_0}$
(D) $-a_x \frac{E_x}{Z_0} - a_y \frac{E_y}{Z_0}$

Consider the following statements :

1. electrical field is perpendicular to direction of propagation
2. electrical field is along the direction of propagation
3. magnetic field is perpendicular to direction of propagation
4. magnetic field is along the direction of propagation

Which of these statements are correct ?

(A) 1 and 3
(B) 1 and 4
(C) 2 and 3
(D) 2 and 4
MCQ 7.3.67
IES EC 2001
In a uniform plane wave, the value of $E/H$ is
(A) $\sqrt{\mu/\varepsilon}$  
(B) $\sqrt{\varepsilon/\mu}$  
(C) 1  
(D) $\sqrt{\mu\varepsilon}$

MCQ 7.3.68
IES EC 2001
The phenomenon of microwave signals following the curvature of earth is known as
(A) Faraday effect  
(B) ducting  
(C) wave tilt  
(D) troposscatter

MCQ 7.3.69
IES EC 2001
Which one of the following statements is NOT correct for a plane wave with $H = 0.5e^{-0.1z}\cos(10^6t - 2x) \mathbf{a}_x \, \text{A/m}$
(A) The wave frequency is $10^6$ r.p.s  
(B) The wavelength is 3.14 m  
(C) The wave travels along $+x$-direction  
(D) The wave is polarized in the $z$-direction

MCQ 7.3.70
IES EE 2012
Skin depth is the distance from the conductor surface where the field strength has fallen to
(A) $\pi$ of its strength at the surface  
(B) $e$ of its strength at the surface  
(C) $(1/e)$ of its strength at the surface  
(D) $(1/\pi e)$ of its strength at the surface

MCQ 7.3.71
IES EE 2012
The vector magnetic potential of a particular wave traveling in free space is given by $\mathbf{A} = \mathbf{a}_x A_x \sin(\omega t - \beta z)$ where $A_x$ is a constant. The expression for the electric field will be
(A) $-\mathbf{a}_z \beta A_x \sin(\omega t - \beta z)$  
(B) $-\mathbf{a}_z \beta A_x \sin(\omega t - \beta z)$  
(C) $-\mathbf{a}_z \omega A_x \cos(\omega t - \beta z)$  
(D) $-\mathbf{a}_z \omega A_x \cos(\omega t - \beta z)$

MCQ 7.3.72
IES EE 2012
The depth of penetration of a wave in a lossy dielectric increases with increasing
(A) conductivity  
(B) permeability  
(C) wavelength  
(D) permittivity

MCQ 7.3.73
IES EE 2011
When a plane wave propagates in a dielectric medium
(A) the average electric energy and the average magnetic energy densities are not equal.  
(B) the average electric energy and the average magnetic energy densities are equal  
(C) the net average energy density is finite  
(D) the average electric energy density is not dependent on the average magnetic energy density

MCQ 7.3.74
IES EE 2011
In free space $\mathbf{H}$ field is given as $\mathbf{H}(z,t) = -\frac{1}{6\pi} \cos(\omega t + \beta z) \mathbf{a}_y$, $\mathbf{E}(z,t)$ is
(A) $20\cos(\omega t + \beta z) \mathbf{a}_x$  
(B) $20\cos(\omega t + \beta z) \mathbf{a}_x$  
(C) $20\sin(\omega t + \beta z) \mathbf{a}_y$  
(D) $20\sin(\omega t + \beta z) \mathbf{a}_y$
MCQ 7.3.75
IES EE 2011
If electric field intensity phasor of an EM wave in free space is \( E = 6e^{-j\omega t}a, \) V/m.
The angular frequency \( \omega \), in rad/s, is
(A) \( 4 \times 3 \times 10^8 \) (B) \( 4y \times 3 \times 10^8 \)
(C) \( i \times 3 \times 10^8 \) (D) \( 10 \times 3 \times 10^8 \)

MCQ 7.3.76
IES EE 2011
**Assertion (A)**: Electromagnetic waves propagate being guided by parallel plate perfect conductor surface.
**Reason (R)**: Tangential component of electric field intensity and normal component of magnetic field intensity are zero on a perfect conductor surface.
(A) Both **Assertion (A)** and **Reason (R)** are individually true and **Reason (R)** is the correct explanation of **Assertion (A)**
(B) Both **Assertion (A)** and **Reason (R)** are individually true but **Reason (R)** is not the correct explanation of **Assertion (A)**
(C) **Assertion (A)** is true but **Reason (R)** is false
(D) **Assertion (A)** is false but **Reason (R)** is true

MCQ 7.3.77
IES EE 2010
A uniform plane wave is propagating in a material for which \( \varepsilon = 4\varepsilon_0, \mu = 7\mu_0 \) and \( \sigma = 0 \). The skin depth for the material is
(A) zero (B) infinity
(C) 28 m (D) 14 m

MCQ 7.3.78
IES EE 2010
Consider the following statements:
1. In conducting medium the field attenuates exponentially with increasing depth.
2. Conducting medium behaves like an open circuit to the electromagnetic field.
3. In lossless dielectric relaxation time is infinite.
4. In charge-free region, the Poisson’s equation becomes Laplace’s equation.
(A) 1, 2 and 3 only (B) 1, 3 and 4 only
(C) 2, 3 and 4 only (D) 1, 2, 3 and 4

MCQ 7.3.79
IES EE 2010
In free space
\( E(Z, t) = 60\pi \cos(\omega t - \beta z)a, \) V/m.
The average power crossing a circular area of \( \pi \) square metres in the plane \( z = \) constant is
(A) \( 16\pi \) watt/m² (B) \( 15\pi \) watt/m²
(C) \( 14\pi \) watt/m² (D) \( 13\pi \) watt/m²

MCQ 7.3.80
IES EE 2009
In free space
\( E(Z, t) = 120\pi \cos(\omega t - \beta Z)a, \) V/m⁻¹
What is the average power in W/m⁻² ?
(A) \( 30\pi a_z \) (B) \( 60\pi a_z \)
(C) \( 90\pi a_z \) (D) \( 120\pi a_z \)
MCQ 7.3.81
The electric field of a uniform plane wave is given by:
\[ \mathbf{E} = 35 \sin(3\pi \times 10^8 t - \pi Z) \mathbf{a}_x + 45 \cos(3\pi \times 10^8 t - \pi Z) \mathbf{a}_y \text{ Vm}^{-1} \]
What is the corresponding magnetic field \( \mathbf{H} \)?
(A) \( \frac{10}{377} \sin(3\pi \times 10^8 t - \pi Z) \mathbf{a}_x + \frac{10}{377} \cos(3\pi \times 10^8 t - \pi Z)(- \mathbf{a}_y) \text{ Am}^{-1} \)
(B) \( \frac{10}{377} \sin(3\pi \times 10^8 t - \pi Z)(- \mathbf{a}_x) + \frac{10}{377} \cos(3\pi \times 10^8 t - \pi Z)\mathbf{a}_y \text{ Am}^{-1} \)
(C) \( \frac{10}{377} \sin(3\pi \times 10^8 t - \pi Z) \mathbf{a}_y + \frac{10}{377} \cos(3\pi \times 10^8 t - \pi Z)(- \mathbf{a}_x) \text{ Am}^{-1} \)
(D) \( \frac{10}{377} \sin(3\pi \times 10^8 t - \pi Z)(- \mathbf{a}_y) + \frac{10}{377} \cos(3\pi \times 10^8 t - \pi Z)\mathbf{a}_x \text{ Am}^{-1} \)

MCQ 7.3.82
Consider the following statements in connection with electromagnetic waves:
1. Conducting medium behaves like an open circuit to the electromagnetic field.
2. At radio and microwave frequencies the relaxation time is much less than the period.
3. In loss-less dielectric the relaxation time is finite.
4. Intrinsic impedance of a perfect dielectric medium is a pure resistance.
Which of these statements is/are correct?
(A) 1 only
(B) 1 and 2 only
(C) 2 and 3 only
(D) 2, 3 and 4

MCQ 7.3.83
What causes electromagnetic wave polarization?
(A) Refraction
(B) Reflection
(C) Longitudinal nature of electromagnetic wave
(D) Transverse nature of electromagnetic wave

MCQ 7.3.84
Assertion (A): The velocity of electromagnetic waves is same as velocity of light.
Reason (R): Electrons also travel with the same velocity as photons.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 7.3.85
Fields are said to be circularly polarized if their magnitudes are
(A) Equal and they are in phase
(B) Equal and they differ in phase by ±90°
(C) Unequal and they differ in phase by ±90°
(D) Unequal and they are in phase
MCQ 7.3.86 Which of the following is zero as applied to electromagnetic field?
(A) grad div \( \mathbf{A} \)  
(B) div grad \( V \)  
(C) div curl \( \mathbf{A} \)  
(D) curl curl \( \mathbf{A} \)

MCQ 7.3.87 What is the Poynting’s vector on the surface of a long straight conductor of radius \( b \) and conductivity \( \sigma \) which carries current \( I \) in the \( z \)-direction?
(A) \(-\frac{I^2}{2\pi b^2} i_z\)  
(B) \(-\frac{I^2}{2\pi b^2} i_z\)  
(C) \(-\frac{I}{2\pi b} i_z\)  
(D) \(-\frac{I}{2\pi b} i_z\)

MCQ 7.3.88 Consider the following statements regarding EM wave
1. An EM wave incident on a perfect dielectric is partially transmitted and partially reflected
2. An EM wave incident on a perfect conductor is fully reflected
3. When an EM wave is incident from a more dense medium to less dense medium at an angle equal to or exceeding the critical angle, the wave suffers total internal reflection
Which of the statements given above are correct?
(A) Only 1 and 2  
(B) Only 2 and 3  
(C) Only 1 and 3  
(D) 1, 2 and 3

MCQ 7.3.89 A uniform plane wave has a wavelength of 2 cm in free space and 1 cm in a perfect dielectric. What is the relative permittivity of the dielectric?
(A) 2.0  
(B) 0.5  
(C) 4.0  
(D) 0.25

MCQ 7.3.90 With the increase in frequency of an electromagnetic wave in free space, how do the velocity \( v_e \) and characteristic impedance \( Z_c \) change?
(A) \( v_e \) increases and \( Z_c \) decreases  
(B) \( v_e \) decreases and \( Z_c \) increases  
(C) Both \( v_e \) and \( Z_c \) increase  
(D) Both \( v_e \) and \( Z_c \) remain unchanged

MCQ 7.3.91 The \( E \) field of a plane electromagnetic wave travelling in a non-magnetic non-conducting medium is given by \( \mathbf{E} = a_5 \cos(10^5 t + 30z) \). What is the dielectric constant of the medium?
(A) 30  
(C) 9  
(B) 10  
(D) 81

MCQ 7.3.92 In the wave equation \( \nabla^2 \mathbf{E} = \mu_e \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_o \frac{\partial \mathbf{E}}{\partial t} \) which term is responsible for attenuation of the wave?
MCQ 7.3.93
Consider the following statements:

1. Poisson’s equation finds application in vacuum tube and gaseous discharge problems.
2. Gauss’s law is useful for determining field and potential distribution about bodies having unsymmetrical geometry.
3. For the propagation of electro-magnetic waves, the time varying electric fields must support time varying magnetic fields.
4. The unit of Poynting’s vector is W/m²

Which of the statements given above are correct?
(A) 1, 2 and 3  
(B) 1, 3 and 4  
(C) 2, 3 and 4  
(D) 1, 2 and 4

MCQ 7.3.94
What is the phase velocity of plane wave in a good conductor?

(A) \( \sqrt{\pi f \mu \sigma} \)  
(B) \( \sqrt{\frac{\pi f}{(\mu \sigma)}} \)  
(C) \( \sqrt{\frac{\pi f}{(\mu \sigma)}} \)  
(D) \( 2 \sqrt{\frac{\pi f}{(\mu \sigma)}} \)

MCQ 7.3.95
The instantaneous electric field of a plane wave propagating in \( z \)-direction is

\[ E(t) = [a_x E_1 \cos \omega t - a_y E_2 \sin \omega t] e^{-\gamma z} \]

This wave is

(A) Linearly polarised  
(B) Elliptically polarised  
(C) Right hand circularly polarised  
(D) Left hand circularly polarised

MCQ 7.3.96
Assertion (A) : Skin depth is the depth by which electromagnetic wave has been increased to 37% of its original value.

Reason (R) : The depth of penetration of wave in a lossy dielectric increases with increasing wavelength.

(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true but R is NOT the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true

MCQ 7.3.97
Which one of the following is the correct electromagnetic wave equation in terms of vector potential \( A \)?

(A) \( \nabla^2 A - \frac{\partial^2 A}{\partial t^2} = -J \)  
(B) \( \nabla^2 A - \frac{\mu}{\varepsilon} \frac{\partial^2 A}{\partial t^2} = -\mu J \)  
(C) \( \nabla^2 A - \frac{\partial^2 A}{\partial t^2} = -\mu J \)  
(D) \( \nabla^2 A - \frac{\mu}{\varepsilon} \frac{\partial^2 A}{\partial t^2} = -\mu J \)
MCQ 7.3.98 Which one of the following statements is correct? The wavelength of a wave propagating in a wave guide is
(A) smaller than the free space wavelength
(B) greater than the free space wavelength
(C) directly proportional to the group velocity
(D) inversely proportional to the phase velocity

MCQ 7.3.99 Which one of the following statements is correct? For a lossless dielectric medium, the phase constant for a travelling wave, \( \beta \) is proportional to
(A) \( \varepsilon_r \)
(B) \( \sqrt{\varepsilon_r} \)
(C) \( 1/\varepsilon_r \)
(D) \( 1/\sqrt{\varepsilon_r} \)

MCQ 7.3.100 In a lossless medium the intrinsic impedance \( \eta = 60\pi \) and \( \mu_r = 1 \). What is the value of the dielectric constant \( \varepsilon_r \)?
(A) 2
(B) 1
(C) 4
(D) 8

MCQ 7.3.101 An electromagnetic field is said to be conservative when
(A) \( \nabla \cdot E = \mu \varepsilon (\partial^2 E/\partial t^2) \)
(B) \( \nabla \cdot H = \mu \varepsilon (\partial^2 H/\partial t^2) \)
(C) Curl on the field is zero
(D) Divergence of the field is zero

MCQ 7.3.102 Given that \( \mathbf{H} = 0.5 \exp[-0.1x] \sin(10^6 t - 2x) \mathbf{a}_x (\text{A/m}) \), which one of the following statements is not correct?
(A) Wave is linearly polarized along \( \mathbf{a}_x \)
(B) The velocity of the wave is \( 5 \times 10^5 \text{ m/s} \)
(C) The complex propagation constant is \( (0.1 + j2) \)
(D) The wave is travelling along \( \mathbf{a}_x \)

MCQ 7.3.103 For a conducting medium with conductivity \( \sigma \), permeability \( \mu \), and permittivity \( \varepsilon \), the skin depth for an electromagnetic signal at an angular frequency \( \omega \) is proportional to
(A) \( \sigma \)
(B) \( 1/\omega \)
(C) \( 1/\sqrt{\sigma} \)
(D) \( 1/\mu \)

MCQ 7.3.104 The electric field of a uniform plane wave is given by \( E = 10 \sin(10\omega t - \pi z) \mathbf{a}_x + 10 \cos(\omega t - \pi z) \mathbf{a}_y (\text{V/m}) \)
The polarization of the wave is
(A) Circular
(B) Elliptical
(C) Linear
(D) Undefined
MCQ 7.3.105

In free space \( \mathbf{H}(z,t) = 0.20 \cos(4 \times 10^7 t - \beta z) \mathbf{a}_y \) A/m. The expression for \( \mathbf{E}(z,t) \) is

(A) \( \mathbf{E}(z,t) = 37.7 \cos(4 \times 10^7 t - \beta z) \mathbf{a}_y \)

(B) \( \mathbf{E}(z,t) = 2.65 \times 10 \cos(4 \times 10^7 t - \beta z) \mathbf{a}_y \)

(C) \( \mathbf{E}(z,t) = 37.7 \cos(4 \times 10^7 t - \beta z) \mathbf{a}_x \)

(D) \( \mathbf{E}(z,t) = -37.7 \cos(4 \times 10^7 t - \beta z) \mathbf{a}_y \)

MCQ 7.3.106

A plane wave whose electric field is given by \( \mathbf{E} = 100 \cos(\omega t - 6\pi x) \mathbf{a}_z \) passes normally from a material ‘A’ having \( \varepsilon_r = 4, \mu_r = 1 \) and \( \sigma = 0 \) to a material ‘B’ having \( \varepsilon_r = 9, \mu_r = 4 \) and \( \sigma = 0 \). Match items in List I with List II and select the correct answer:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Intrinsic impedance of medium ‘B’</td>
<td>1. ( 6\pi )</td>
</tr>
<tr>
<td>b Reflection coefficient</td>
<td>2. ( 80\pi )</td>
</tr>
<tr>
<td>c Transmission coefficient</td>
<td>3. ( 1/7 )</td>
</tr>
<tr>
<td>d Phase shift constant of medium ‘A’</td>
<td>4. ( 8/7 )</td>
</tr>
</tbody>
</table>

Codes:

(A) 4 1 2 3

(B) 2 3 4 1

(C) 4 3 2 1

(D) 2 1 4 3

MCQ 7.3.107

In free space \( \mathbf{E}(z,t) = 50 \cos(\omega t - \beta z) \mathbf{a}_x \) V/m and \( \mathbf{H}(z,t) = 5/12\pi \cos(\omega t - \beta z) \mathbf{a}_y \) A/m. The average power crossing a circular area of radius \( \sqrt{24} \) m in plane \( z = \) constant is

(A) 200 W

(B) 250 W

(C) 300 W

(D) 350 W

MCQ 7.3.108

Consider a plane electromagnetic wave incident normally on the surface of a good conductor. The wave has an electric field of amplitude 1 V/m and the skin depth for the conductor is 10 cm.

Assertion (A) : The amplitude of electric field is \( (1/e^2)(V/m) \) after the wave has travelled a distance of 20 cm in the conductor.

Reason (R) : Skin depth is the distance in which the wave amplitude decays to \( (1/e) \) of its value at the surface.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true
Three media are characterised by

1. \( \varepsilon_r = 8, \mu_r = 2, \sigma = 0 \)
2. \( \varepsilon_r = 1, \mu_r = 9, \sigma = 0 \)
3. \( \varepsilon_r = 4, \mu_r = 4, \sigma = 0 \)

\( \varepsilon_r \) is relative permittivity, \( \mu_r \) is relative permeability and \( \sigma \) is conductivity.

The value of the intrinsic impedances of the media 1, 2 and 3 respectively are

(A) 188 \( \Omega \), 377 \( \Omega \) and 1131 \( \Omega \)
(B) 377 \( \Omega \), 1131 \( \Omega \) and 188 \( \Omega \)
(C) 188 \( \Omega \), 1131 \( \Omega \), and 377 \( \Omega \)
(D) 1131 \( \Omega \), 188 \( \Omega \), and 377 \( \Omega \)

A plane EM wave \( (E_x, H_y) \) travelling in a perfect dielectric medium of surge impedance ‘\( Z \)’ strikes normally on an infinite perfect dielectric medium of surge impedance 2\( Z \). If the refracted EM wave is \( (E_x, H_y) \), the ratios of \( E_x/E_r \) and \( H_y/H_r \) are respectively

(A) 3 and –3
(B) \( 3/2 \) and \( 1/3 \)
(C) \( 3/4 \) and \( 3/2 \)
(D) \( 3/4 \) and \( 2/3 \)

For a perfect conductor, the field strength at a distance equal to the skin depth is \( X\% \) of the field strength at its surface. The value ‘\( X\% \)’ is

(A) Zero
(B) 50\%
(C) 36\%
(D) 26\%

***********
SOLUTIONS 7.1

SOL 7.1.1 Option (B) is correct.
Given magnetic field intensity in the non magnetic medium is
\[ H = 3\cos(\omega t - kz)\mathbf{a}_z \text{ A/m} \]
The negative coefficient of \( z \) in \( (\omega t - kz) \) shows that the wave is propagating in \(+ \mathbf{a}_z\) direction.

SOL 7.1.2 Option (C) is correct.
From the property of phasor, we know that the instantaneous electric field is the real part of \( \{E, e^{j\omega t}\} \).
i.e.
\[ E(x, t) = \text{Re}\{E, e^{j\omega t}\} \]
where \( E \) is the phasor form of electric field.

Given the electric field intensity in time domain,
\[ E(x, t) = E_0 e^{-\alpha x} \sin(\omega t - \beta x)\mathbf{a}_y \]
\[ = E_0 e^{-\alpha x} \left[ \frac{e^{j(\omega t - \beta x)} - e^{-j(\omega t - \beta x)}}{2j} \right] \mathbf{a}_y \]
\[ = -\frac{jE_0}{2} e^{-\alpha x} e^{j(\omega t - \beta x)} \mathbf{a}_y + C.C. \]
where \( C.C. \) is complex conjugate of the \( 1^{st} \) part.

So, using the property of complex conjugates we get
\[ E(x, t) = 2\text{Re}\left\{-\frac{jE_0}{2} e^{-\alpha x} e^{j(\omega t - \beta x)} \mathbf{a}_y \right\} \]
\[ = \text{Re}\left\{-jE_0 e^{-\alpha x} e^{-j\beta x} e^{j\omega t} \mathbf{a}_y \right\} \]
Comparing it with equation (1), we get
\[ E = -2jE_0 e^{-(\alpha + \beta)x} \mathbf{a}_y \text{ V/m} \]

SOL 7.1.3 Option (C) is correct.
Given the magnetic field intensity,
\[ H = 10\cos(6 \times 10^7 t - ky)\mathbf{a}_z \text{ A/m} \]
Comparing it with the general equation of magnetic field.
\[ H = H_0\cos(\omega t - ky)\mathbf{a}_z \text{ A/m} \]
We get,
\[ \omega = 6 \times 10^7 \]
So, the wave no is,
\[ k = \frac{\omega}{c} = \frac{6 \times 10^7}{3 \times 10^8} = 0.2 \text{ (c is the velocity of wave in free space)} \]
SOL 7.1.4 Option (D) is correct.

Given magnetic field intensity in the non magnetic medium is

\[ H = 1.5 \cos(10^9 t - 5z) \mathbf{a}_z \, \text{A/m} \]

Comparing it with the general equation of magnetic field intensity

\[ H = H_0 \cos(\omega t - \beta z) \mathbf{a}_z \, \text{A/m} \]

We get,

\[ \omega = 10^9 \, \text{rad/sec} \]

and

\[ \beta = 5. \]

So, the phase velocity of the wave in the medium is given as

\[ v_p = \frac{\omega}{\beta} = \frac{10^9}{5} = 4 \times 10^8 \, \text{m/s} \]

SOL 7.1.5 Option (A) is correct.

Wavelength of an electromagnetic wave with phase constant \( \beta \) in a medium is defined as

\[ \lambda = \frac{2\pi}{\beta} \]

So, the phase constant of the wave in terms of wavelength can be given as

\[ \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{12.6} = 0.5 \, \text{rad/m} \quad (\lambda = 12.6 \, \text{m}) \]

SOL 7.1.6 Option (A) is correct.

Given the electric field intensity in the nonmagnetic material as

\[ E = 8 \cos(4 \times 10^8 t - 2x) \mathbf{a}_y \, \text{V/m} \]

Comparing it with the general equation of electric field

\[ E = E_0 \cos(\omega t - \beta x) \mathbf{a}_y \, \text{A/m} \]

We get,

\[ \omega = 4 \times 10^8 \, \text{rad/s} \]

and

\[ \beta = 2 \, \text{rad/m} \]

So, the phase velocity of the wave in the medium is given by

\[ v_p = \frac{\omega}{\beta} = 3 \times 10^8 \, \text{m/s} \]

Since the medium is non magnetic so, \( \mu = \mu_0 \) and the relative permittivity of the medium is given as

\[ \varepsilon_r = \left( \frac{c}{v_p} \right)^2 = \left( \frac{3 \times 10^8}{2 \times 10^8} \right)^2 = 2.25 \]

SOL 7.1.7 Option (D) is correct.

The general equation of electric field intensity of an EM wave propagating in \( \mathbf{a}_z \) direction in a medium is given as

\[ E = E_0 \cos(\omega t - \beta z) \mathbf{a}_z \, \text{A/m} \]

Comparing it with the given expression of electric field intensity, we get

\[ \omega = 5 \times 10^8 \, \text{rad/s} \]

So, the time period of the EM wave is

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{5 \times 10^8} = 12.57 \, \text{ns} \]
For View Only

Option (D) is correct.

The general equation of electric field intensity of an EM wave propagating in \( \mathbf{a}_z \) direction in a medium is given as

\[
\mathbf{E} = E_0 \cos(\omega t - \beta x)\mathbf{a}_z \text{ A/m}
\]

Comparing it with the given expression of electric field intensity, we get

\[
\omega = 4 \times 10^8 \text{ rad/s}
\]

So, the time period of the wave in air is given as

\[
T = \frac{2\pi}{\omega} = \frac{2\pi}{4 \times 10^8} = 15.71 \text{ ns}
\]

Since in one time period the wave travels its one wavelength (\( \lambda \)) so, time taken by the wave to travel \( \lambda/4 \) distance is

\[
t = \frac{T}{4} = 4.93 \text{ ns}
\]

Option (D) is correct.

Intrinsic impedance of any material is given as

\[
\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}
\]

where \( \mu \) is permeability, \( \sigma \) is conductivity and \( \varepsilon \) is permittivity of the medium.

Since the given material is lossless, nonmagnetic and dielectric so, we have

\[
\sigma = 0 \quad \text{(lossless)}
\]

\[
\mu = \mu_0 \quad \text{(non magnetic)}
\]

and

\[
\varepsilon = \varepsilon_r\varepsilon_0 = (2.25)\varepsilon_0 \quad (\varepsilon_r = 2.25)
\]

Therefore the intrinsic impedance of the material is

\[
\eta = \sqrt{\frac{3.230}{1.5}} = 323.3 \Omega \quad \left(\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \Omega\right)
\]

Option (B) is correct.

Given,

Frequency of the wave propagation, \( f = 0.5 \text{ MHz} = 0.5 \times 10^6 \text{ Hz} \)

Conductivity of medium, \( \sigma = 3 \times 10^7 \text{ S/m} \)

Relative permeability of medium, \( \mu_r = \varepsilon_r \approx 1 \)

So, the angular frequency of the wave propagation is

\[
\omega = 2\pi f = 2\pi \times 0.5 \times 10^6 = \pi \times 10^6
\]

and we get

\[
\frac{\sigma}{\omega\varepsilon} = \frac{3 \times 10^7}{\pi \times 10^6 \times 8.85 \times 10^{-12}} = 0.1 \times 10^{13} >> 1
\]

Therefore, the phase constant of the propagating wave is given as

\[
\beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad (\sigma/\omega\varepsilon >> 1)
\]

\[
= \sqrt{\frac{\pi \times 10^6 \times 4\pi \times 10^{-7} \times 3 \times 10^7}{2}}
\]
So, the wavelength of the radio wave in the medium is
\[ \lambda = \frac{2\pi}{\beta} = 2.8 \text{ mm} \]

**SOL 7.1.11** Option (C) is correct.

Attenuation constant for a plane wave with angular frequency \( \omega \) in a certain medium is given as
\[ \alpha = \omega \sqrt{\frac{\mu \varepsilon}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} - 1 \right] \tag{1} \]

Since for a poor conductor, conductivity is very low i.e.
\[ \sigma << \omega \varepsilon \]
or,
\[ \frac{\sigma}{\omega \varepsilon} << 1 \]

So, in equation (1) using binomial expansion we get,
\[ \alpha = \omega \sqrt{\frac{\mu \varepsilon}{2}} \left[ 1 + \frac{1}{2} \left( \frac{\sigma}{\omega \varepsilon} \right)^2 - 1 \right] \tag{\sigma/\omega \varepsilon << 1} \]
\[ = \omega \sqrt{\frac{\mu \varepsilon}{2}} \frac{1}{2} \frac{\sigma}{\omega \varepsilon} = \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} \]

Therefore, the skin depth of the poor conductor is
\[ \delta = \frac{1}{\alpha} = \frac{2}{\sigma} \sqrt{\frac{\varepsilon}{\mu}} \]

which is independent of frequency (\( \omega \)).

**SOL 7.1.12** Option (B) is correct.

Wave equation for a plane wave propagating in \( +a_z \) direction is given as
\[ \frac{\partial^2 f}{\partial t^2} - v_p^2 \frac{\partial^2 f}{\partial z^2} = 0 \]
where \( v_p \) is the velocity of wave propagation

Now from Assertion (A) the electric field is
\[ \mathbf{E} = E_0 \sin(z) \cos(ct) \mathbf{a}_z \]
It represents the electric field of a plane wave if it satisfies the wave equation
i.e.
\[ \frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \frac{\partial^2 \mathbf{E}}{\partial z^2} = 0 \]
where \( c \) is velocity of wave in free space

From the given expression of field intensity we have
\[ \frac{\partial \mathbf{E}}{\partial t} = -cE_0 \sin(z) \sin(ct) \]
or,
\[ \frac{\partial^2 \mathbf{E}}{\partial t^2} = -c^2 E_0 \sin(z) \cos(ct) \]
and
\[ \frac{\partial \mathbf{E}}{\partial z} = E_0 \cos(z) \cos(ct) \]
or,
\[ \frac{\partial^2 \mathbf{E}}{\partial z^2} = -E_0 \sin(z) \cos(ct) \]

Thus, we get
\[ \frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \frac{\partial^2 \mathbf{E}}{\partial z^2} = 0 \]

Since, the electric field \( \mathbf{E} \) satisfies the wave equation so it represents the field of a
plane wave.
Therefore, A and R both are true and R is the correct explanation of A.

**SOL 7.1.13**

Option (A) is correct.
Given the magnetic field intensity in free space is
\[ H = 0.1 \cos(10^9 t - \beta y) a_y \text{ A/m} \] (1)
The general equation of magnetic field intensity of the EM wave propagating in \( a_y \) direction is given as
\[ H = H_0 \cos(\omega t - \beta y) a_y \text{ A/m} \] (2)
Comparing equations (1) and (2) we get,
direction of wave propagation, \( a_y = a_y \)
and angular frequency, \( \omega = 10^9 \text{ rad/sec} \)
So, the phase constant of the wave is
\[ \beta = \frac{\omega}{c} = \frac{10^9}{3 \times 10^8} \text{ (c is velocity of wave in free space)} \]
\[ = 3.33 \text{ rad/m} \]
Now, electric field intensity in free space is defined as
\[ E = -\eta_0 a_y \times H \]
where \( \eta_0 \) is intrinsic impedance in free space and \( a_y \) is direction of wave propagation.
So,
\[ E = -37.7(a_y) \times 0.1 \cos(10^9 t - \beta y) a_z \]
\[ = -37.7 \cos(10^9 t - 3.33) a_z \]
Therefore, electric field intensity of the wave at \( y = 1 \text{ cm at } t = 0.1 \text{ ns} \) is
\[ E = -37.7 \cos[10^9(10^{-10}) - (3.33)(10^{-2})] a_z \]
\[ = -46.6 a_z \text{ V/m} \]

**SOL 7.1.14**

Option (C) is correct.
Given the magnetic field intensity of the plane wave in free space is
\[ H = (2 + j5) (4a_y + 2ja_z) e^{jxt} \text{ A/m} \]
From the Maxwell’s equation, the maximum electric field intensity of the plane wave is given as
\[ |E|_{\text{max}} = \eta_0 |H|_{\text{max}} \]
where \( \eta_0 \) is intrinsic impedance in air and \( |H|_{\text{max}} \) is the maximum magnetic field intensity of the plane wave.
Now, the maximum magnetic field intensity of the plane wave is given as
\[ |H|_{\text{max}} = \sqrt{H \cdot H^*} \]
where \( H^* \) is the complex conjugate of the magnetic field phasor.
So,
\[ |H|_{\text{max}} = \sqrt{[(2 + j5)(4a_y + 2ja_z)] \cdot [(2 - j5)(4a_y - 2ja_z)]} \]
\[ = \sqrt{(2 + j5)(2 - j5)(4)(4) - (2j)^2(2j)} \]
\[ = \sqrt{29 \times 20} = 24.1 \text{ A/m} \]
Therefore, the maximum electric field intensity of the plane wave is
\[ |E|_{\text{max}} = \eta_0 |H|_{\text{max}} = 377 \times 24.1 = 9.08 \text{ kV/m} \]
Option (D) is correct.

Given the instantaneous electric field in the free space is

\[ E = (5a_y - 6a_z)\cos(\omega t - 50z) \, \text{V/m} \]

So, the phasor form of electric field intensity is

\[ \mathbf{E}_s = (5a_y - 6a_z)e^{-j\omega t} \, \text{V/m} \]

The phasor form of magnetic field is given in the terms of electric field intensity as

\[ \mathbf{H}_s = \frac{1}{\eta_0}(\mathbf{a}_k) \times (\mathbf{E}) \]

where \( \mathbf{a}_k \) is the unit vector in the direction of wave propagation and \( \eta_0 \) is the intrinsic impedance in free space.

So,

\[ \mathbf{H}_s = \frac{1}{\eta_0}(a_z) \times (5a_y - 6a_z)e^{-j\omega t} \, \text{V/m} \]

\[ = \frac{1}{\eta_0}(-5a_x - 6a_y)e^{-j\omega t} \, \text{V/m} \]

\[ = -\frac{1}{\eta_0}(15a_z + 36a_y)e^{-j\omega t} \, \text{V/m} \]

Option (C) is correct.

For any electromagnetic wave propagating in a medium electric field leads magnetic field by an angle \( \theta_n \), where \( \theta_n \) is the phase angle of intrinsic impedance given as

\[ \tan 2\theta_n = \frac{\sigma}{\omega\varepsilon} \]

Now, for a perfect conductor

\[ \sigma = \frac{1}{\rho} \approx \infty \]

i.e.

\[ 2\theta_n = \infty \]

\[ \theta_n = 45^\circ \]

So, electric field leads magnetic field by 45°

or in other words magnetic field lags electric field by 45°.

Option (B) is correct.

Given, the electric field intensity of the wave in phasor form

\[ \mathbf{E}_s = (5a_y + 10a_z)e^{-j(\omega t - 2z)} \, \text{V/m} \]

So we get the direction of wave propagation as

\[ \mathbf{a}_s = \frac{4a_x - 2a_z}{\sqrt{20}} = \frac{2a_x - a_z}{\sqrt{5}} \]

Therefore, the phasor form of magnetic field intensity of the plane wave is given as

\[ \mathbf{H}_s = \frac{1}{\eta_0} \mathbf{a}_s \times \mathbf{E}_s \]

where \( \eta_0 \) is intrinsic impedance in free space

\[ = \frac{1}{120\pi}(\frac{2a_x - a_z}{\sqrt{5}}) \times (5a_y + 10a_z)e^{-j(\omega t - 2z)} \, \text{V/m} \]

\[ = -29.66e^{-j(\omega t - 2z)} \, \text{mA/m} \]

Option (A) is correct.

The time average power density of the EM wave is given as
\[ P_{ave} = \frac{E^2}{2\eta_0} a_x \]

where \( E \) is the magnitude of the electric field intensity of the wave, \( a_x \) is the unit vector in the direction of wave propagation and \( \eta_0 \) is the intrinsic impedance in the free space. So, we get

\[ P_{ave} = \frac{\sqrt{5^2 + 10^2}}{2(120\pi)} \left( \frac{4a_x - 2a_z}{\sqrt{20}} \right) \]

\[ = 18.9a_x - 34.15a_z \text{ Watt/m}^2 \]

**SOL 7.1.19** Option (A) is correct.

As the given electric field vector has the amplitude

\[ E_0 = (-2\sqrt{3}a_x + \sqrt{3}a_y - a_z) \]

So in the same direction the wave will be polarized.

**SOL 7.1.20** Option (B) is correct.

From Maxwells’ equation we have

\[ \nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \]

Given

\[ \nabla \times \mathbf{E} = 100\beta \sin(\omega t - \beta z) a_y \]

or,

\[ \nabla \times \mathbf{E} = 100\beta \sin(\omega t - \beta z) a_y \]

So,

\[ \frac{-\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} = 100\beta \sin(\omega t - \beta z) a_y \]

Therefore the magnetic flux density vector is

\[ \mathbf{B} = \int 100\beta \sin(\omega t - \beta z) a_y \, dt \]

\[ = \frac{100\beta}{\omega} \cos(\omega t - \beta z) a_y \]

\[ = \frac{100}{\sqrt{\mu_0 \varepsilon_0}} \cos(\omega t - \beta z) a_y \]

\[ = 3 \times 10^6 \cos(\omega t - \beta z) a_y \]

\[ \text{(} \beta = \frac{\omega}{\sqrt{\mu_0 \varepsilon_0}} \text{)} \]

**SOL 7.1.21** Option (B) is correct.

Poynting vector in an \( EM \) field is defined as

\[ \mathbf{P} = \mathbf{E} \times \mathbf{H} \]

where \( \mathbf{E} \) is electric field intensity and \( \mathbf{H} \) is the magnetic field intensity in the region.

Now, the electric field intensity in the region is given as

\[ \mathbf{E} = 100\cos(\omega t - \beta z) a_x \]

and as calculated in previous question the magnetic field intensity in the region is

\[ \mathbf{B} = 3 \times 10^6 \cos(\omega t - \beta z) a_y \]

So, the poynting vector in the field is

\[ \mathbf{P} = \mathbf{E} \times \frac{\mathbf{B}}{\mu_0} \]

\[ \text{(} \mathbf{H} = \frac{\mathbf{B}}{\mu_0} \text{)} \]
For View Only

\[ w_e = \frac{1}{4} \varepsilon_0 \mathbf{E} \cdot \mathbf{E}^* \]

where \( \mathbf{E} \) is the electric field intensity in phasor form and \( \mathbf{E}^* \) is its conjugate. Therefore, the average stored energy density in the region is

\[ w_e = \frac{25 \varepsilon_0}{4} \sin^2 \pi x \]

SOL 7.1.23 Option (A) is correct.

Given the electric field

\[ \mathbf{E} = 10 \sin \pi y \sin (6\pi \times 10^8 t - \sqrt{3} \pi x) \mathbf{a}_z \text{ V/m} \]

In phasor form, \( \mathbf{E}_s = 10 \sin \pi y e^{-j\pi/2} e^{-j\sqrt{3}\pi x} \mathbf{a}_z \).

So, from Maxwell’s equation, the magnetic flux density in the phasor form is given as

\[ \mathbf{B}_s = \frac{1}{j\omega} (\nabla \times \mathbf{E}_s) \]

where \( \omega = 6\pi \times 10^8 \) as determined from the given expression of \( \mathbf{E} \).

\[ \mathbf{B}_s = -\frac{10\sqrt{3} \pi}{6\pi \times 10^8} (\sin \pi y) e^{-j\pi/2} e^{-j\sqrt{3}\pi x} \mathbf{a}_y + \frac{j \pi}{6\pi \times 10^8} (\cos \pi y) e^{-j\pi/2} e^{-j\sqrt{3}\pi x} \mathbf{a}_y \]

Therefore, the time average energy density stored in the magnetic field will be

\[ w_m = \frac{1}{4\mu_0} (\mathbf{B}_s \cdot \mathbf{B}_s^*) \quad \text{where } \mathbf{B}_s^* \text{ is the conjugate of } \mathbf{B}_s \]

or,

\[ w_m = \frac{10^5}{144\pi} (25 + 50 \sin^2 \pi x) \]

SOL 7.1.24 Option (A) is correct.

For an EM wave propagating in two mediums, the wavelengths of the wave in two mediums are related as

\[ \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \]

where \( \lambda_1 \) and \( \lambda_2 \) are the wavelengths of EM wave in two mediums with permittivity \( \varepsilon_1 \) and \( \varepsilon_2 \) respectively. So, the wavelength of plane wave in free space is given as

\[ \frac{\lambda}{\lambda} = \sqrt{\frac{1}{\varepsilon_r}} \]

\[ \lambda_0 = \lambda_1 \sqrt{\varepsilon_r} \]

where \( \lambda \) is the wavelength of the wave in the medium with relative permittivity \( \varepsilon_r \). So,

\[ \lambda_0 = 20\sqrt{9} = 60 \text{ cm} \quad (\lambda = 20 \text{ cm}, \varepsilon_r = 9) \]
SOL 7.1.25 Option (D) is correct.
Given,
Conductivity of the glass, \( \sigma = 10^{-12} \text{ S/m} \)
and relative permittivity of the glass, \( \varepsilon_r = 2.25 \)
So, the permittivity of glass is
\[ \varepsilon = \varepsilon_0 \varepsilon_r = 2.25 \varepsilon_0 \]
Therefore, the time taken by the charge to flow out to the surface is
\[ \tau \approx \frac{\varepsilon}{\sigma} = \frac{(2.25) \times (8.85 \times 10^{-12})}{10^{-12}} \]
\[ \approx 20 \text{ sec} \]

SOL 7.1.26 Option (A) is correct.
The reflection coefficient of the wave propagating from medium 1 to medium 2 is defined as
\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \]
where \( \eta_1 \) and \( \eta_2 \) are the intrinsic impedance of the two mediums respectively. So, the reflection coefficient for the wave propagating from free space to a dielectric medium is given as
\[ \Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} \]
where \( \eta \) is intrinsic impedance of the dielectric medium and \( \eta_0 \) is intrinsic impedance in free space. Since the intrinsic impedance of the dielectric medium is given as
\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\pi \varepsilon_0}} = \frac{\eta_0}{2} \]
So, we have
\[ \Gamma = \frac{\eta_0/2 - \eta_0}{\eta_0/2 + \eta_0} \]
\[ = \frac{1/2 - 1}{1/2 + 1} = -\frac{1}{3} \]
Therefore, the magnitude of electric field of reflected wave is
\[ E_r = \Gamma E_0 = -\frac{E_0}{3} \quad (E_0 \text{ is the magnitude of incident field)} \]

***********
SOLUTIONS 7.2

SOL 7.2.1  Option (D) is correct.

Time period of wave propagating in a medium is given as:

\[ T = \frac{2\pi}{\omega} \]

where \( \omega \) is the angular frequency of the wave.

Given the magnetic field intensity in the free space is

\[ H = 0.3 \cos(\omega t - \beta y) \mathbf{a}_x \text{ A/m} \]

So, at \( t = T/8 \) the magnetic field intensity is

\[ H = 0.3 \cos \left( \omega \frac{T}{8} - \beta y \right) \mathbf{a}_x = 0.3 \cos \left( \frac{\pi}{4} - \beta y \right) \mathbf{a}_x \]

or,

\[ H = 0.5 \cos(\beta y - \pi/4) \]

Therefore we get the plot of \( H \) versus \( y \) as shown below.

![Plot of H vs y](image)

SOL 7.2.2  Option (D) is correct.

Phase velocity of the medium, \( v_p = 7.5 \times 10^7 \text{ m/s} \)
Relative permeability, \( \mu_r = 4.8 \)
Conductivity \( \sigma = 0 \) (lossless medium)

Since phase velocity of an EM wave in a medium is defined as

\[ v_p = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \]

where \( c \) is the velocity of wave in air, \( \mu_r \) is the relative permeability of the medium and \( \varepsilon_r \) is the relative permittivity of the medium. So, we have

\[ 7.5 \times 10^7 = \frac{3 \times 10^8}{\sqrt{(4.8)\varepsilon_r}} \]

or,

\[ \varepsilon_r = 3.33 \]

Now the intrinsic impedance of the medium is given as
Given the electric field intensity in the phasor form is
\[ E_s = 5e^{0.3x} a_z \text{ V/m} \]
and the general equation of electric field phasor of an EM wave propagating in \( a_z \) direction is
\[ E_s = E_0 e^{-\beta x} a_z \text{ V/m} \]
So, comparing the equations (1) and (2) we get, 
\[ a = -a_z \]
and phase constant,
\[ \beta = 0.3 \text{ rad/m} \]
and from the Maxwell’s equation, the magnetic field phasor of the wave is given as
\[ H_s = \frac{1}{\eta}(a_z) \times E_s \]
where \( \eta \) is the intrinsic impedance of the medium and \( a_z \) is the unit vector in the direction of wave propagation.

So,
\[ H_s = \frac{1}{45.24}(-a_z) \times (5e^{0.3x} a_z) \]
\[ = \frac{5}{45.24} e^{0.3x} a_y = 11.05 e^{0.3x} a_y \text{ mA/m} \]

and the angular frequency of the wave is given as
\[ \omega = \beta v_p = (0.3)(7.5 \times 10^7) = 2.25 \times 10^7 \]

So, the magnetic field intensity of the EM wave in time domain is
\[ H(x, t) = \text{Re}\{H_s e^{j \omega t}\} = 11.05 \cos(\omega t + 0.3x) a_y \\
= 11.05 \cos(2.25 \times 10^7 t + 0.3x) a_y \text{ mA/m} \]

**SOL 7.2.3**
Option (D) is correct.

Given the electric field intensity of the propagating wave,
\[ E = E_0 e^{\sigma x} \sin(10^8 t - \beta x) a_y \text{ V/m} \]
The general equation of electric field intensity of plane wave propagating in \( a_y \) direction is given by
\[ E = E_0 e^{-\sigma x} \sin(\omega t - \beta x) a_y \text{ V/m} \]
Comparing equation (1) and (2) we get,
\[ \alpha = \frac{1}{3} \text{ NP/m and } \omega = 10^8 \text{ rad/sec} \]
So, the attenuation constant of a propagating wave is given as
\[ \alpha = \omega \sqrt{\frac{\mu \sigma}{2}} \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2 + 1} \]
Let
\[ x_0 = \sqrt{\frac{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}{2}} \]
Therefore,
\[ \alpha = \omega \sqrt{\frac{\mu_0 \varepsilon_0}{2}} \mu_r \varepsilon_r (x_0 - 1) \]
or,
\[ (x_0 - 1) = \frac{2\alpha^2}{\omega^2 \mu_0 \varepsilon_0 \mu_r \varepsilon_r} \]
Now, we put $\alpha = 1/3$, $\mu_r = \varepsilon_r = 4$, $\omega = 10^8$ so, we get

$$x_0 - 1 = 2 \frac{(1/3)^2 c_0^2}{(10^8)^2 \times (4)^2}, \quad \left( c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s} \right)$$

$$x_0 = \frac{1}{8} + 1$$

$$x_0 = \frac{9}{8}$$

\[
\sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} = \frac{9}{8},
\]

\[
\frac{\sigma}{\omega \varepsilon} = \sqrt{\frac{81}{64}} - 1
\]

Thus, loss tangent $\frac{\sigma}{\omega \varepsilon} = 0.52$

**SOL 7.2.4** Option (C) is correct.

From the field intensity we get,

$$\omega = 10^9 \pi$$

and it is given that, $\mu_r = 0.5$, $\sigma = 0.01 \text{ S/m}$, $\varepsilon_r = 8$.

So, the phase constant,

\[
\beta = \omega \sqrt{\frac{\mu \varepsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega t} \right)^2} \right] + 1}
\]

\[
= 10^9 \pi \sqrt{\frac{\mu_0 \varepsilon_0 \pi (0.5)}{2 \left[ \sqrt{1 + \left( \frac{0.01}{10^9 \pi 8 \varepsilon_0} \right)^2} \right] + 1}}
\]

\[
= 20.95
\]

Let the distance travelled by the wave be $z$ to have a phase shift of $10^\circ$.

So,

\[
\beta z = 10^9 \pi = \frac{10 \pi}{180} \text{ rad}
\]

\[
z = \frac{\pi}{9 \times (20.95)} = 16.66 \text{ mm}
\]

**SOL 7.2.5** Option (A) is correct.

The attenuation constant of a propagating wave in a medium is defined as

\[
\alpha = \omega \sqrt{\frac{\mu \varepsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} \right] - 1}
\]

Now, from the given data we have

$\mu_r = 0.5$, $\sigma = 0.01 \text{ S/m}$, $\varepsilon_r = 8$.

So,

\[
\alpha = 10^9 \pi \sqrt{\frac{\mu_0 \varepsilon_0 \pi (0.5)}{2 \left[ \sqrt{1 + \left( \frac{0.01}{10^9 \pi 8 \varepsilon_0} \right)^2} \right] - 1}}
\]

\[
= 0.9425
\]

Initially the amplitude of the electric field = 0.5

So, after travelling distance $z$ amplitude of wave $= 0.5 e^{-ax}$.

Therefore, the distance travelled by the wave for which the amplitude of the wave
Reduced by 40% is evaluated as

\[ 0.5e^{-\alpha z} = 0.5 \times \frac{60}{100} \]  

\[ e^{-0.9425z} = 0.6 \]

or,

\[ z = \frac{1}{0.9425} \ln \left( \frac{1}{0.6} \right) = 542 \text{ mm} \]

**SOL 7.2.6**  
Option (A) is correct.  

Given the field intensities of the plane wave as

\[ E(x, t) = 900 \cos(5 \times 10^8 \pi t - \beta x) a_y \text{ V/m} \]

\[ H(x, t) = 1.9 \cos(5 \times 10^8 \pi t - \beta x) a_z \text{ V/m} \]

So, we get

\[ |E| = 900, \quad |H| = 1.9, \quad \omega = 5 \times 10^8 \pi \]

Now, the intrinsic impedance in the medium is

\[ \eta = \frac{|E|}{|H|} = \frac{900}{1.9} = 473.7 \]

and phase constant of the wave in the medium is

\[ \beta = \frac{\omega}{v_p} = \frac{5 \times 10^8 \pi}{7 \times 10^7} = 0.224 \text{ m}^{-1} \]

Since, for a perfect dielectric \( \sigma = 0 \)

Therefore,

\[ \eta = \sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} \]  

\[ \beta = \frac{\omega}{v_p} = \frac{\omega}{c} \sqrt{\mu_r \varepsilon_r} \]  

Comparing the equation (1) and (2) we get,

\[ \mu_r = \left( \frac{\beta \eta_0 c}{\omega \varepsilon_0} \right) \right] = \left[ \frac{(0.224)(473.7)(3 \times 10^8)}{(5 \times 10^8 \pi)(377)} \right] \]

\[ = 5.37 \]

Again from equation (1)

\[ \varepsilon_r = \left( \frac{\varepsilon_0}{\eta_0} \right)^2 \mu_r = \left( \frac{377}{473.7} \right)^2 \times 8.37 = 4.4 \]

**SOL 7.2.7**  
Option (B) is correct.  

General equation of electric field intensity of a plane wave propagating in free space in \(-a_z\) direction having amplitude \(E_0\) and frequency \(\omega\) is given as:

\[ E = E_0 \cos(\omega t + \beta x) a_y \]

where \(\beta\) is phase constant of the wave and \(a_y\) is the unit vector in the direction of polarization of wave and since the EM wave is polarized in \(+a_z\) direction. So,

\[ a_y = a_z \]

and we get,

\[ E = E_0 \cos(\omega t + \frac{\omega x}{v_c}) a_z \]  

\[ \text{in free space } \beta = \frac{\omega}{v_c} \]

Therefore, the magnetic field intensity of the wave is given as

\[ H = \frac{1}{\eta_0}(a_y) \times (E) \]

where \(a_y\) is the unit vector in the direction of wave propagation and \(\eta_0\) is the intrinsic impedance of the wave in the medium.
So, \[ H = \frac{1}{\eta_0} (-a_z) \times \left[ E_0 \cos(\omega t - \frac{\omega}{c} x) a_z \right] \quad (a_z = -a_z) \]

\[ = \frac{E_0}{\eta_0} \cos(\omega t + \frac{\omega}{c} x) a_y \]

**SOL 7.2.8** Option (B) is correct.

General equation of electric field intensity of a plane wave propagating in free space is given as:

\[ E = E_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) a_n \quad (1) \]

where \( a_n \) is unit vector in direction of polarization, \( \mathbf{k} \) is the wave number in the direction of wave propagation with amplitude \( k = \beta = \frac{\omega}{c} \), and \( \mathbf{r} = xa_x + ya_y + za_z \) is the position vector.

Since, the wave is propagating in the direction from origin to point \((1,1,1)\).

So, \[ k = \left( \frac{\omega}{c} \right) (a_x + a_y + a_z) - 0 = \frac{\omega}{c} (a_x + a_y + a_z) \sqrt{3} \]

and since the field is polarized parallel to \( x-z \) plane

So, \[ a_n = \frac{ma_x + na_y}{\sqrt{m^2 + n^2}} \]

where \( m \) and \( n \) are constants.

Now, the electric field of wave is always perpendicular to the direction of propagation of EM wave. So, we have

\[ \mathbf{k} \cdot \mathbf{a}_n = 0 \]

\[ \left[ \frac{\omega}{c} \left( \frac{a_x + a_y + a_z}{\sqrt{3}} \right) \right] \cdot \left[ \frac{ma_x + na_y}{\sqrt{m^2 + n^2}} \right] = 0 \]

\[ m + n = 0 \]

\[ m = -n \]

Therefore, the unit vector in the direction of polarization of the wave is

\[ \mathbf{a}_n = \frac{ma_x + (-m)n a_z}{\sqrt{m^2 + (-n)^2}} = \frac{a_x - a_z}{\sqrt{2}} \quad (m = -n) \]

Putting all the values in equation (1), we get the electric field of the wave as

\[ E = E_0 \cos \left[ \omega t - \frac{\omega}{c} \left( \frac{a_x + a_y + a_z}{\sqrt{3}} \right) \cdot \left( xa_x + ya_y + za_z \right) \left( \frac{a_x - a_z}{\sqrt{2}} \right) \right] \]

\[ = E_0 \cos \left[ \omega t - \frac{\omega}{\sqrt{3} c} (x + y + z) \left( \frac{a_x - a_z}{\sqrt{3}} \right) \right] \]

**SOL 7.2.9** Option (C) is correct.

Skin depth \( (\delta) \) of any medium is defined as the reciprocal of attenuation constant \( (\alpha) \) of a plane wave in the medium

i.e. \[ \delta = \frac{1}{\alpha} \]

The attenuation constant of the plane wave in the medium is given as

\[ \alpha = \omega \sqrt{\frac{\mu \varepsilon}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} - 1 \right]} \]

Now, \[ \frac{\sigma}{\omega \varepsilon} = \frac{2}{2\pi \varepsilon_0 \varepsilon_0} = \frac{2}{2\pi \times 50 \times 10^9 \times 80 \times 8.85 \times 10^{-12}} \]
i.e., \( \frac{\sigma}{\omega \varepsilon} >> 1 \)

So,

\[
\alpha = \sqrt{\frac{\omega \mu \sigma}{2}} \quad (\sigma/\omega \varepsilon >> 1)
\]

\[
= \sqrt{\frac{2\pi \times 50 \times 10^3 \times 4 \times 4\pi \times 10^{-7} \times 2}{2}} = 0.4\pi
\]

Therefore,

\[
\delta = \frac{1}{\alpha} = \frac{1}{0.4\pi} = 0.796 \text{ m}
\]

**SOL 7.2.10**
Option (B) is correct.

For the microwave experiment the angular frequency is

\[
\omega = 2\pi f = 2\pi \times 10 \times 10^9 = 2\pi \times 10^{10}
\]

So,

\[
\frac{\sigma}{\omega \varepsilon} = \frac{6.25 \times 10^7}{2\pi \times 10^{10} \times 1 \times 8.85 \times 10^{-12}} = 1.12 \times 10^4 >> 1
\]

Therefore, the skin depth of the material is

\[
\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\frac{2}{\omega \mu \sigma}}} \quad (\sigma/\omega \varepsilon >> 1)
\]

\[
= \sqrt{\frac{2\pi \times 10^{10} \times 1 \times 4\pi \times 10^{-7} \times 6.25 \times 10^7}{2}} = 6.36 \times 10^{-7} \text{ m} = 0.636 \mu \text{m}
\]

Thus, for the successful experiment, width of coating must be greater than skin depth

i.e.,

\[
t > 0.636
\]

\[
t > 0.64 \mu \text{m}
\]

**SOL 7.2.11**
Option (A) is correct.

Given, the electric field intensity of the plane wave

\[
\mathbf{E} = 3 \cos(10^7 t - 0.2y) \mathbf{a}_x + 2 \sin(10^7 t - 0.2y) \mathbf{a}_z \text{ V/m}
\]

Comparing it with the general equation of electric field of a plane wave, we get

Angular frequency,

\[
\omega = 10^7
\]

Phase constant,

\[
\beta = 0.2
\]

So, the phase velocity of the propagating wave is

\[
v_p = \frac{\omega}{\beta} = \frac{10^7}{0.2} = 5 \times 10^7 \text{ m/s}
\]

or,

\[
\frac{c}{\sqrt{\varepsilon_r}} = 5 \times 10^7
\]

where \( c \) is velocity of wave in air and \( \varepsilon_r \) is the relative permittivity of the medium.

So,

\[
\varepsilon_r = \left( \frac{3 \times 10^8}{5 \times 10^7} \right)^2 = 36
\]

Therefore, permittivity of the medium is

\[
\varepsilon = \varepsilon_r \varepsilon_0 = 36 \varepsilon_0
\]

Now, the complex permittivity of the medium is
SOL 7.2.12 Option (B) is correct.

Conductivity of all the metals are in the range of mega siemens per meter and frequency of the visible waves are in the range of $10^{15}$ Hz. So, we can assume

Conductivity of a metal $\approx 10^6 \text{ S/m}$

Frequency of a visible wave $\approx 10^{15}$ Hz

Now, the attenuation constant of a wave in a certain medium is given as:

$$\alpha = \omega \sqrt{\mu \varepsilon / 2 \sqrt{1 + (\sigma / \omega \varepsilon)^2} - 1}$$

Since for a metal, $\sigma >> \omega \varepsilon$

So,

$$\alpha = \omega \sqrt{\mu \varepsilon / 2} \sqrt{\sigma / \omega \varepsilon} = \sqrt{\omega \mu \sigma / 2}$$

Therefore, the skin depth of a metal is

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{10^{15} \times 4\pi \times 10^{-7} \times 10^6}{2 \pi}} = \sqrt{\frac{1}{2\pi} \times 10^{-7} \approx 1 \text{ nm}}$$

Thus, the skin depth is in the range of nanometers for a metal and that’s why the wave (visible wave) can’t penetrate inside the metal and the metals are opaque.

i.e. (A) and (R) both are true and (R) is the correct explanation of (A).

SOL 7.2.13 Option (D) is correct.

Since, the wave is propagating in free space so, the velocity of the wave is $3 \times 10^8 \text{ m/s}$ and the amplitude of magnetic field intensity in $z = 0$ plane is given as

$$H_0 = \frac{E_0}{\mu_0}$$

Therefore, the plot of magnetic field intensity $H_0$ versus time $t$ in $z = 0$ plane is as shown in the figure below:
Since, the wave is propagating in $+\mathbf{a}_z$ direction so, an amplitude which exists in the plane $z = 0$ at any time $t$ must exist in the plane 

$$z = (1 \times 10^{-6} - t) \times 3 \times 10^8 \text{ m at } t = 1 \mu\text{sec}.$$ 

So, the amplitude of $H_0$ will be equal to the $H_1$ at $t = 1 \mu\text{sec}$ for the plane 

$$z = (10^6 - t) \times 3 \times 10^8 \text{ m}$$ 

Thus, the plot of $H_1$ versus $z$ will be as shown in figure below

![Plot of $H_1$ versus $z$](image)

**SOL 7.2.14** Option (D) is correct.

Velocity of the wave in free space is 

$$c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

So, the velocity of the wave in dielectric 1 is 

$$v_{p1} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{c}{2}$$

The velocity of wave in dielectric 2 is 

$$v_{p2} = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{c}{3}$$

The velocity of wave in dielectric 3 is 

$$v_{p3} = \sqrt{\frac{\mu_0}{3\varepsilon_0}} = \frac{c}{\sqrt{3}}$$

Therefore, the time $t$ taken by the wave to strike the interface at $x = 5 \text{ m}$ is 

$$t = t_1 + t_2 + t_3$$

$$= \frac{6}{3 \times 10^8} + \frac{3}{c/2} + \frac{2}{c/3}$$

$$= (0.04 + 0.06 + 0.08) \times 10^{-6} = 0.06 \mu\text{sec}$$

**SOL 7.2.15** Option (D) is correct.

Intrinsic impedance of 1st medium is 

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$$

and the intrinsic impedance of 2nd medium is 

$$\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$$

So, the reflection coefficient at the interface of the two medium is given as 

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
\( \Gamma = \sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}} \) \\
\( \sqrt{\frac{1}{\varepsilon_2}} + \sqrt{\frac{1}{\varepsilon_1}} \)

or, 
\( \frac{1}{5} = \sqrt{\frac{1}{\varepsilon_2}} - \sqrt{\frac{1}{\varepsilon_1}} \) \\
\( \sqrt{\frac{1}{\varepsilon_2}} + \sqrt{\frac{1}{\varepsilon_1}} \)  
(given \( \Gamma = \frac{1}{5} \))

\( 5 + 1 \) \\
\( 5 - 1 \) = \( \frac{2}{2} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \)  
(by rationalisation)

\( \frac{6}{4} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \) \\
\( \frac{\varepsilon_1}{\varepsilon_2} = \frac{9}{4} \)

**SOL 7.2.16** Option (D) is correct.

Given

Frequency of the propagating wave, \( f = 50 \text{ MHz} = 50 \times 10^6 \text{ Hz} \)

Skin depth of the dielectric medium, \( \delta = 0.32 \text{ mm} = 0.32 \times 10^{-3} \text{ m} \)

Permittivity of dielectric, \( \mu = 6.28 \times 10^{-7} \)

So, the conductivity of the dielectric medium is given as

\[ \sigma = \frac{1}{\pi \mu \delta^2} = \frac{1}{(3.14) \times (50 \times 10^6) \times (6.28 \times 10^{-7}) \times (0.32 \times 10^{-3})^2} = 0.99 \times 10^5 \text{ S/m} \]

**SOL 7.2.17** Option (A) is correct.

Frequency of the wave, \( f = 8 \text{ GHz} = 8 \times 10^9 \text{ Hz} \)

Distance travelled by the wave, \( z = 0.175 \text{ mm} = 0.175 \times 10^{-3} \text{ m} \)

Permittivity of dielectric, \( \mu = 6.28 \times 10^{-7} \)

and as calculated in previous question the conductivity of the dielectric medium is

\( \sigma = 0.99 \times 10^5 \text{ S/m} \)

So, the attenuation constant of the wave in the dielectric medium is

\[ \alpha = \sqrt{\pi \mu \sigma} = \sqrt{(3.14) \times (8 \times 10^9) \times (6.28 \times 10^{-7}) \times (0.99 \times 10^5)} = 3.95 \times 10^4 \text{ NP/m} \]

Therefore, the reducing factor of the field intensity in dB after travelling distance \( z \) is

\[ 20 \log_{10} e^{-\alpha z} = 20 \log_{10} e^{-3.95 \times 10^4 \times (0.175 \times 10^{-3})} = -40 \text{ dB} \]

**SOL 7.2.18** Option (A) is correct.

Given the electric field intensity in phasor form

\[ E_i = E_0 (a_y - ja_x) e^{-j\omega t} \]

So, the instantaneous expression of electric field intensity will be,
The electric field $E$ can be expressed as:

$$E = \text{Re}\left\{E_0(a_y - ja_z)e^{j(x-z)t}\right\}$$

$$= \text{Re}\left\{E_0(a_y - ja_z)[\cos(\omega t) + jsin(\omega t)]\right\}e^{-j\omega t}$$

$$= E_0(a_y\cos(\omega t) + a_z\sin(\omega t))e^{-j\omega t}$$

Therefore, the magnitude of the field is

$$|E| = \sqrt{(E_0\cos(\omega t))^2 + (E_0\sin(\omega t))^2}$$

or,

$$|E_1| + |E_2| = E_0$$

which is a circular equation i.e. the wave is circularly polarized.

Now, the instantaneous angle $\theta$ that the field $E$ makes with $y$-axis is given as

$$\tan\theta = \frac{E_0\sin(\omega t)}{E_0\cos(\omega t)}$$

or,

$$\theta = \omega t$$

Therefore as the time increases, $E$ rotates from $y$ to $z$ as shown in figure below:

![Diagram showing rotation of electric field from y to z](image)

and since the direction of wave propagation is in $+a_x$ direction so, the rotation from $y$ to $z$ obeys the right hand rule. Thus, we conclude that the field is Right hand circularly polarized.

**SOL 7.2.19**

Option (B) is correct.

Given the phasor form of electric field intensity,

$$E_0 = 4(a_y - ja_z)e^{-j\omega t}$$

So, the electric field intensity of the reflected wave will be

$$E_n = \Gamma(4(a_y - ja_z))e^{j\omega t}$$

where $\Gamma$ is the reflection coefficient at the interface. Therefore,

$$E_n = 4(-a_y + ja_z)e^{j\omega t} \quad \text{(for perfect conductor $\Gamma = -1$)}$$

and the instantaneous expression of the electric field of reflected wave will be

$$\hat{E} = \text{Re}\left\{4(-a_y + ja_z)(\cos(\omega t) + jsin(\omega t))\right\}e^{j\omega t}$$

$$= 4(-\cos(\omega t)a_y - \sin(\omega t)a_z)e^{j\omega t}$$

Therefore, the magnitude of the reflected field is

$$|E_1| + |E_2| = 4$$

or,

$$\sqrt{(4\cos(\omega t))^2 + (4\sin(\omega t))^2}$$

which is a circular equation i.e. the wave is circularly polarized.

Now, the instantaneous angle $\theta$ that $E$ makes with $z$-axis is given as

$$\tan\theta = \frac{-4\sin(\omega t)}{-4\cos(\omega t)}$$
\[ \theta = \omega t \]

So, as time increases, electric field \( \mathbf{E} \) rotates from \( z \) to \( x \) as shown in the figure below:

Since the direction of wave propagation is along \( -a_y \), so, the rotation from \( z \) to \( x \) follows left hand rule. Thus, we conclude that the EM wave is LHC (left hand circularly) polarized.

**SOL 7.2.20**

Option (C) is correct.

Given the electric field intensity of incident wave,

\[ \mathbf{E}_{i\alpha} = 10a_y e^{-j(0y + 8x)} \]

So, the direction of wave propagation is

\[ \mathbf{K} = 6a_y + 8a_x \]

Since the wave is incident on the perfect conductor so, the magnitude of the reflected wave is given as,

\[ \mathbf{E}_{r\alpha} = -\mathbf{E}_{r\alpha} = -10a_z \]

\( (\Gamma = -1 \text{ for perfect conductor}) \)

The direction of wave propagation of reflected wave will be along \((6a_y - 8a_x)\) as shown in figure below:

Therefore, the field intensity of the reflected wave is

\[ \mathbf{E}_{r\alpha} = -10a_z e^{-j(0y - 8x)} \]

Thus, the net electric field intensity of the total wave in free space after reflection will be

\[ \mathbf{E}_s = \mathbf{E}_{i\alpha} + \mathbf{E}_{r\alpha} = 10a_y e^{-j(0y + 8x)} + \left[-10a_z e^{-j(0y - 8x)}\right] \]

\[ = 10a_y e^{-j0y} (e^{-j8x} - e^{-j8x}) = -j10a_z e^{-j8y} \sin 4x \text{ V/m} \]
SOL 7.2.21 Option (B) is correct.

Given, the electric field intensity of the incident wave,
\[ E_i = 25 \mathbf{a}_x e^{-j(6z + 8y)} \text{ V/m} \]

So, the direction of the wave propagation is
\[ \mathbf{k} = 6 \mathbf{a}_x + 8 \mathbf{a}_y \]

Since the wave is incident on a perfect conductor so, the magnitude of the electric field of the reflected wave is
\[ E_r = -E_i \quad \text{(reflection coefficient, } \Gamma = -1) \]

The reflected wave will propagate in \(6 \mathbf{a}_x - 8 \mathbf{a}_y\) direction as shown in figure below:

![Perfect Conductor Diagram]

So, we get the electric field intensity of reflected wave as
\[ E_r = -25 \mathbf{a}_x e^{-j(6z - 8y)} \text{ V/m} \]

Since, the magnetic field intensity of a plane wave in terms of electric field intensity is defined as
\[ \mathbf{H} = \frac{1}{\eta_0} (\mathbf{a}_k \times \mathbf{E}) \]

where \(\mathbf{a}_k\) is unit vector in the direction of wave propagation and \(\eta_0\) is the intrinsic impedance of free space. So, the magnetic field intensity of the reflected wave is given as
\[ H_r = \frac{1}{\eta_0} (\mathbf{a}_k \times E_r) \]

where,
\[ \mathbf{a}_k = \frac{\mathbf{k}}{\kappa} = \frac{6 \mathbf{a}_z - 8 \mathbf{a}_y}{\sqrt{(6 \mathbf{a}_z - 8 \mathbf{a}_y)^2}} = (0.6 \mathbf{a}_z - 0.8 \mathbf{a}_y) \]

So, we get
\[ H_r = \frac{1}{120 \pi} [(0.6 \mathbf{a}_z - 0.8 \mathbf{a}_y) \times (-25 \mathbf{a}_x e^{-j(6z - 8y)})] \]
\[ = \frac{1}{120 \pi} [(-15 \mathbf{a}_y - 20 \mathbf{a}_z e^{-j(6z - 8y)})] \]
\[ = -(\frac{a_y}{4 \pi} + \frac{a_z}{3 \pi}) e^{-j(2z - 3y)} \text{ A/m} \]

SOL 7.2.22 Option (D) is correct.

Given, the magnetic field intensity of the EM wave propagating in free space,
\[ \mathbf{H} = 0.2 \cos(\omega t - \beta y) \mathbf{a}_z \text{ A/m} \]

So, the time average power density of the EM wave is given as
\[ \mathcal{P}_{\text{ave}} = \frac{1}{2} \eta_0 H^2 a_n, \]

where \( \eta_0 \) is the intrinsic impedance in free space and \( H \) is the magnitude of magnetic field intensity in free space.

So,

\[ \mathcal{P}_{\text{ave}} = \frac{1}{2}(120\pi)(0.1)^2 a_n \quad \text{(} \eta_0 = 120\pi, H = 0.1 \text{)} \]

Therefore, the total power passing through the square plate of side 20 cm is given as

\[ P_{\text{total}} = \int \mathcal{P}_{\text{ave}} \cdot dS \]

\[ = \mathcal{P}_{\text{ave}} \cdot S a_n \]

where \( S \) is the area of the square plate given as \( S = (0.2)^2 = 0.04 \text{ m}^2 \) \quad \text{(Side of square = 0.2 m)}

and \( a_n \) is the unit vector normal to the plate given as

i.e.

\[ a_n = \frac{a_x + a_y}{\sqrt{2}} \]

So,

\[ P_{\text{total}} = (0.6\pi a_n) \cdot \left[ 0.04 \left( \frac{a_x + a_y}{\sqrt{2}} \right) \right] \]

\[ = 0.09331 \text{ Watt} = 53.31 \text{ mW} \]

SOL 7.2.23  Option (D) is correct.

Given, the electric field intensity of the incident wave,

\[ E_\alpha = 5e^{-j\beta_1} \text{ V/m} \]

So, we get the phase constant of the wave as

\[ \beta_1 = 5 \]

or,

\[ \frac{\omega}{c} \sqrt{\mu_0 \varepsilon_0} = 5 \quad (\beta = \frac{\omega}{c}) \]

\[ \frac{\omega}{c} \sqrt{(4)(1)} = 5 \]

\[ \omega = \frac{5c}{2} \]

Now, the intrinsic impedance of the lossless medium is given as

\[ \eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}} = 2 \sqrt{\frac{\mu_0}{\varepsilon_0}} = 2\eta_0 = 754 \]

and the intrinsic impedance of lossy medium is

\[ \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \]

where, the magnitude of the intrinsic impedance is given as

\[ |\eta_2| = \sqrt{\frac{\mu_2}{\varepsilon_2}} \left[ 1 + \left( \frac{\omega \varepsilon_2}{\sqrt{\mu_2}} \right)^2 \right]^{1/4} = \sqrt{\frac{\mu_0/4\varepsilon_0}{\varepsilon_2}} \left[ 1 + \left( \frac{5}{2} \right)^2 \right]^{1/4} \]

\[ = \frac{60\pi}{(15.18)^{1/4}} \]
and the phase angle of the intrinsic impedance is
\[ \tan 2\theta_i = \frac{\sigma_i}{\omega\varepsilon_i} = 3.77 \]
or
\[ \theta_i = 37.57° \]
So, the reflection coefficient of the wave is given as
\[ \Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} = \frac{95.48 / 37.57° - 754}{95.48 / 37.57° + 754} = 0.1886/171.08° \]
Therefore, the standing wave ratio is
\[ S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.8186}{1 - 0.8186} = 12.025 \]

SOL 7.2.24 Option (A) is correct.
The general expression for phasor form of electric field vector is
\[ E = E_0 e^{-j(\beta x + \beta y + \beta z)} \]
Comparing the given field with this expression we get,
\[ \beta_x + \beta_y + \beta_z = 0.01\pi(-3x + \sqrt{3}y - 2z) \]
So, the propagation vector is
\[ k = \sqrt{(\beta_x + \beta_y + \beta_z)} = 0.01\pi(-3a_x + \sqrt{3}a_y - 3a_z) \]
Therefore, the direction of the propagation of the wave is
\[ a_x = \frac{k}{k} = \frac{-3a_x + \sqrt{3}a_y - 2a_z}{\sqrt{9 + 3 + 4}} = \frac{1}{4}(-5a_x + \sqrt{2}a_y - 2a_z) \]

SOL 7.2.25 Option (C) is correct.
As calculated in previous question we have the propagation vector from the given data as
\[ k = 0.04\pi(-2a_x - 3a_y + \sqrt{3}a_z) \]
and the direction of wave propagation is
\[ a_x = \frac{k}{k} = \frac{0.04\pi(-2a_x - 3a_y + \sqrt{3}a_z)}{0.04\pi(-2a_x - 3a_y + \sqrt{3}a_z)} = \frac{-2a_x - 3a_y + \sqrt{3}a_z}{4} \]
Therefore, the phase constant along the direction of propagation is
\[ \beta = k \cdot a_x = \left[0.04\pi(-2a_x - 3a_y + \sqrt{3}a_z)\right] \cdot \left(\frac{-2a_x - 3a_y + \sqrt{3}a_z}{4}\right) = 0.16\pi \]
So, the wavelength along the direction of wave propagation is
\[ \lambda = \frac{2\pi}{\beta} = 25.5 \text{ m} \]

**SOL 7.2.26** Option (C) is correct

From the given expression of magnetic field vector we get,
\[ \beta_x x + \beta_y y + \beta_z z = 0.04\pi(\sqrt{3} x - 2y - 3z) \]

So, the propagation vector of the plane wave is
\[ \mathbf{k} = \nabla \left( \beta_x x + \beta_y y + \beta_z z \right) = 0.04\pi(\sqrt{3} a_x - 2a_y - 3a_z) \]

and the direction of wave propagation is
\[ \mathbf{a}_k = \frac{\mathbf{k}}{\beta} = \frac{0.04\pi(\sqrt{3} a_x - 2a_y - 3a_z)}{(\sqrt{3} a_x - 2a_y - 3a_z)} \]

Therefore, the phase constant along the direction of wave propagation is
\[ \beta = \mathbf{k} \cdot \mathbf{a}_k = 0.16\pi \]

Since the wave is propagating in free space so it’s phase velocity will be
\[ v_p = 3 \times 10^8 \text{ m/s} \]
or,
\[ \frac{\omega}{\beta} = 3 \times 10^8 \]

So, the frequency of the plane wave is
\[ f = \frac{(3 \times 10^8)(0.16\pi)}{2\pi} = 2.4 \times 10^7 \text{ Hz} \]

\[ (\omega = 2\pi f) \]

**SOL 7.2.27** Option (D) is correct.

From the given expression of the field vector, we have the propagation vector,
\[ \mathbf{k} = \frac{\pi}{25}(\sqrt{3} a_x - 2a_y - 3a_z) \]

So the phase constants along \( x \), \( y \) and \( z \)-axes are
\[ \beta_x = \frac{\sqrt{3} \pi}{25}, \beta_y = -\frac{2\pi}{25}, \beta_z = -\frac{3\pi}{25} \]

Therefore, the apparent wave lengths along the three axes are
\[ \lambda_x = \frac{2\pi}{\beta_x} = \frac{2\pi}{\sqrt{3} \frac{\pi}{25}} = \frac{50}{\sqrt{3}} = 28.87 \text{ m} \]
\[ \lambda_y = \frac{2\pi}{|\beta_y|} = \frac{2\pi}{\frac{2\pi}{25}} = +25 \text{ m} \]
\[ \lambda_z = \frac{2\pi}{|\beta_z|} = \frac{2\pi}{\frac{3\pi}{25}} = +50 \text{ m} = 26.7 \text{ m} \]

**SOL 7.2.28** Option (A) is correct.

As determined in previous question, the propagation vector of the plane wave is
Therefore, the direction of wave propagation is

\[ \mathbf{a}_k = \frac{k}{k} = \frac{\sqrt{3} \mathbf{a}_x - 2 \mathbf{a}_y - 3 \mathbf{a}_z}{4} \]

So the phase constant along the direction of wave propagation is

\[ \beta = \mathbf{k} \cdot \mathbf{a}_k = 0.16\pi \]

Therefore, the angular frequency of the propagating wave is

\[ \omega = \nu_p \beta = (3 \times 10^8) \times (0.16\pi) \text{ (In free space } \nu_p = 3 \times 10^8 \text{ m/s)} \]

\[ = 1.51 \times 10^8 \text{ rad/sec} \]

So, for the determined values of apparent phase constants in previous question, the apparent phase velocities are given as

\[ v_{px} = \frac{\omega}{\beta_x} = \frac{1.51 \times 10^8}{\left(\frac{\sqrt{3} \pi}{25}\right)} = 6.93 \times 10^8 \text{ m/s} \]

\[ v_{py} = \frac{\omega}{\beta_y} = \frac{1.51 \times 10^8}{\left(\frac{2\pi}{25}\right)} = 6 \times 10^8 \text{ m/s} \]

and

\[ v_{pz} = \frac{\omega}{\beta_z} = \frac{1.51 \times 10^8}{\left(\frac{2\pi}{25}\right)} = 4 \times 10^8 \text{ m/s} \]

**SOL 7.2.29**

Option (D) is correct.

The necessary condition for the vector field \( \mathbf{E} = E_0 e^{-j\beta} \) to represent the electric field intensity of a uniform plane wave is

\[ \mathbf{k} \cdot \mathbf{E}_0 = 0 \]

where \( \mathbf{k} \) is the propagation vector of the wave and \( E_0 \) is the amplitude of the electric field intensity of the plane wave. Now, we check all the given options for this condition.

(A) From given data we have

\[ \mathbf{k} = \sqrt{3} \mathbf{a}_x + \mathbf{a}_z \]

\[ E_0 = j \mathbf{a}_x - 2 \mathbf{a}_y + j\sqrt{3} \mathbf{a}_z \]

So,

\[ \mathbf{k} \cdot \mathbf{E}_0 = -2\sqrt{3} + j\sqrt{3} \neq 0 \]

(B) From given data we have

\[ E_0 = \mathbf{a}_x - j2\mathbf{a}_y - \sqrt{3} \mathbf{a}_z \]

\[ \mathbf{k} = \mathbf{a}_x + \sqrt{3} \mathbf{a}_z \]

So,

\[ \mathbf{k} \cdot \mathbf{E}_0 = 1 - 3 \neq 0 \]

(C) From given data we have

\[ E_0 = (\sqrt{3} + j\frac{\sqrt{3}}{2}) \mathbf{a}_x + (1 + j\frac{\sqrt{3}}{2}) \mathbf{a}_y - j\sqrt{3} \mathbf{a}_z \]

\[ \mathbf{k} = j\sqrt{3} \mathbf{a}_x + 3 \mathbf{a}_y + 2 \mathbf{a}_z \]

So,

\[ \mathbf{k} \cdot \mathbf{E}_0 = 3 + j\sqrt{3} + 3 + \frac{3j\sqrt{3}}{2} - j2\sqrt{3} \neq 0 \]

(D) From given data we have
\[ E_0 = \left(-\sqrt{3} - j\frac{1}{2}\right)a_x + \left(1 - j\sqrt{3}\right)a_y + j\sqrt{3}a_z \]

\[ k = \sqrt{3}a_x + 3a_y + 2a_z \]

So,
\[ k \cdot E_0 = -3 - j\frac{3\sqrt{3}}{2} + 3 - j\frac{3\sqrt{3}}{2} + j2\sqrt{3} = 0 \]

So the vector represents electric field vector of a uniform plane wave.

**SOL 7.2.30**

Option (D) is correct.

For the field vectors \( E_s \) and \( H_s \) defined as
\[ E_s = E_0 e^{-j\beta} \]
\[ H_s = H_0 e^{-j\beta} \]

The condition that it represents the field vectors of a uniform plane wave is
\[ E_0 \cdot H_0 = 0, \quad E_0 \cdot k = 0 \text{ and } H_0 \cdot k = 0 \]

where \( k \) is the propagation vector of the plane wave.

Now, we check the all given pairs for this condition

In Option (D)
\[ E_0 = -ja_x - 2a_y + j\sqrt{3}a_z \]
\[ H_0 = a_x - 2a_y - \sqrt{3}a_z \]

and
\[ k = \sqrt{3}a_x + a_z \]

So
\[ E_0 \cdot H_0 = -j + j4 - \beta = 0 \]
\[ E_0 \cdot k = -j\sqrt{3} + j\sqrt{3} = 0 \]
\[ H_0 \cdot k = \sqrt{3} - \sqrt{3} = 0 \]

Therefore, it represents the field vectors of a uniform plane wave.

**SOL 7.2.31**

Option (B) is correct.

For a propagating electromagnetic wave, the field satisfies the following Maxweell’s equation.
\[ \nabla \cdot E = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \neq 0 \]

Now, we check the condition for the given fields as below.
\[ P = 60\sin(\omega t + 10x)a_z \]

So,
\[ \nabla \cdot P = 0 \]

and
\[ \nabla \times P = -600\cos(\omega t + 10x)a_y \neq 0 \]

i.e. \( P \) is a possible EM field.

again,
\[ Q = \frac{10}{\rho}\cos(\omega t - 2\rho)a_\phi \]

So,
\[ \nabla \cdot Q = 0 \]

and
\[ \nabla \times Q = \frac{1}{\rho \frac{\partial}{\partial \rho}}[10\cos(\omega t - 2\rho)]a_\phi \neq 0 \]

i.e. \( Q \) is a possible EM field
\[ R = 3\rho^2 \cot \phi a_\rho + \frac{1}{\rho} \cos \phi a_\phi \]

So,
\[ \nabla \cdot R = \frac{1}{\rho \frac{\partial}{\partial \rho}}(3\rho^2 \cot \phi)\frac{\sin \phi}{\rho} \neq 0 \]
i.e. $R$ is not a possible EM field.

$$S = \frac{1}{r} \sin \theta \sin (\omega t - 6r) a_\phi$$

So,

$$\nabla \cdot S = \frac{1}{r^2 \sin \theta} \sin (\omega t - 6r) \frac{\partial (\sin^2 \phi)}{\partial r} \neq 0$$

i.e. $S$ is not an EM field.

Thus, the possible EM fields are $P$ and $Q$.

**SOL 7.2.32**

Option ( ) is correct.

Since, 20% of the energy in the incident wave is reflected at the boundary. So, we have,

$$|\Gamma|^2 = \frac{20}{100}$$

or,

$$|\Gamma| = \sqrt{0.2} = \pm 0.447$$

Where $\Gamma$ is the reflection coefficient at the medium interface. Therefore, we have

$$\eta_b - \eta_i \over \eta_b + \eta_i = \pm 0.447$$

$$\eta_b \sqrt{\frac{\mu_2}{\varepsilon_{r_2}} - \eta_b} \sqrt{\frac{\mu_1}{\varepsilon_{r_1}}} = \pm 0.447$$

$$\eta_b \sqrt{\frac{\mu_2}{\varepsilon_{r_2}}} + \eta_b \sqrt{\frac{\mu_1}{\varepsilon_{r_1}}} = \pm 0.447$$

$$\sqrt{\frac{\mu_2}{\mu_1}} + \sqrt{\frac{\mu_1}{\mu_2}} = \pm 0.447$$

$$\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} = \pm 0.447$$

$$\frac{\mu_1}{\mu_2} = \frac{1 \pm 0.447}{1 + 0.447}$$

$$\frac{\mu_1}{\mu_2} = 2.62 \text{ or } 0.38$$

So,

$$\varepsilon_{r_2} = \left( \frac{\mu_2}{\mu_1} \right) = 0.056 \text{ or } 17.9$$

**********
SOL 7.3.1 Option (C) is correct.

Electric field of the propagating wave in free space is given as

\[ E_i = (8a_x + 6a_y + 5a_z) \, e^{j(\omega t + 3x - 4y)} \, \text{V/m} \]

So, it is clear that wave is propagating in the direction \((-3a_x + 4a_y)\).

Since, the wave is incident on a perfectly conducting slab at \(x = 0\). So, the reflection coefficient will be equal to \(-1\).

i.e.

\[ E_r = (-1) \, E_i = -8a_x - 6a_y - 5a_z \]

Again, the reflected wave will be as shown in figure below:

```
  3a_x + 4a_y
  `-\-------------------`
  Conducting slab
  `-\-------------------`
        Free space
              \(x = 0\)

  -3a_x + 4a_y
```

i.e. the reflected wave will be along the direction \(3a_x + 4a_y\). Thus, the electric field of the reflected wave will be

\[ E_r = (-8a_x - 6a_y - 5a_z) \, e^{j(\omega t - 3x + 4y)} \, \text{V/m} \]

SOL 7.3.2 Option (B) is correct.

Given, the electric field intensity of the EM wave as

\[ E = 10(a_y + ja_x) \, e^{-j25z} \]

So, we conclude that the wave is propagating in \(a_x\) direction and the \(y\) and \(z\)-components of the field are same. Therefore, the wave is circularly polarized.

Now, the angle formed by the electric field with the \(z\)-axis is given as

\[ \theta = \omega t \]

So, with increase in time the tip of the field magnitude rotates from \(z\) to \(y\)-axis and as the wave is propagating in \(a_x\) direction so, we conclude that the wave is left circular (i.e., left circular polarization).

The phase constant of the field is given as

\[ \beta = \frac{\omega}{c} \]

\[ 25 = \frac{2\pi f}{c} \]  \((\beta = 25)\)
SOL 7.3.3 Option (C) is correct. 
Power radiated from any source is constant.

SOL 7.3.4 Option (C) is correct. 
Intrinsic impedance of EM wave 
\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{120\pi}{2} = 30\pi \]
Time average power density of the EM wave is given as 
\[ P_{\text{ave}} = \frac{1}{2} EH = \frac{1}{2} \frac{E^2}{\eta} = \frac{1}{2} \frac{1}{60\pi} = \frac{1}{120\pi} \quad (E = 1 \text{ V/m}) \]

SOL 7.3.5 Option (B) is correct. 
In the given problem

\[ \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \]
\[ \eta_1 = \sqrt{\frac{\mu_0}{\varepsilon}} = 120\pi \]
\[ \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 40\pi \]
\[ \eta_1 = \frac{1}{3} \]
\[ \eta_2 = \frac{40\pi}{120\pi} = \frac{1}{3} \]

Reflection coefficient at the medium interface is given as 
\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -\frac{1}{2} \]
As, given the electric field component of the incident wave is 
\[ E_i = 24\cos(3 \times 10^8 - \beta y)\hat{a}_x \]
So, we conclude that the incident wave is propagating along \( \hat{a}_x \) direction and the angular frequency of the wave is 
\[ \omega = 3 \times 10^8 \text{ rad/s} \]
So, the phase constant of the wave is given as 
\[ \beta = \frac{\omega}{c} = \frac{3 \times 10^8}{3 \times 10^8} = 1 \]
Therefore, the reflected wave will be propagating in \( -\hat{a}_x \) direction and its electric field component is given as 
\[ E_r = \Gamma E_\theta \cos(3 \times 10^8 + y) \quad (\beta = 1 \text{ rad/m}) \]
where \( E_\theta \) is the maximum value of the field component of incident wave.
i.e., 
\[ E_\theta = 24\hat{a}_x \]
So, we have 
\[ E_\theta = -\frac{1}{2} \left[ 24 \cos(3 \times 10^8 + y)\hat{a}_x \right] \]
\[ = -12 \cos(3 \times 10^8 + y)\hat{a}_x \]
Therefore, the magnetic field component of the reflected wave is given as
\[
H_r = \frac{1}{\eta} (a_y \times E_r)
\]
where \(\eta\) is the intrinsic impedance of medium 1, and \(a_y\) is the unit vector in the direction of wave propagation. So, we get
\[
H_r = \frac{1}{120\pi} \left[ a_y \times (-12 \cos(3 \times 10^8 + y)a_x) \right]
\]
\[
= \frac{1}{5\pi} \cos(3 \times 10^8 + y)a_x
\]

**SOL 7.3.6** Option (C) is correct.
The intrinsic impedance of the wave is defined as
\[
\eta = \sqrt{\mu \over \varepsilon}
\]
where \(\mu\) is permeability and \(\varepsilon\) is permittivity of the medium.
Now, the reflection coefficient at the medium interface is given as
\[
\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}
\]
Substituting values for \(\eta_1\) and \(\eta_2\) we have
\[
\tau = \frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = 1 - \sqrt{9} = \frac{1 - \sqrt{9}}{1 + \sqrt{9}}
\]
(\(\varepsilon_r = 9\))
or,
\[
|\Gamma| = 0.5
\]

**SOL 7.3.7** Option (B) is correct.
Since, the wave is propagating in a direction making an angle 90° with positive \(y\)-axis. So, the \(y\)-component of propagation constant will be zero. As the direction of propagation makes an angle 30° with positive \(x\)-axis so, we have the propagation constant of the wave as
\[
\gamma = \beta \cos 30^\circ x \pm \beta \sin 30^\circ y
\]
where \(\beta\) is the phase constant of the wave. So, we get
\[
\gamma = \frac{2\pi}{\lambda} \sqrt{3} x \pm \frac{2\pi}{\lambda} \frac{1}{2} y = \frac{\pi}{\lambda} \sqrt{3} x \pm \frac{\pi}{\lambda} y
\]
Now, in all the given options the direction of electric field of the wave is given along \(a_y\). So, considering that direction we get the field intensity of the wave as
\[
E = a_y E_0 e^{j(\omega t - \gamma)} = a_y E_0 e^{j\left(\omega t - \left(\frac{\pi\sqrt{3} x \pm \pi}{\lambda}\right)\right)}
\]

**SOL 7.3.8** Option (D) is correct.
Since, the given field intensity have components in \(a_x\) and \(a_y\) direction so, the magnitude of the field intensity of the plane wave is
\[
|H| = H_x^2 + H_y^2 = \left(\frac{5\sqrt{3}}{\eta_0}\right)^2 + \left(\frac{5}{\eta_0}\right)^2 = \left(\frac{10}{\eta_0}\right)^2
\]
So, the time average power density of the EM wave is given as
\[
P_{ave} = \frac{\eta_0 |H|^2}{2} = \frac{\eta_0 (\frac{10}{\eta_0})^2}{2} = \frac{50}{\eta_0} \text{ watts}
\]
Option (D) is correct.
The Brewster angle is given as
\[ \tan \theta_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \]
\[ \tan 60^\circ = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \]
or
\[ \varepsilon_{r_2} = 3 \]

Option (C) is correct.
Given, the electric field intensity of the propagating wave
\[ E = a_x \sin(\omega t - \beta z) + a_y \sin(\omega t - \beta z + \pi/2) \]
So, we conclude that the wave is propagating along \( \mathbf{a}_x \) direction and the field components along \( \mathbf{a}_x \) and \( \mathbf{a}_y \) are equal.
\[ E_x = E_y \]
Therefore, the wave is circularly polarized. Now we will determine the field is either right circular or left circular. The angle between the electric field \( \mathbf{E} \) and \( \mathbf{x} \)-axis is given as
\[ \theta = \tan^{-1}\left(\frac{\cos \omega t}{\sin \omega t}\right) = \frac{\pi}{2} - \omega t \]
So, with increase in time the tip of the field intensity moves from \( y \) to \( x \)-axis and as the wave is propagating in \( \mathbf{a}_z \) direction therefore, the wave is left hand circularly polarized.

Option (B) is correct.
The reflection coefficient at the medium interface is given as
\[ \Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sqrt{\frac{\mu_2}{\mu_1}}} = \frac{1 + \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} = -\frac{1}{3} \]
So, the transmitted power is
\[ P_t = (1 - |\Gamma|^2) P_i \]
\[ P_t = (1 - \frac{1}{9}) P_i = \frac{8}{9} P_i \]
or,
\[ \frac{P_t}{P_i} = \frac{8}{9} \]

Option (D) is correct.
\[ \sin \theta = \frac{1}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{2}} \]
or
\[ \theta = 45^\circ = \frac{\pi}{4} \]
The configuration is shown below. Here \( A \) is point source.
Now \( AO = 1 \text{ m} \)
From geometry \( BO = 1 \text{ m} \)
Thus, \( \text{area} = \pi r^2 = \pi \times OB = \pi \text{ m}^2 \)

SOL 7.3.13 Option (C) is correct.
Given, the electric field of the EM wave in medium 1 as \( \mathbf{E}_1 = 4a_x + 3a_y + 5a_z \)
As the medium interface lies in the plane \( x = 0 \) so, the tangential and normal components of the electric field are \( \mathbf{E}_{1t} = 3a_y + 5a_z \)
and \( \mathbf{E}_{1n} = 4a_x \)
Now, from the boundary condition we know that the tangential component of electric field is uniform. So, we get \( \mathbf{E}_{2t} = \mathbf{E}_{1t} = 3a_y + 5a_z \)
Again from the boundary condition the normal component of displacement vector are equal.
\[ \begin{align*}
D_{2n} &= D_{1n} \\
\varepsilon_2 E_{2n} &= \varepsilon_1 E_{1n} \\
4\varepsilon_2 E_{2n} &= 3\varepsilon_1 4a_x \\
E_{2n} &= 3a_x
\end{align*} \]
Thus, the net electric field intensity in medium 2 is \( \mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n} = 3a_y + 3a_y + 5a_z \)

SOL 7.3.14 Option (C) is correct.
From the expression of the magnetic field intensity of the EM wave, we have
Angular frequency, \( \omega = 50,000 \)
Phase constant, \( \beta = 0.004 \)
So, the phase constant of the wave is given as \( v_p = \frac{\omega}{\beta} = \frac{5 \times 10^4}{4 \times 10^{-2}} = 1.25 \times 10^7 \text{ m/s} \)

SOL 7.3.15 Option (C) is correct.
Refractive index of glass \( n_g = \sqrt{\mu_r \varepsilon_r} = 1.5 \)
Frequency \( f = 10^{14} \text{ Hz} \)
\( c = 3 \times 10^8 \text{ m/sec} \)
The wavelength of the 10^{14} Hz beam of light is \( \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^{14}} = 3 \times 10^{-6} \)
So, wavelength of the light beam in glass is given as \( \lambda_g = \frac{\lambda}{n_g} = \frac{3 \times 10^{-6}}{1.5} = 2 \times 10^{-6} \text{ m} \)

SOL 7.3.16 Option (B) is correct.
The time average poynting vector of the EM wave is defined as \( \mathbf{P}_{ave} = \frac{1}{2} \text{Re} \left[ \mathbf{E} \times \mathbf{H}^* \right] \)
where, $E_s$ is the phasor form of the electric field intensity and $H_s^*$ is the complex conjugate of the phasor form of magnetic field intensity. So, we have

\[ E_s \times H_s^* = (a_x + ja_y) e^{jkz-j\omega t} \times \frac{k}{\omega \mu} (-ja_x + a_y) e^{-jkz+j\omega t} \]

\[ = a_x \left[ \frac{k}{\omega \mu} - (-j) \frac{k}{\omega \mu} \right] = 0 \]

Thus,

\[ P_{ave} = \frac{1}{2} \text{Re} [E_s \times H_s^*] = 0 \]

**SOL 7.3.17** Option (D) is correct.

We have

\[ \text{VSWR} = \frac{E_{max}}{E_{min}} = 5 = \frac{1 - |\Gamma|}{1 + |\Gamma|} \]

or

\[ |\Gamma| = \frac{2}{3} \]

As the wave is normally incident on the interface so, the reflection coefficient will be real (either positive or negative). Now, for a wave propagating from medium 1 to medium 2 having permittivities $\varepsilon_1$ and $\varepsilon_2$ respectively.

(i) If $\varepsilon_2 > \varepsilon_1$, the reflection coefficient is negative

(ii) If $\varepsilon_2 < \varepsilon_1$ then, the reflection coefficient is positive.

Since, the given EM wave is propagating from free space to the dielectric material with $\varepsilon > \varepsilon_0$, therefore

\[ \Gamma = -\frac{2}{3} \]

or,

\[ \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{2}{3} \]

or,

\[ \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi} = -\frac{2}{3} \]

So,

\[ \eta_2 = 24\pi \]

**SOL 7.3.18** Option (A) is correct.

The skin depth ($\delta$) of a material is related to the operating frequency ($f$) as

\[ \delta \propto \frac{1}{\sqrt{f}} \]

Therefore,

\[ \delta_x \propto \frac{1}{\sqrt{f}} \]

\[ \delta_2 = \sqrt{\frac{1}{4}} \]

or

\[ \delta_2 = \sqrt{\frac{1}{4} \times 25} = 25 \text{ cm} \]

**SOL 7.3.19** Option (D) is correct.

The intrinsic impedance of a medium with permittivity $\varepsilon$ and permeability $\mu$ is defined as

\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} \]

So, the reflection coefficient at the boundary interface of the two mediums is given as

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_r} - \sqrt{\varepsilon_r}}{\sqrt{\varepsilon_r} + \sqrt{\varepsilon_r}} \]
\[ = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \frac{1 - \sqrt{4}}{1 + \sqrt{4}} \quad \text{since } \varepsilon_r = 4 \]
\[ = -\frac{1}{3} = 2.333/180^\circ \]

SOL 7.3.20
Option (A) is correct.
We have \( E(z,t) = 10 \cos(2\pi \times 10^7 t - 0.1z) \)
So, we get \( \omega = 2\pi \times 10^7 \)
\( \beta = 0.1\pi \)
Therefore, the phase velocity of the wave is given as
\[ v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{0.1\pi} = 2 \times 10^8 \text{ m/s} \]

SOL 7.3.21
Option (C) is correct.
We have \( E = (0.5a_x + a_y e^{j\tau}) e^{j(\omega t - kz)} \)
So, its components along \( x \) and \( y \)-axis are
\[ |E_x| = 0.5 e^{j(\omega t - kz)} \]
and
\[ |E_y| = e^{j\theta} e^{j(\omega t - kz)} \]
i.e.
\[ |E_x| \neq |E_y| \]
Since, the components are not equal and have the phase difference of \( \pi/2 \) so, we conclude that the EM wave is elliptically polarized.

SOL 7.3.22
Option (B) is correct.
Loss tangent of a medium is defined as
\[ \tan \delta = \frac{\sigma}{\omega \varepsilon} \]
where \( \sigma \) is the conductivity \( \varepsilon \) is permittivity of the medium and \( \omega \) is operating angular frequency. So, we get
\[ \tan \delta = \frac{1.7 \times 10^{-4}}{2\pi \times 3 \times 10^9 \times 78\varepsilon_0} = \frac{1.7 \times 10^{-4} \times 9 \times 10^9}{3 \times 10^9 \times 39} = 2.3 \times 10^{-5} \]

SOL 7.3.23
Option (A) is correct.
We have \( \frac{\partial^2 E_x}{\partial Z^2} = c^2 \frac{\partial^2 E_x}{\partial t^2} \)
As the field component \( E_x \) changes with \( z \) so, we conclude that the EM wave is propagating in \( z \)- direction.

SOL 7.3.24
Option (B) is correct.
The required condition is
\[ |I_c| = |I_d| \]
i.e. the conduction current equals to the displacement current. So, we get
\[ |J_c| = |J_d| \]
Chap 7 Electronagnetics Waves

SOL 7.3.25
Option (A) is correct.
VSWR (voltage standing wave ratio) of the transmission line is defined as
\[ S = \frac{1 + |\Gamma|}{1 - |\Gamma|} \]
where \( \Gamma \) is the reflection coefficient of the transmission line. So, we get
\[ 3 = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (\text{VSWR} = 3) \]
or
\[ |\Gamma| = 0.5 \]
Therefore, the ratio of the reflected power strength to the incident power is given as
\[ \frac{P_r}{P_i} = |\Gamma| = 0.25 \]
Thus, 25% of incident power is reflected.

SOL 7.3.26
Option (C) is correct.
The fig is as shown below:

As per snell law
\[ \frac{\sin \theta_i}{\sin \theta_q} = \frac{1}{\sqrt{\varepsilon_r}} \]
or
\[ \sin 30^\circ = \frac{1}{\sqrt{\varepsilon_r}} \]
\[ \frac{1}{\sqrt{\varepsilon_r}} = \frac{1}{\sqrt{\varepsilon_r}} \]
or
\[ \varepsilon_r = 2 \]

SOL 7.3.27
Option (A) is correct.
Since, the phase constant is defined as
\[ \beta = \frac{2\pi}{\lambda} = \omega \sqrt{\mu \varepsilon} \]
So, the wavelength in terms of permittivity of the medium can be given as
\[ \lambda = \frac{2\pi}{\omega \sqrt{\mu \varepsilon}} \]
or, \[ \lambda \propto \frac{1}{\sqrt{\varepsilon}} \]

So, we get \[ \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \]

**SOL 7.3.28** Option (C) is correct.

A scalar wave equation must satisfy following relation
\[ \frac{\partial^2 E}{\partial t^2} - v_p^2 \frac{\partial^2 E}{\partial z^2} = 0 \]

where \[ v_p = \frac{\omega}{\beta} \]

(Phase velocity of the wave)

Basically \( \omega \) is the multiply factor of frequency, \( f \) and \( \beta \) is multiply factor of \( z \) or \( x \) or \( y \).

So, we can conclude that expression given in option (C) does not satisfy equation (1) (i.e. the wave equation).

**SOL 7.3.29** Option (D) is correct.

In a lossless dielectric \((\sigma = 0)\) medium, impedance is given by
\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} \]

where \( \mu \) is permeability and \( \varepsilon \) is permittivity of the medium. So, we get
\[ \eta = \sqrt{\frac{\mu_0 \mu}{\varepsilon_0 \varepsilon_r}} = 120\pi \times \frac{\mu_r}{\varepsilon_r} \]
\[ = 120\pi \times \sqrt{\frac{9}{8}} = 288.4 \Omega \]

**SOL 7.3.30** Option (D) is correct.

Intrinsic impedance of a medium is given as
\[ \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \]

Since, copper is good conductor i.e. \( \sigma >> \omega\varepsilon \) so, we get
\[ \eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma} / 45^\circ} \]

Thus, the impedance will be complex with an inductive component.

**SOL 7.3.31** Option (B) is correct.

Given, the electric field intensity of the EM wave as
\[ \mathbf{E} = 24e^{j(\omega t + \beta z)} a_y \text{ V/m} \]

Now, the time average poynting vector for the EM wave is defined as
\[ \mathbf{P} = \frac{1}{2} (\mathbf{E}_x \times \mathbf{H}_y^* = \frac{\mathbf{E}_x}{2\eta} \mathbf{a}_k \]

where \( \eta \) is the intrinsic impedance of the medium and \( \mathbf{a}_k \) is the direction of wave propagation. Since, from the given expression of the field intensity we conclude that the wave is propagating along \(- \mathbf{a}_z\) So, we have
\[ \mathbf{P} = \frac{(24)^2}{2 \times 120\pi} (- \mathbf{a}_z) = -\frac{24}{\pi} \mathbf{a}_z \]
\( (\mathbf{a}_k = - \mathbf{a}_z, |\mathbf{E}| = 24 \text{ V/m}) \)
SOL 7.3.32 Option (A) is correct.
Given the propagation constant of the wave
\[ \gamma = \alpha + j\beta = 0.1\pi + j0.2\pi \]
So, we get
\[ \beta = 0.2\pi \]
or,
\[ \frac{2\pi}{\lambda} = 0.2\pi \]
Therefore, wavelength of the propagating wave is
\[ \lambda = \frac{2}{0.2} = 5 \text{ m} \]

SOL 7.3.33 Option (C) is correct.
The depth of penetration or skin depth is defined as
\[ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \]
i.e.
\[ \delta \propto \frac{1}{\sqrt{f}} \]
or,
\[ \delta \propto \frac{1}{\sqrt{\lambda}} \]  \( (\lambda = c/f) \)
So, the depth of penetration (skin depth) increases with increase in wavelength.

SOL 7.3.34 Option (B) is correct.
Given, the electric field intensity of the wave
\[ E(z, t) = E_0 e^{j(\omega t + \beta z)} a_x + e_0 e^{j(\omega t + \phi)} a_y \]  \( \cdots (1) \)
Generalizing
\[ E(z) = a_x E_1(z) + a_y E_2(z) \]  \( \cdots (2) \)
Comparing (1) and (2) we can see that \( E_1(z) \) and \( E_2(z) \) are in space quadrature but in time phase so, their sum \( E \) will be linearly polarized along a line that makes an angle \( \phi \) with \( x \)-axis as shown below.

SOL 7.3.35 Option (B) is correct.
Skin depth of the conducting medium at frequency, \( f_1 = 10 \text{ MHz} \) is given as
\[ \delta = \frac{1}{\sqrt{\pi f_1 \mu \sigma}} \]
or
\[ \frac{10^{-8}}{\sqrt{\pi \times 10 \times 10^6 \times \mu \sigma}} = \frac{1}{\sqrt{\pi}} \]  \( (f_1 = 10 \text{ MHz}) \)
or,
\[ \mu \sigma = \frac{10^{-3}}{\pi} \]
Now, phase velocity at another frequency \( f_2 = 1000 \text{ MHz} \) is
\[ v_p = \frac{4\pi f_2}{\mu \sigma} \]
Putting \( \mu \sigma = 10^{-3}/\pi \) in the above expression, we get
\[ v_p = \sqrt{\frac{4 \times \pi \times 1000 \times 10^6 \times \pi}{10^{-3}}} \approx 3 \times 10^6 \text{ m/sec} \]

SOL 7.3.36 Option (C) is correct.
Reflected power \( P_r \) of a plane wave in terms of incident power \( P_i \) is defined as
\[ P_r = |\Gamma|^2 P_i \]  \( \cdots (1) \)
where, \( \Gamma \) is the reflection coefficient at the medium interface given as
\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (2) \]

where \( \eta_1 \) and \( \eta_2 \) are the intrinsic impedance of the two mediums (air and glass) respectively. Since, the refractive index of the glass is 1.5

i.e. \[ n_2 = c\sqrt{\mu_2\varepsilon_2} = 1.5 \quad (3) \]

where
\[ \mu_2 = \mu_0 \quad \text{(Permeability of glass)} \]
\[ \varepsilon_2 = \varepsilon_r\varepsilon_0 \quad \text{(Permittivity of glass)} \]

So, putting these values in equation (3) we get
\[ \sqrt{\varepsilon_r} = 1.5 \]

and
\[ \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{\varepsilon_0} = \frac{\eta_0}{1.5} \]

Therefore, from equation (2) we have
\[ \Gamma = \frac{\eta_0 - \eta_0}{\eta_0 + \eta_0} = \frac{1 - 1.5}{1 + 1.5} = -\frac{1}{5} \quad \text{(for free space } \eta = \eta_0) \]

Thus, from equation (1) the reflected power is given as
\[ P_r = \left( \frac{1}{5} \right)^2 \times P_i \]

or,
\[ P_r \cdot P_i = 4\% \]

**SOL 7.3.37** Option (B) is correct.

Skin depth of a material is defined as
\[ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \]

Putting the given values in the expression, we get
\[ \delta = \frac{1}{\sqrt{3.14 \times 1 \times 10^9 \times 4\pi \times 10^{-7} \times 10^6}} = 15.9 \mu m \]

**SOL 7.3.38** Option (C) is correct.

The energy density in a medium having electric field intensity \( E \) is defined as
\[ w_E = \frac{1}{2}\varepsilon |E|^2 \]

where \( \varepsilon \) is permittivity of the medium.

So, due to the field \( E = 100\sqrt{\pi} \) V/m in free space, the energy density is
\[ w_E = \frac{1}{2}(8.85 \times 10^{-12})(100\sqrt{\pi})^2 \]
\[ = 1.39 \times 10^{-7} \text{ J/m}^3 = 189 \text{ nJ/m}^3 \]

**SOL 7.3.39** Option (C) is correct.

For a uniform plane wave propagating in free space, the fields \( E \) and \( H \) are everywhere normal to the direction of wave propagation \( \mathbf{a}_k \) and their direction are related as
\[ \mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H \]

i.e. the angle between electric field \( (\mathbf{a}_E) \) and magnetic field vector \( (\mathbf{a}_H) \) is always 90°.

**SOL 7.3.40** Option (A) is correct.

The incidence angle of an EM wave for which there is no reflection is called
Brewster’s angle. For the vertically polarized wave (parallel polarized wave) the Brewster angle is defined as

$$\tan \theta_{B||} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

So, for the given dielectric medium we get

$$\tan \theta_{B||} = \sqrt{\frac{9}{4}}$$

or,

$$\theta_{B||} = \tan^{-1}\left(\frac{3}{2}\right)$$

SOL 7.3.41

Option (A) is correct.

Given, the electric field component of the EM wave propagating in free space,

$$E = 10 \cos(10^7t + kz) \hat{a}_y \text{ V/m}$$

The general equation of electric field component of an EM wave propagating in $\hat{a}_z$ direction is given as

$$E = E_0 \cos(\omega t + kz) \hat{a}_z \text{ V/m}$$

So, we conclude that the EM wave is propagating in $\hat{a}_z$ direction.

$$\omega = 10^7 \text{ rad/s}$$

or

$$2\pi f = 10^7$$

$$f = \frac{10^7}{2\pi}$$

So,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^7} \times \frac{2\pi}{2\pi} = 188.5 \text{ m}$$

i.e. wavelength of the wave is

wave amplitude, $E_0 = 10 \text{ V/m}$

wave number,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{188.5} = \frac{1}{30}$$

$$= 0.233 \text{ rad/m}$$

The wave doesn’t attenuate as it travels. So, statement (2) and (3) are correct.

SOL 7.3.42

Option (A) is correct.

The incidence angle of a plane wave for which there is no reflection is called Brewster’s angle. For the parallel polarized wave, Brewster’s angle is given as

$$\tan \theta_{B||} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

where $\varepsilon_1$ and $\varepsilon_2$ are the permittivity of two mediums respectively.

So, for the given parallel polarized plane wave the incidence angle ($\theta_i$) for no reflection is given as

$$\tan \theta_i = \sqrt{\frac{\varepsilon_0}{\varepsilon_1 \varepsilon_0}}$$

or,

$$\theta_i = \tan^{-1}\left(\frac{1}{9}\right)$$

Therefore, the angle $\alpha$ for no reflection is

$$\alpha = 90^\circ - \theta_i = 83.66^\circ$$
SOL 7.3.43 Option (C) is correct.

Given, the characteristic impedance of air,

\[ \eta = 360 \, \Omega \]

\[ E_x = 3 \sin (\omega t - \beta z) \, V/m \]

\[ E_y = 6 \sin (\omega t - \beta z + 75^\circ) \, V/m \]

So, the time average power per unit area is

\[ P_{ave} = \frac{1}{2} \left( \left| E_x \right|^2 + \left| E_y \right|^2 \right) \]

\[ = \frac{1}{2} \times \frac{(3^2 + 6^2)}{360} = 6.25 \times 10^{-2} \, W/m^2 = 82.5 \, mW/m^2 \]

SOL 7.3.44 Option (B) is correct.

Operating frequency \( f = 3 \, GHz = 3 \times 10^9 \, Hz \)

Medium parameters,

\[ \mu = 4\pi \times 10^{-7} \, H/m \]

\[ \varepsilon = 10^{-9} / 36\pi \]

\[ \sigma = 5.8 \times 10^7 \, S/m \]

So, we have intrinsic impedance defined as

\[ |\eta| = \sqrt{\frac{\mu}{\varepsilon}} \times \sqrt{\frac{1}{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}} = \sqrt{\frac{4\pi \times 10^{-7}}{(10^{-9} / 36\pi)^2}} \]

\[ = 2.02 \times 10^2 \, \Omega \]

The phase angle of intrinsic impedance is given as

\[ \theta_\eta = \frac{1}{2} \tan^{-1}\left(\frac{\sigma}{\omega \varepsilon}\right) = \frac{1}{2} \tan^{-1}\left(\frac{5.8 \times 10^7}{2\pi \times 3 \times 10^9 \times \frac{10^{-9}}{36\pi}}\right) \]

\[ = \frac{\pi}{4} \]

So,

\[ \eta = |\eta| e^{i\theta} = 0.22 e^{i\pi/4} \, \Omega \]

SOL 7.3.45 Option (C) is correct.

The Skin depth of a conductor is defined as

\[ \delta = \frac{1}{\sqrt{\pi \mu \sigma}} \]

So, statement 2 and 3 are correct while are incorrect.

SOL 7.3.46 Option (C) is correct.

For circular polarization the two orthogonal field components must have the same magnitude and has a phase difference of 90°.

So, all the three statements are necessary conditions.

SOL 7.3.47 Option (B) is correct.

Velocity of light in any dielectric medium is defined as

\[ v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} \]
where \( c \) is velocity of light in vacuum and \( \varepsilon_r \) is dielectric constant of the medium. Since \( \varepsilon_r > 1 \),

So, \( v < c \)

Therefore, both A and R are true and R is correct explanation of A.

SOL 7.3.48 Option (D) is correct.

Given, the electric field of a plane wave,

\[
E = 50 \sin(10^8 t + 2\pi z) a_y \text{ V/m}
\]

Comparing it with the general expression electric field of a plane wave travelling in \( a_z \) direction given as

\[
E = E_0 \sin(\omega t - \beta z) a_y
\]

We get the direction of propagation of the given plane wave is \( -a_z \).

SOL 7.3.49 Option (D) is correct.

The poynting vector is the instantaneous power flow per unit area in an EM wave and defined as

\[
\mathbf{P} = \mathbf{E} \times \mathbf{H}
\]

So, \( \mathbf{E} \times \mathbf{H} \) is rate of energy flow (power flow) per unit area.

SOL 7.3.50 Option (C) is correct.

Given the electric field,

\[
E = (a_x + j a_y) e^{-j\beta z}
\]

So, it is clear that \( y \)-component of field leads the \( x \)-component by 90° and the wave propagates in \( z \)-direction. The components are same. So, the tip of electric field traverse in circular path in the clockwise direction and wave propagates in \( z \)-direction as shown in figure.

Therefore, it is negative circularly polarized wave or (left hand polarized wave).

SOL 7.3.51 Option (D) is correct.

Consider the reflector is of angle \( \theta = 90^\circ \) for which the incident and reflected wave is shown in figure.
So, it is clear that the incident and reflected wave both makes same angle $\alpha$ with the $x$-axis i.e. reflected wave in same direction.

**SOL 7.3.52** Option (C) is correct.

Poynting vector represents the instantaneous power density vector associated with the EM field at a given point.

i.e. $\mathbf{P} = \mathbf{E} \times \mathbf{H}$

**SOL 7.3.53** Option (A) is correct.

Given, the electric field intensity of the wave in free space,

$$\mathbf{E} = 50\sin(10^7t + kz)\hat{a}_y \text{ V/m}$$

Comparing it with the general expression of electric field defined as

$$\mathbf{E} = E_0\sin(\omega t - \beta z)\hat{a}_y \text{ V/m}$$

We get,

1. The wave propagates in $-\hat{a}_z$ direction along $z$-axis.
2. The wavelength is given as

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \times 2\pi}{10^7} = 188.5 \text{ m}$$

3. Wave number, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{188.5} = 0.233$

4. The wave doesn’t attenuate as it travels.

**SOL 7.3.54** Option (A) is correct.

An electromagnetic wave incident on a conducting medium has the depth of penetration (skin depth) defined as

$$\delta = \frac{1}{\alpha}$$

i.e. inversely proportion to attenuation constant.

**SOL 7.3.55** Option (B) is correct.

Since, after reflection the phase of both $x$ and $y$ components will be reversed so the reflected wave will be also right circularly polarised.

**SOL 7.3.56** Option (C) is correct.

Given,

Electric field intensity of the wave $\mathbf{E} = 10\cos(6\pi \times 10^8 t - bx)\hat{a}_y$
Permeability of medium, \( \mu = \mu_0 \)
Permittivity of medium, \( \varepsilon = 81\varepsilon_0 \)
From the expression of the electric field, we get the angular frequency as
\[
\omega = 6\pi \times 10^8
\]
The phase velocity of the wave is given as
\[
v_p = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \times 81\varepsilon_0}} = \frac{3 \times 10^8}{9} = \frac{10^8}{3} \quad (c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s})
\]
So, the phase constant of the EM wave is
\[
\beta = \frac{\omega}{v_p} = \frac{6\pi \times 10^8}{10^8/3} = 18\pi \text{ rad/m}
\]
**SOL 7.3.57**
Option (B) is correct.

Given, the phase velocity of the plane wave in dielectric is 0.4 times its value in free space
i.e. \( v_p = 0.4c \) \( \text{(1)} \)
Since, the phase velocity of a medium having permittivity \( \varepsilon \) and permeability \( \mu \) is defined as
\[
v_p = \frac{1}{\sqrt{\mu \varepsilon}}
\]
So, putting it in equation (1) we get
\[
\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 0.4c \quad (\mu = \mu_0 \mu_r, \varepsilon = \varepsilon_0) \quad (c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}})
\]
**SOL 7.3.58**
Option (A) is correct.

Given, the electric field in free space,
\[
E(x, t) = 60\cos(\omega t - 2x)\mathbf{a}_y \text{ V/m}
\]
So, we get the magnitude of the electric field as
\[
E_0 = 60
\]
The time average power density in the electric field is given as
\[
\mathcal{P}_{\text{ave}} = \frac{1}{2} \frac{E_0^2}{\eta_0} = \frac{1}{2} \times \frac{(60)^2}{120\pi}
\]
Therefore, the average power through the circular area of radius 4 m is
\[
\mathcal{P}_{\text{ave}} = (\mathcal{P}_{\text{ave}}) \times (\pi r^2)
\]
\[
= \frac{1}{2} \times \frac{(60)^2}{120\pi} \times \pi(4)^2 = 120 \text{ W}
\]
**SOL 7.3.59**
Option (C) is correct.

The gyro frequency is the frequency whose period is equal to the period of revolution of an electron in its circular orbit under the influence of earth’s magnetic field. So, the radio wave at frequency near \( f_g \) is attenuated by the earth’s magnetic field. (Since, there is a resonance phenomena and oscillating electron receive more and more energy from incident wave.)
SOL 7.3.60 Option (C) is correct.

The relation between electric and magnetic field of the reflected, transmitted and incident wave is given below.

\[ E_i = \eta_1 H_i \]
\[ E_r = -\eta_1 H_r \]
\[ E_t = \eta_2 H_i \]

So, (1) and (3) are correct while (2) is incorrect.

SOL 7.3.61 Option (D) is correct.

From snell’s law,

\[ \frac{n_1 \sin \theta_1}{\sqrt{\varepsilon_0 \mu_0 \sin \theta_1}} = \frac{\sqrt{\mu_0} \sin \theta_2}{\sqrt{\mu_0 (2\varepsilon_2) \sin 60^\circ}} = \frac{\sin \theta_2}{\sin 25^\circ} = \sin \theta_2 \]

which is not possible so there will be no transmitted wave.

SOL 7.3.62 Option (C) is correct.

(1) Consider \( E_1 \) is \( x \)-component and \( E_2 \) is \( y \)-component so, when \( E_1 \) and \( E_2 \) will be in same phase. The wave will be linearly polarized. \( (a \rightarrow 1) \)

(2) When \( E_1 \) and \( E_2 \) will have any arbitrary phase difference then it will be elliptically polarized. \( (d \rightarrow 2) \)

(3) When \( E_1 \) leads \( E_2 \) by \( 90^\circ \) then \( \omega t \) increases counter clockwise and so the wave is right circularly polarized. \( (c \rightarrow 3) \)

(4) When \( E_1 \) lags \( E_2 \) by \( 90^\circ \) then the tip of field vector \( E \) will traverse circularly in clockwise direction and left circularly polarized. \( (b \rightarrow 4) \)

SOL 7.3.63 Option (D) is correct.

An incident wave normal to a perfect conductor is completely reflected in the reverse direction. The magnetic field intensity of reflected wave is same as the incident wave whereas the electric field intensity of reflected wave has the \( 180^\circ \) phase difference in comparison to the incident field. \( (\Gamma = -1 \) for conducting surface). 

SOL 7.3.64 Option (B) is correct.

(a) Propagation constant for a perfect conductor is

\[ \gamma = \alpha + j\beta \]

where \( \alpha = \beta = \sqrt{\frac{\omega \mu \sigma}{2}} \) \( a \rightarrow 1 \)

(b) Radiation intensity of an antenna is defined as

\[ U(\theta, \phi) = r^2 P_{ave} \]

\[ = r^2 \left| \frac{E}{2\eta} \right|^2 = \left( \frac{r^2}{2\eta} \right) \left| \frac{E}{f} \right|^2 \]

\( b \rightarrow 2 \)

(c) Wave impedance of an EM wave is defined as
SOL 7.3.65 Option (B) is correct.
Given,
Electric field intensity, \( E = E_x a_x + E_y a_y \)
The direction of wave propagation, \( a_z = a_z \)
So, the magnetic field intensity of the EM wave is given as
\[
H = \frac{a_z}{\eta} \times E
\]
where, \( \eta \) is the intrinsic impedance of the medium. Putting the expression for electric field in equation, we get
\[
H = \frac{a_z}{\eta} \times (E_x a_x + E_y a_y) = \frac{1}{\eta} (E_x a_x - E_y a_y)
\]

SOL 7.3.66 Option (B) is correct.
An EM wave propagating in free space consists of electric and magnetic field intensity both perpendicular to direction of propagation.

SOL 7.3.67 Option (B) is correct.
In a uniform plane wave the field intensities are related as
\[
E = \eta H
\]
where \( \eta \) is intrinsic impedance given as
\[
\eta = \sqrt{\frac{j \omega \mu}{\sigma + j \omega \varepsilon}}
\]
Assume the medium is perfectly dielectric \( (\sigma = 0) \). So, we get
\[
\eta = \sqrt{\frac{\mu}{\varepsilon}}
\]
or,
\[
\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}
\]

SOL 7.3.68 Option (A) is correct.
The higher frequency (microwave) signal is continuously refracted on the ground as shown in figure.

![Diagram](image_url)

This phenomenon is called ducting.

SOL 7.3.69 Option (D) is correct.
Given, the magnetic field intensity of a plane wave,
\[
H = 0.5 e^{-0.1x} \cos(10^5 t - 2x) a_x
\]
The general expression for magnetic field intensity of a plane wave travelling in positive $x$-direction is

$$H = H_0 e^{-\alpha x} \cos(\omega t - \beta x) \mathbf{a}_z$$  \hspace{1cm} (2)

Comparing the equation (1) and (2) we get,

Wave frequency, $\omega = 10^6$ rad/sec

Wavelength, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2} = 3.14$ m

and the wave travels in $+x$-direction.

Since, the magnetic field intensity points toward $a_z$ direction and the wave propagates in $+a_x$ direction. So, direction of electric field intensity will be

$$a_E = -a_k \times a_H = -(a_x \times a_z) = a_y$$

Therefore, the wave is polarized in $a_y$ direction (direction of electric field intensity).

SOL 7.3.70 Option (C) is correct.

Skin depth ($\delta$) is the distance through which the wave amplitude decreases to a factor $e^{-1}$ or $1/e$.

SOL 7.3.71 Option (D) is correct.

From Maxwell’s equation, For a varying magnetic field $B$, the electric field intensity $E$ is defined as

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Since, the magnetic flux density $B$ in terms of magnetic vector potential is given as

$$B = \nabla \times A$$

So, from the two equations we have

$$E = -\frac{\partial A}{\partial t}$$ \hspace{1cm} (For $\nabla V = 0$)

Given,

$$A = a_x A_1 \sin(\omega t - \beta z)$$

So,

$$E = -\frac{\partial}{\partial t}[a_x A_1 \sin(\omega t - \beta z)] = -a_x \omega A_x \cos(\omega t - \beta z)$$

SOL 7.3.72 Option (C) is correct.

The depth of penetration of wave (skin depth) in a lossy dielectric (conductor) is given as

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

So, the skin depth increases when

(1) permeability decreases
(2) conductivity decreases
(3) frequency decreases

Since, the wavelength of the wave is given as

$$\lambda = \frac{v_p}{f} \hspace{1cm} \text{i.e.} \hspace{1cm} \lambda \propto \frac{1}{f}$$

So, as $\lambda$ increases, $f$ decreases and therefore, skin depth increases.
For View Only

SOL 7.3.73 Option (A) is correct.
For a good conductor,
\[ \alpha = \beta = \sqrt{\frac{\pi f \mu \sigma}{\gamma}} \]
Since, the skin depth is defined as
\[ \delta = \frac{1}{\alpha} \]
or,
\[ \delta = \frac{1}{\beta} \quad (\alpha = \beta) \]
Now, the phase constant of the wave is given as
\[ \beta = \frac{2\pi}{\lambda} \]
So, we have \( \delta = \frac{\lambda}{2\pi} \) It is defined for a good conductor.

SOL 7.3.74 Option (B) is correct.
Given, the magnetic field intensity of the wave propagating in free space,
\[ \mathbf{H}(z, t) = -\frac{1}{6\pi} \cos(\omega t + \beta z) \mathbf{a}_y \]
So, we conclude.
direction of propagation, \( \mathbf{a}_k = -\mathbf{a}_z \)
direction of magnetic field, \( \mathbf{a}_H = \mathbf{a}_y \)
So, the direction of electric field intensity is given as
\[ \mathbf{a}_E = \mathbf{a}_H \times \mathbf{a}_E = \mathbf{a}_y \times (-\mathbf{a}_z) = -\mathbf{a}_x \]
and the electric field amplitude is given as,
\[ \mathbf{E} = \eta \mathbf{H} \]
\[ = (120\pi) \left( -\frac{1}{6\pi} \cos(\omega t + \beta z) \right) \]
\[ = -20 \cos(\omega t + \beta z) \]
So, the electric field vector of EM wave is
\[ \mathbf{E}(z, t) = 10 \cos(\omega t + \beta z) \mathbf{a}_x \]

SOL 7.3.75 Option (B) is correct.
Given, the electric field intensity of EM wave in phase form as
\[ \mathbf{E}_z = 10e^{-j\psi} \mathbf{a}_x \]
So, we get
the phase constant, \( \beta = 4 \text{ rad/m} \)
Since, the wave is propagating in free space, therefore, the angular frequency \( \omega \) of the wave is given as
\[ \omega = c \beta = (3 \times 10^8)(4) = 4 \times 3 \times 10^8 \text{ rad/s} \]

SOL 7.3.76 Option (B) is correct.
A and R both true and R is correct explanation of A.

SOL 7.3.77 Option (A) is correct.
Skin depth of a material is defined as
\[ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \]
Since, conductivity of the material is $\sigma = 0$.
So, we get $\delta \rightarrow \infty$

**SOL 7.3.78**
Option (A) is correct.

1. In a conducting medium as the wave travels its amplitude is attenuated by the factor $e^{-\alpha z}$ (i.e. attenuated exponentially).
2. Conducting medium doesn’t behave as on open circuit to the EM field.
3. In lossless dielectric ($\sigma = 0$) relaxation time is defined as

$$T_r = \frac{\varepsilon}{\sigma} \rightarrow \infty$$

4. In charge free region ($\rho_v = 0$). Poissons equation is generalised as

$$\nabla^2 V = -\frac{\rho_e}{\varepsilon}$$

$$\nabla^2 V = 0$$

which is Laplace equation. Therefore only statement 2 is incorrect.

**SOL 7.3.79**
Option (A) is correct.

For a given electric field in free space the average power density is defined as

$$P_{ave} = \frac{1}{2} \left| \frac{\mathbf{E}}{\eta_0} \right|^2 = \frac{1}{2} \left| \frac{60\pi}{120\pi} \right|^2 = 15\pi \text{ Watt/m}^2$$

**SOL 7.3.80**
Option (A) is correct.

Given,

$$\mathbf{E} = 120\pi \cos(\omega t - \beta z) \mathbf{a}_z$$

Since, the wave is propagating in $\mathbf{a}_z$ directions so, the magnetic flux density of the propagating wave is

$$\mathbf{H} = \frac{\mathbf{a}_z \times \mathbf{E}}{\eta_0} = \frac{\mathbf{a}_z \times \left[120\pi \cos(\omega t - \beta z) \mathbf{a}_z\right]}{\eta_0} = \cos(\omega t - \beta z) \mathbf{a}_y \quad (\mathbf{a}_k = \mathbf{a}_z)$$

Therefore, the average power density of an EM wave is defined as

$$P_{ave} = \frac{1}{2} \text{Re}\{\mathbf{E} \times \mathbf{H}^*\} = \frac{1}{2} \left| \left[120\pi \cos(\omega t - \beta z) \mathbf{a}_z\right] \times \left[\cos(\omega t - \beta z) \mathbf{a}_y\right]\right|$$

$$= 60\pi \mathbf{a}_z$$

**SOL 7.3.81**
Option (B) is correct.

Given, the electric field intensity is

$$\mathbf{E} = 10\sin(3\pi \times 10^8 t - \pi z) \mathbf{a}_z + 10\cos(3\pi \times 10^8 t - \pi z) \mathbf{a}_y$$

So, the magnetic field intensity is given as

$$\mathbf{H} = \frac{\mathbf{a}_z \times \mathbf{E}}{\eta_0}$$

(Direction of propagation is $\mathbf{a}_k = \mathbf{a}_z$)

$$= 5\sin(3\pi \times 10^8 t - \pi z) \mathbf{a}_y + \frac{10}{3\pi} \cos(3\pi \times 10^8 t - \pi z)(-\mathbf{a}_z)$$

**SOL 7.3.82**
Option (D) is correct.

1. For a perfect conducting medium the transmission coefficient is zero but a medium having finite conductivity transmission coefficient has some finite value. So it doesn’t behave like an open circuit to the electromagnetic field.
2. Relaxation time in a medium is defined as

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
\[ T_0 = \frac{\xi}{\sigma} \]

Which in turn given the values in the range of \(10^{-20}\) sec. While the radio frequency wave has the time period \(T\) in the range of nsec to psec \((10^{-9} \text{ to } 10^{-12})\). So the relation time at radio frequency/microwave frequency is much less than the period.

(3) For a lossless dielectric \((\sigma = 0)\) and so,

\[ T_r = \frac{\xi}{\sigma} \rightarrow \infty \]

(4) Intrinsic impedance of a perfect dielectric \((\sigma = 0)\) is

\[ \eta = \sqrt{\frac{H}{\xi}} \]

which is a pure resistance.

So, the statement (2), (3) and (4) are correct.

**SOL 7.3.83**

Option (D) is correct.

The polarization of a uniform plane wave described the time varying behaviour of the electric field intensity vector so for polarization the field vector must be transverse to the propagation of wave.

i.e. Transverse nature of electromagnetic wave causes polarization.

**SOL 7.3.84**

Option (B) is correct.

In free space electrons and photon both have the same velocity \(3 \times 10^8 \text{ m/s}\). So, the velocity of electromagnetic waves is same as velocity of light.

So A and R both are true and R is the correct explanation of A.

**SOL 7.3.85**

Option (A) is correct.

Fields are said to be circularly polarized if their components have same magnitudes but they differ in phase by \(\pm 90^\circ\).

**SOL 7.3.86**

Option (C) is correct.

From Maxwell’s equation for an EM field, the divergence of the magnetic flux density is zero.

i.e.

\[ \nabla \cdot B = 0 \]

\[ \nabla \cdot (\nabla \times A) = 0 \]

\[ \text{div curl } A = 0 \]

**SOL 7.3.87**

Option (B) is correct.

Electric field intensity due to the current element is defined as

\[ E = \frac{J}{\sigma} = \frac{1}{\pi b^2 \sigma} a_r \]

The magnetic flux density due to the current element is given as

\[ H = \frac{1}{2\pi b} a_\phi \]

So, the poynting vector of the field is

\[ \mathcal{P} = E \times H \]

\[ = -\frac{I^2}{2\pi b^2 \sigma} a_\rho = -\frac{I^2}{2\sigma \pi b} \hat{i} \]
SOL 7.3.88 Option (D) is correct.
All the three statements are correct.

SOL 7.3.89 Option (C) is correct.
Wavelength of a plane wave in any medium is defined as
\[ \lambda = \frac{v_p}{f} \]
where \( v_p \) = phase velocity
\( f \) = frequency of the wave
Since,
\[ v_p = \frac{c}{\sqrt{\varepsilon_r}} \]
So,
\[ \lambda \propto \frac{1}{\sqrt{\varepsilon_r}} \]
\[ \frac{\lambda_{\text{air}}}{\lambda_{\text{dielectric}}} = \sqrt{\frac{\varepsilon_r_{\text{dielectric}}}{\varepsilon_r_{\text{air}}}} \]
\[ \frac{2}{1} = \sqrt{\frac{\varepsilon_r_{\text{dielectric}}}{\varepsilon_r_{\text{air}}}} \]
\[ \varepsilon_r = 4 \]

SOL 7.3.90 Option (D) is correct.
The velocity of an \( EM \) wave in free space is given as
\[ v_c = C = 3 \times 10^8 \text{ m/s} \]
and the characteristic impedance (intrinsic impedance) is given as
\[ Z_c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \]
So both the terms are independent of frequency of the wave i.e. remain unchanged.

SOL 7.3.91 Option (D) is correct.
Given, electric field intensity
\[ \mathbf{E} = 5 \cos(10^9 t + 30z) \mathbf{a}_x \]
So, we conclude that,
\( \omega = 10^9 \), and \( \beta = 30 \)
and since
\[ \beta = \frac{\omega}{v_p} \]
\[ \beta = \frac{\omega}{\frac{c}{\sqrt{\varepsilon_r}}} \]
\[ \varepsilon_r = \left( \frac{\beta c}{\omega} \right)^2 = \left( \frac{30 \times 3 \times 10^8}{10^9} \right)^2 = 81 \]

SOL 7.3.92 Option (C) is correct.
For attenuation of the wave the medium must have some finite conductivity \( \sigma \). In the given wave equation the term \( \mu \sigma \frac{\partial \mathbf{E}}{\partial t} \) involves \( \sigma \) so this term is responsible for the attenuation of the wave.

SOL 7.3.93 Option (A) is correct.
The statement 1, 3 and 4 are correct while statement 2 is incorrect as Gauss’s law is applicable only for symmetrical geometry.
SOL 7.3.94 Option (D) is correct.
In a good conductor \( \beta = \sqrt{\pi \mu \sigma} \)
So, phase velocity \( v_p = \frac{\omega}{\beta} = 2 \sqrt{\frac{\pi \mu}{\rho \sigma}} \)

SOL 7.3.95 Option (A) is correct.
Given, the electric field intensity of the plane wave is
\[ E(t) = [E_1 \cos \omega t \hat{a}_x - E_2 \sin \omega t \hat{a}_y] e^{-jkz} \]
Since the components of the field are
\[ |E_x| = E_1 \]
and
\[ |E_y| = E_2 \]
i.e. \( |E_x| \neq |E_y| \)
So, the wave is elliptically polarized.

SOL 7.3.96 Option (D) is correct.
For a lossy dielectric, skin depth is defined as
\[ \delta = \frac{\lambda}{2\pi} \]
So, as the wavelength increases the depth of penetration of wave also increases.
i.e. Reason (R) is correct.
The Skin depth is the depth by which electric field strength reduces to \( \frac{1}{e} = 37\% \) of its original value.
i.e. Assertion (A) is false.

SOL 7.3.97 Option (D) is correct.
The electromagnetic equation in terms of vector potential \( \mathbf{A} \) is given as
\[ \nabla^2 \mathbf{A} - \mu_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J} \]

SOL 7.3.98 Option (B) is correct.
The wavelength of an EM wave propagating in a waveguide is defined as
\[ \lambda = \frac{\lambda'}{\sqrt{1 - \left( \frac{f}{f_c} \right)^2}} \]
where \( \lambda' \) is the wavelength of the wave in unbounded medium (free space), \( f_c \) is the cutoff frequency of the waveguide and \( f \) is the operating frequency.
Now, for a propagating wave in the waveguide, the operating frequency is higher than the cutoff frequency.
i.e. \( f > f_c \Rightarrow \frac{f}{f_c} < 1 \)
Putting it in equation (1) we get
\[ \lambda < \lambda' \]
i.e. Wavelength of a propagating wave in a wave guide is smaller than the free space wavelength.

SOL 7.3.99 Option (A) is correct.
For a lossless dielectric medium
\[ \sigma = 0 \]
and propagation constant,
\[ \gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\varepsilon)} \]
\[ \alpha + j\beta = j\omega\sqrt{\mu\varepsilon} \]
\[ \beta = \omega\sqrt{\mu\varepsilon} \quad \text{i.e.} \quad \beta \propto \sqrt{\varepsilon_r} \]

**SOL 7.3.100** Option (C) is correct.

For a lossless medium \((\sigma = 0)\) intrinsic impedance is defined as
\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_r}{\varepsilon_r}} \]
\[ 60\pi = 120\pi \sqrt{\frac{1}{\varepsilon_r}} \]
\[ \varepsilon_r = 4 \]

**SOL 7.3.101** Option (C) is correct.

A field is said to be conservative if the curl of the field is zero.

**SOL 7.3.102** Option (B) is correct.

Given, the magnetic field intensity,
\[ H = 0.5e^{-0.1t}\sin(10^6 t - 2x)a_x \text{ A/m} \]

Comparing it with general expression of magnetic field intensity of wave propagating in \(a_x\) direction given as
\[ H = H_0e^{-\alpha x}\sin(\omega t - \beta x)a_z \]

We get
(i) the direction of wave propagation is \(a_z\)
(ii) \(\alpha = 0.1, \beta = 2\)

So, propagation constant \(\gamma = \alpha + j\beta = 0.1 + j2\)

(iii) phase velocity, \(v_p = \frac{\omega}{\beta} = \frac{10^6}{2} = 5 \times 10^5 \text{ m/s}\)

(iv) \(a_y = a_x, \quad a_z = a_z\)

So, direction of polarization,
\[ a_y = -(a_y \times a_y) = -(a_y \times a_z)a_z \text{ i.e. wave is polarized along } a_y. \]

**SOL 7.3.103** Option (C) is correct.

Skin depth of any conducting medium is defined as
\[ \delta = \frac{1}{\sqrt{\pi f\mu\sigma}} \]

So, at a given frequency \(\omega = 2\pi f\)
\[ \delta \propto \frac{1}{\sqrt{\mu}} \text{ and } \delta \propto \frac{1}{\sqrt{\sigma}} \]

**SOL 7.3.104** Option (B) is correct.

Given, the electric field intensity of the plane wave,
\[ E = 10\sin(10\omega t - \pi z)a_x + 10\cos(\omega t - \pi z)a_y \]

So, the field components are
\[ E_x = 10\sin(10\omega t - \pi z) \]
\[ E_y = 10 \cos(\omega t - \pi z) \]

and since, \[ |E_x| = |E_y| \]

So the polarization is circular.

**SOL 7.3.105** Option (D) is correct.

In free space electric field intensity is defined as

\[ E = -\eta (a_x \times H) \]

where \( a_x \) is unit vector in the direction of propagation.

Given, \[ H = 0.10 \cos(4 \times 10^7 t - \beta z) a_x \text{ A/m} \]

So, the direction of propagation, \( a_x = a_z \)

and we have,

\[ E = -377[ a_x \times (0.10 \cos(4 \times 10^7 t - \beta z) a_x)] \]

\[ = -37.7 \cos(4 \times 10^7 t - \beta z) a_y \]

\( (\eta = 377 \Omega) \)

**SOL 7.3.106** Option (A) is correct.

Given the electric field in medium \( A \)

\[ E = 100 \cos(\omega t - 6\pi x) z \]

In medium \( A \), \( \varepsilon_r = 4, \quad \mu_r = 1, \quad \sigma = 0 \)

In medium \( B \), \( \varepsilon_r = 9, \quad \mu_r = 4, \quad \sigma = 0 \)

So,

(a) intrinsic impedance of medium ‘\( B \)’ is

\[ \eta_B = \sqrt{\frac{\mu_r}{\varepsilon_r}} = \sqrt{\frac{4 \mu_0}{9 \varepsilon_0}} = \frac{3}{2} \times 120\pi = 80\pi \quad (a \rightarrow 2) \]

(b) Intrinsic impedance of medium ‘\( A \)’ is

\[ \eta_A = \sqrt{\frac{\mu_r}{\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{2} \times 120\pi = 60\pi \]

So, reflection coefficient,

\[ \Gamma = \frac{\eta_B - \eta_A}{\eta_B + \eta_A} = \frac{80\pi - 60\pi}{80\pi + 60\pi} = \frac{1}{5} \quad (b \rightarrow 3) \]

(c) Transmission coefficient,

\[ \tau = \frac{2\eta_B}{\eta_B + \eta_A} = \frac{2 \times 80\pi}{80\pi + 60\pi} = \frac{8}{7} \quad (c \rightarrow 4) \]

(d) Phase shift constant of medium \( A \) is given from the field equation as

\[ \beta = 12\pi \quad (d \rightarrow 1) \]

**SOL 7.3.107** Option (A) is correct.

Average power density in an \( EM \) wave is defined as

\[ P_{ave} = \frac{1}{2} \text{Re}(E_x \times H_x) = \frac{1}{2} \times 50 \times \frac{5}{12\pi} = 3.316 \]

So, the average power crossing a circular area of radius \( \sqrt{24} \text{ m} \) is

\[ P_{ave} = P_{ave}(\pi r^2) = (3.316)(\pi(\sqrt{24})^2) = 250 \text{ Watt} \]

**SOL 7.3.108** Option (B) is correct.

Electric field amplitude, \( E_0 = 1 \text{ V/m} \)

Skin depth, \( \delta = 10 \text{ cm} = 0.1 \text{ m} \)

So, the attenuation constant of the wave in the conductor is
Now, the electric field intensity after travelling a distance $z$ inside a conductor is
\[ E = E_0 e^{-\alpha z} \]
where, $E_0$ is the field intensity at the surface of the conductor. So, the distance travelled by the wave for which amplitude of electric field changes to $(1/e)^2$ (V/m) is given as
\[ E = \frac{E_0}{e^2} \]
\[ E_0 e^{-10z} = \frac{E_0}{e^2} \]
\[ 10z = 2 \]
\[ z = 20 \text{ cm} \]
Alternatively, since the skin depth is the distance in which the wave amplitude decays to $(1/e)$ of its value at surface. So, for the amplitude to be $(1/e^2)$ of the field at its surface the wave penetrates a length of $20$ cm. So A and R both are true and R is correct explanation of A.

**SOL 7.3.109** Option (C) is correct.
For any media having conductivity, $\sigma = 0$, the intrinsic impedance is given as
\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r} \eta_0} \]
For media 1, \[ \eta_1 = \sqrt{\frac{2}{8}} (377) = 188 \Omega \]
For media 2, \[ \eta_2 = \sqrt{\frac{9}{377}} (377) = 1131 \Omega \]
for media 3, \[ \eta_3 = \sqrt{\frac{4}{1}} (377) = 377 \Omega \]

**SOL 7.3.110** Option (B) is correct.
For an EM wave a medium incident on another medium, reflection coefficient is defined as
\[ \Gamma = \frac{E_r}{E_i} = -\frac{H_r}{H_i} \]
and
\[ \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2Z - Z}{2Z + Z} = \frac{1}{3} \]
So,
\[ \frac{E_r}{E_i} = -\frac{H_r}{H_i} = \frac{1}{3} \]
\[ E_r = 3 \text{ and } H_r = -3 \]

**SOL 7.3.111** Option (B) is correct.
For a perfect conductor conductivity $\sigma = \infty$
So, the skin depth of the perfect conductor is
\[ \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0 \]

**********
EXERCISE 8.1

MCQ 8.1.1 Assertion (A) : A sinusoidal voltage \( v_t = V_0 \cos(2 \times 10^8 \pi t) \) is applied to the input terminal of a transmission line of length 20 cm such that the wave propagates with the velocity \( c = 3 \times 10^8 \) m/s on the line. Its output voltage will be in the same phase to the input voltage.

Reason : Transmission line effects can be ignored if \( \frac{l}{\lambda} \leq 0.01 \).

where \( l \) is the length of transmission line and \( \lambda \) is the wavelength of the wave.

(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

MCQ 8.1.2 A transmission line is formed of coaxial line with an inner conductor diameter of 1 cm and an outer conductor diameter of 4 cm. If the conductor has permeability \( \mu = 2 \mu_0 \) and conductivity \( \sigma = 11.6 \times 10^7 \) S/m then its resistance per unit length for the operating frequency of 4 GHz will be

(A) 4.95 \( \Omega \)/m
(B) 78.8 \( \Omega \)/m
(C) 0.788 \( \Omega \)/m
(D) 0.495 \( \Omega \)/m

MCQ 8.1.3 A transmission line formed of co-axial line with inner and outer diameters 1.5 cm and 3 cm respectively is filled with a dielectric of permeability \( \mu = 2 \mu_0 \). Its line parameter \( L' \) will be equal to

(A) 277 nH/m
(B) 2.77 nH/m
(C) 872 nH/m
(D) 8.7 nH/m

MCQ 8.1.4 A co-axial transmission line is filled with a dielectric having conductivity, \( \sigma = 2 \times 10^{-3} \) S/m. If the inner and outer radius of the co-axial line are 1/4 cm and 1/2 cm respectively then the conductance per unit length of the transmission line will be

(A) 9.1 mS/m
(B) 1.45 mS/m
(C) 911 S/m
(D) 145 S/m

MCQ 8.1.5 If Permittivity of the dielectric filled inside the coaxial transmission line having inner and outer diameter 2 cm and 5 cm respectively is \( \varepsilon = 9 \varepsilon_0 \) then the capacitance
per unit length of the line will be
(A) 361 pF/m  (B) 3.61 nF/m
(C) 5.74 nF/m  (D) 57.4 pF/m

**MCQ 8.1.6**
A parallel plate transmission line consists of 1.2 cm wide conducting strips having conductivity, \( \sigma = 1.16 \times 10^8 \) S/m and permeability \( \mu = \mu_0 \) is operating at 4 GHz frequency. What will be the line parameter \( R' \)?
(A) 1.38 \( \Omega \)/m  (B) 0.69 \( \Omega \)/m
(C) 0.97 \( \Omega \)/m  (D) 1.97 \( \Omega \)/m

**MCQ 8.1.7**
A parallel plate transmission line is formed by copper strips of width \( w = 1.2 \) cm separated by a distance \( d = 0.3 \) cm. If the dielectric filled between the plates has permeability, \( \mu = 2\mu_0 \) then what will be the inductance per unit length of the transmission line?
(A) 157 nH/m  (B) 1.57 \( \mu \)H/m
(C) 0.78 nH/m  (D) 78.1 \( \mu \)H/m

**MCQ 8.1.8**
The space between the strips of a parallel plate transmission line is filled of a dielectric of permittivity, \( \varepsilon_r = 1.3 \) and conductivity, \( \sigma \approx 0 \). If the width of the strips is \( w \) cm and the separation between them is 0.6 cm then the line parameters \( G' \) and \( C' \) will be respectively
(A) 0, 0.02 nF/m  (B) 0.02 mS/m, 0.14 nF/m
(C) 0, 0.18 nF/m  (D) 1.8 mS/m, 0

**MCQ 8.1.9**
Which one of the following statement is not correct for a transmission line?
(A) Attenuation constant of a lossless line is always zero.
(B) Characteristic impedance of both lossless and distortionless line is real
(C) Attenuation constant of a distortionless line is always zero.
(D) Both (A) and (C).

**MCQ 8.1.10**
Inductance and capacitance per unit length of a lossless transmission line are 250 nH/m and 0.2 nF/m respectively. The velocity of the wave propagation and characteristic impedance of the transmission line are respectively.
(A) \( 2 \times 10^8 \) m/s, 100 \( \Omega \)  (B) \( 3 \times 10^8 \) m/s, 50 \( \Omega \)
(C) \( 2 \times 10^3 \) m/s, 50 \( \Omega \)  (D) \( 3 \times 10^8 \) m/s, 100 \( \Omega \)

**MCQ 8.1.11**
A 1 GHz parallel plate transmission line consists of brass strips of conductivity \( \sigma = 6.4 \times 10^7 \) S/m separated by a dielectric of permittivity \( \varepsilon = 6\varepsilon_0 \). If the axial component and transverse component of the electric field in the transmission line is \( E_x \) and \( E_y \) respectively then \( E_x/E_y \) equals to
(A) \( 2.16 \times 10^{-4} \)  (B) \( 4.167 \times 10^{-5} \)
(C) \( 1.25 \times 10^{-4} \)  (D) \( 7.22 \times 10^{-5} \)
MCQ 8.1.12 A transmission line operating at a frequency $6 \times 10^8$ rad/s has the parameters $R' = 2 \Omega/m$, $L' = 4 \mu H/m$, $G' = 8 \mu S/m$, $C' = 6 \mu F/m$. The propagation constant, $\gamma$ will be

(A) $(0.5 + j1.2) m^{-1}$

(B) $(0.10 + j2.4) m^{-1}$

(C) $(1.2 + j0.5) m^{-1}$

(D) $(2.4 + j0.10) m^{-1}$

MCQ 8.1.13 The parameters of a transmission line are given as $R' = 10 \Omega/m$, $L' = 0.1 \mu H/m$, $C' = 10 \mu F/m$, $G' = 40 \mu S/m$. If the transmission line is operating at a frequency, $\omega = 1.2 \times 10^9$ rad/s then the characteristic impedance of the line will be

(A) $50 - j2 \Omega$

(B) $4 - j100 \Omega$

(C) $100 - j4 \Omega$

(D) $100 + j4 \Omega$

MCQ 8.1.14 After travelling a distance of 20 m along a transmission line, the voltage wave remains 13% of its source amplitude. What is the attenuation constant of the transmission line?

(A) 0.13 NP/m

(B) 0.10 NP/m

(C) 0.20 NP/m

(D) 0.06 NP/m

MCQ 8.1.15 Amplitude of a voltage wave after travelling a certain distance down a transmission line is reduced by 87%. If the propagation constant of the transmission line is $(0.3 + j2.9)$ then the phase shift in the voltage wave is

(A) $61^\circ$

(B) $561^\circ$

(C) $73^\circ$

(D) $273^\circ$

MCQ 8.1.16 A parallel plate lossless transmission line consists of brass strips of width $w$ and separated by a distance $d$. If both $w$ and $d$ are doubled then it’s characteristic impedance will

(A) halved

(B) doubled

(C) not change

(D) none of these

MCQ 8.1.17 Phase velocity of voltage wave in a distortion less line having characteristic impedance, $Z_0 = 0.2 k\Omega$ and attenuation constant, $\alpha = 10 mNP/m$ is $v_p = \_\_\_\_ \times 10^4$ m/s. The line parameters $R'$ and $L'$ will be respectively

(A) $1 \Omega/m$, $0.5 \mu H/m$

(B) $10 k\Omega/m$, $2 \mu H/m$

(C) $2 \Omega/m$, $1 \mu H/m$

(D) $1 \Omega/m$, $2 \mu H/m$

MCQ 8.1.18 A distortionless line has parameters $R' = 4 \Omega/m$ and $G' = \_\_\_\_ \times 10^{-4}$ S/m. The attenuation constant and characteristic impedance of the transmission line will be respectively

(A) 25 NP/m, 0.01 $\Omega$

(B) 100 NP/m, $4 \times 10^{-2}$ $\Omega$

(C) $4 \times 10^{-2}$ NP/m, 100 $\Omega$

(D) 0.01 NP/m, 25 $\Omega$
### MCQ 8.1.19
A transmission line operating at 5 GHz frequency has characteristic impedance $Z_0 = 80 \Omega$ and the phase constant $\beta = 1.5 \text{ rad/m}$. The inductance per unit length of the transmission line will be

- (A) 3.81 nH/m
- (B) 38.1 nH/m
- (C) 2.61 nH/m
- (D) 26.1 nH/m

### MCQ 8.1.20
A $\Omega$ transmission line is connected to a 330 $\Omega$ resistance and to a 50 V DC source with zero internal resistance. The voltage reflection coefficients at the load end and at the source and of the transmission line are respectively

- (A) $-1, 1/3$
- (B) $-1, -1$
- (C) $1/3, 1/3$
- (D) $1/3, -1$

### MCQ 8.1.21
The voltage wave in a lossless transmission line has the maximum magnitude of 6 volt and minimum magnitude of 2.4 volt. The reflection coefficient of the transmission line is

- (A) 0.43
- (B) 2.33
- (C) 1.40
- (D) 0.71

### MCQ 8.1.22
An insulating material of permittivity $\varepsilon = \varepsilon_0$ is used in a 25 $\Omega$ lossless co-axial line. If the inner radius of the coaxial line is 0.6 mm then what will be its outer radius?

- (A) 6.004 mm
- (B) 2.1 mm
- (C) 3.002 mm
- (D) 4.2 mm

### MCQ 8.1.23
A lossless transmission line of characteristic impedance $Z_0 = 35 \Omega$ is connected to a load impedance $Z_L = (25 \Omega)$. What will be the standing wave ratio an the line?

- (A) 0.57
- (B) 3.65
- (C) 0.27
- (D) 1.22

### MCQ 8.1.24
A purely resistance load $Z_L$ is connected to a $\Omega$ lossless transmission line. Such that it has a voltage standing wave ratio of 3. The possible value of $Z_L$ will be

- (A) 50 $\Omega$
- (B) 450 $\Omega$
- (C) (A) and (B) both
- (D) none of these

### MCQ 8.1.25
A voltage generator with $v_g(t) = 3 \cos(\pi \times 10^9 t)$ volt is applied to a 50 $\Omega$ lossless air spaced transmission line. If the line length is 10 cm and it is terminated in a load impedance $Z_L = (25 \Omega)$ then the input impedance of the transmission line will be

- (A) $(50 - j50.8) \Omega$
- (B) $(12.5 - j12.7) \Omega$
- (C) $(25.4 - j25) \Omega$
- (D) $(25 - j25.4) \Omega$
MCQ 8.1.26 The wavelength on a lossless transmission line terminated in a short circuit is $\lambda$. What is the minimum possible length of the transmission line for which it appears as an open circuit at it’s input terminals?
(A) $\lambda$
(B) $\lambda/2$
(C) $4\lambda$
(D) $\lambda/4$

MCQ 8.1.27 A lossless transmission line is operating at a frequency of 4 MHz. When the line is short circuited at it’s output end, the input impedance appears to be equivalent to an inductor with inductance of 32 nH but when the line is open circuited at it’s output end, the input impedance appears to be equivalent to a capacitor with capacitance of 20 pF. What is the characteristic impedance of the transmission line?
(A) 10 $\Omega$
(B) 1.6 k$\Omega$
(C) $-40 \Omega$
(D) 40 $\Omega$

MCQ 8.1.28 A $\lambda/4$ section of a 50 $\Omega$ lossless transmission line terminated in a 150 $\Omega$ resistive load is preceded by another $\lambda/4$ section of a 200 $\Omega$ lossless line as shown in figure. What is the input impedance, $Z_{in}$?

\[ \begin{array}{c}
\text{Line 2} \\
\lambda/4 \\
\end{array} \]

(A) 600 $\Omega$
(B) 400 $\Omega$
(C) 267 $\Omega$
(D) 300 $\Omega$

MCQ 8.1.29 A transmission line of length $l$ is short circuited at one end and open circuited at the other end. The voltage standing wave pattern in the transmission line will be

(A) ![Open circuit end](open.png) $l$ ![Short circuit end](short.png)
(B) ![Open circuit end](open.png) $l$ ![Short circuit end](short.png)
MCQ 8.1.30  Phase velocity of a voltage wave in a transmission line of length \( l \) is \( v_p \). If the transmission line is open circuited at one end and short circuited at the other end, then the natural frequency of the oscillation of the wave will be

(A) \( \frac{nv_p}{2l} \); \( n = 0, 1, \ldots, \infty \)

(B) \( \frac{(2n+1)v_p}{4l} \); \( n = 0, 1, \ldots, \infty \)

(C) \( \frac{(2n+1)v_p}{4l} \); \( n = 1, 2, 3, \ldots, \infty \)

(D) \( \frac{nv_p}{2l} \); \( n = 1, 2, 3, \ldots, \infty \)

MCQ 8.1.31  At an operating frequency of 500 Hz, length of a transmission line is given by \( l = \frac{\lambda}{4} \). For the same transmission line the length at 1 kHz will be given by

(A) \( l = \frac{\lambda}{8} \)

(B) \( l = \frac{\lambda}{4} \)

(C) \( l = \frac{\lambda}{2} \)

(D) none of these

MCQ 8.1.32  A lossless transmission line is terminated in a short circuit. The minimum possible length of the line for which it appears as a short circuit at its input terminals is

(A) \( \lambda/2 \)

(B) \( \lambda/4 \)

(C) \( \lambda \)

(D) 0

MCQ 8.1.33  A transmission line is operating at wavelength ‘\( \lambda \)’. If the distance between successive voltage minima is 10 cm and distance between load and first voltage minimum is 7.5 cm, then the distance between load and first voltage maxima is

(A) \( \lambda/8 \)

(B) \( 3\lambda/8 \)

(C) \( 5\lambda/8 \)

(D) \( \lambda/4 \)

************
A z-polarized transverse electromagnetic wave (TEM) propagating along a parallel plate transmission line filled of perfect dielectric in +a direction. Let the electric and magnetic field of the wave be \(E\) and \(H\) respectively. Which of the following is correct relation for the fields.

(A) \(\frac{\partial E}{\partial y} = 0\)  
(B) \(\frac{\partial H}{\partial z} = 0\)  
(C) Both (A) and (B)  
(D) none of these

Statement for Linked Question 2 - 3:
A load impedance \(Z_L = (0.3 - j0.5) \, k\Omega\) is being connected to a lossless transmission line of characteristic impedance \(Z_0 = 0.5 \, k\Omega\) operating at wavelength \(\lambda = 2 \, cm\).

The distance of the first voltage maximum from the load will be

(A) 0.44 cm  
(B) 2.44 cm  
(C) 1.56 cm  
(D) 0.44 cm

The distance of the first current maximum from the load will be

(A) 3.56 cm  
(B) 0.56 cm  
(C) 1.44 cm  
(D) 2.56 cm

Distance of the first voltage maximum and first current maximum from the load on a 50 \(\Omega\) lossless transmission line are respectively 4.5 cm and 1.5 cm. If the standing wave ratio on the transmission line is \(S = 3\) then the load impedance connected to the transmission line will be

(A) \((90 - j120)\,\Omega\)  
(B) 10 \(\Omega\)  
(C) \((30 - j40)\,\Omega\)  
(D) \((40 - j30)\,\Omega\)

Total length of 50 \(\Omega\) lossless transmission line terminated in a load impedance \(Z_L = (30 + j15)\,\Omega\) is \(l = 7\lambda/20\) as shown in figure. The total input impedance across the terminal \(AB\) will be
MCQ 8.2.6

Assertion (A): The input impedance of a quarter wavelength long lossless line terminated in a short-circuit is infinity.

Reason (R): The input impedance at the position where the magnitude of the voltage on a distortionless line is maximum is purely real.

(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Statement for Linked Question 7 - 8:

A voltage generator with \( v_g(t) = 25 \cos(\frac{4\pi}{10} t - 30) \) and an internal impedance \( Z_g = 30 \Omega \) is applied to a 30Ω lossless transmission line that has a relative permittivity \( \varepsilon_r = 2.25 \) and length, \( l = 6 \text{ m} \).

MCQ 8.2.7

If the line is terminated in a load impedance, \( Z_L = (30 - j10) \Omega \), then what will be the input impedance of the transmission line?

(A) \( (0.05 - j0.01) \Omega \)  \hspace{1cm} (B) \( (50.62 + j23.48) \Omega \)

(C) \( (92.06 - j21.80) \Omega \) \hspace{1cm} (D) \( (23.14 + j5.48) \Omega \)

MCQ 8.2.8

The input voltage of the transmission line will be

(A) \( 4.4 \cos(8\pi \times 10^2 t + 22.56^\circ) \)

(B) \( 4.4 \cos(8\pi \times 10^2 t - 37.44^\circ) \) V

(C) \( 4.4 \cos(8\pi \times 10^2 t - 22.56^\circ) \) V

(D) \( 4.4 \cos(8\pi \times 10^2 t - 30^\circ) \) V

Statements for Linked Question 9 - 10:

Two equal load impedances of 150Ω are connected in parallel through a pair of transmission line, and the combination is connected to a feed transmission line as shown in figure. All the lines are lossless and have characteristic impedance \( Z_0 = 100 \Omega \).
MCQ 8.2.9  The effective load impedance of feedline \( Z_L \) equals to
(A) \((7.04 - j17.24)\ \Omega\)
(B) \((35.20 + j8.62)\ \Omega\)
(C) \((35.20 - j8.62)\ \Omega\)
(D) \((8.62 + j35.20)\ \Omega\)

MCQ 8.2.10  The total input impedance of the feedline (line 3) will be
(A) \((2.15 - j1.13)\ \Omega\)
(B) \((215.14 - j113.4)\ \Omega\)
(C) \((215.14 + j113.4)\ \Omega\)
(D) \((107.57 - j6.7)\ \Omega\)

MCQ 8.2.11  A 0.3 GHz voltage generator with \( V_{sg} = 150\ \text{volt} \) and an internal resistance \( Z_g = 100\ \Omega \) is connected to a 100 \( \Omega \) lossless transmission line of length \( l = 0.375\lambda \). If the line is terminated in a load impedance \( Z_L = (100 - j100)\ \Omega \) then what will be the current flowing in the load?
(A) \(0.67\ \text{cos}(3 \times 10^8 t - 108.4^\circ)\)
(B) \(0.67\ \text{cos}(6\pi \times 10^8 t - 108.4^\circ)\)
(C) \(75\ \text{cos}(3 \times 10^8 t - 108.4^\circ)\)
(D) \(0.67\ \text{cos}(6\pi \times 10^8 t - 135^\circ)\)

MCQ 8.2.12  A voltage generator \( V_{sg} = 150\ \text{V} \) with an internal resistance \( Z_g = 100\ \Omega \) is connected to a load \( Z_L = 150\ \Omega \) through a 0.15\( \lambda \) section of a 100 \( \Omega \) lossless transmission line. What is the average power delivered to the transmission line?
(A) 54 Watt
(B) 30 Watt
(C) 27 Watt
(D) 60 Watt

MCQ 8.2.13  A voltage generator \( V_{sg} = 500\ \text{volt} \) with an internal resistance \( Z_g = 100\ \Omega \) is applied to a configuration of lossless transmission lines as shown in figure. The power delivered to the loads \( Z_{L1} \) and \( Z_{L2} \) will be respectively

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
MCQ 8.2.14
The input impedance of an infinitely long transmission line is equal to it’s characteristic impedance. The transmission line will be
(A) slightly lossy
(B) lossless
(C) Distortion less
(D) (B) and (C) both

MCQ 8.2.15
An infinitely long lossy transmission line with characteristic impedance $Z_0 = 200 \, \Omega$ is fed by a $\lambda/2$ section of 80 $\Omega$ lossless transmission line as shown in figure. If a voltage generator $V_g = 4 \, V$ with an internal resistance $Z_g = 100 \, \Omega$ is applied to the whole configuration then the average power transmitted to the infinite transmission line will be

(A) 2.2 mWatt  
(B) 22.2 mWatt  
(C) 17.8 mWatt  
(D) $-2.2 \, mWatt$

Common Data for Question 16 - 17:
A unit step voltage generator is applied to a 90 $\Omega$ airspaced lossless transmission line at time, $t = 0$. At any time, $t \geq 0$ the voltage waveform at the sending end of the transmission line is shown in the figure below:
MCQ 8.2.16 The length of the transmission line will be
(A) 1200 m  (B) 600 m
(C) 150 m  (D) 300 m

MCQ 8.2.17 The unit step generator voltage connected to the line has an internal resistance
$R_g = 100 \Omega$. What will be the load impedance connected to the transmission line?
(A) $21.43 \Omega$  (B) $93.16 \Omega$
(C) $42.86 \Omega$  (D) $233 \Omega$

MCQ 8.2.18 At time $t = 0$ unit step voltage generator $V_g$ with an internal resistance $R_g$ is
applied to a $100 \Omega$ shorted transmission line filled with dielectric of permittivity
$\varepsilon = 4\varepsilon_0$ as shown in figure

The voltage waveform for any time $t \geq 0$ at the sending end is shown in figure below

$V_g$ and $R_g$ will be respectively equal to
(A) 30 volt, 19.2 $\Omega$  (B) 38.4 volt, 60 $\Omega$
(C) 60 volt, 38.4 $\Omega$  (D) 19.2 volt, 30 $\Omega$
Statement for Linked Question 19 - 20:
A 2.5 m section of an airspaced lossless transmission line is fed by a unit step voltage generator $V_g = 30 \text{ volt}$ with internal resistance $R_g = 200 \Omega$. The transmission line is terminated in a resistive load $Z_L = 50 \Omega$ and characterized by $Z_0 = 100 \Omega$.

MCQ 8.2.19 The bounce diagram of the transmission line will be

![Bounce Diagram A](image)

![Bounce Diagram B](image)

![Bounce Diagram C](image)

![Bounce Diagram D](image)

MCQ 8.2.20 The instantaneous voltage waveform $v(t)$ at the sending end of the transmission line will be

![Voltage Waveform A](image)

![Voltage Waveform B](image)
**MCQ 8.2.21** The SWR circle ‘L1L2’ is shown on the smith chart for a lossless transmission line. If line is terminated in a load $Z_L = 50 \, \Omega$ then the possible value of the characteristic impedance of the line will be

(A) 125 $\Omega$
(B) 250 $\Omega$
(C) 20 $\Omega$
(D) (A) and (C) Both

**Common Data for Question 22 - 25 :**
A lossless transmission line characterized by $Z_0 = 50 \, \Omega$ is terminated in a load $Z_L = (50 + j75) \, \Omega$

**MCQ 8.2.22** The reflection coefficient of the line will be

(A) $4.4e^{-j0.6\lambda}$
(B) $0.24e^{+j0.6\lambda}$
(C) $4.4e^{+j0.6\lambda}$
(D) $0.24e^{-j0.6\lambda}$

**MCQ 8.2.23** The input impedance at a distance of $0.35\lambda$ from the load will be

(A) $(0.61 - j0.22)\Omega$
(B) $(61 + j2.2)\Omega$
(C) $(61 - j2.2)\Omega$
(D) $(0.61 + j0.022)\Omega$

**MCQ 8.2.24** The shortest length of the transmission line for which the input impedance appears to be purely resistive will be

(A) $0.25\lambda$
(B) $0.456\lambda$
(C) $0.106\lambda$
(D) $0.544\lambda$

**MCQ 8.2.25** The first voltage maximum will occur at a distance of

(A) $0.106\lambda$ from load
(B) $0.144\lambda$ from load
(C) $0.106\lambda$ from Generator
(D) $0.144\lambda$ from generator

**MCQ 8.2.26** A transmission line of characteristic impedance $50 \, \Omega$ is terminated by an inductor as shown in the figure.
A positive wave with constant voltage $V_0 = 1\ \text{volt}$ is incident on the load terminal at $t = 0$. At any time $t$ the resulting negative wave voltage at the load terminal will be

(A) $(1 - 2e^{-25t})\ \text{Volt}$
(B) $(2e^{-25t} - 1)\ \text{Volt}$
(C) $2e^{-25t}\ \text{Volt}$
(D) $(e^{-25t} - 1)\ \text{Volt}$

**MCQ 8.2.27**
A transmission line has the characteristic impedance $Z_0$ and the voltage standing wave ratio is $S$. The line impedance on the transmission line at voltage maximum and minimum are respectively.

(A) $Z_0S, \frac{Z_0}{S}$
(B) $\frac{Z_0}{S}, Z_0S$
(C) $Z_0S, Z_0S$
(D) $Z_0S, \frac{Z_0}{S}$

**MCQ 8.2.28**
Consider the three mediums of intrinsic impedances $\eta_1$, $\eta_2$, and $\eta_3$ respectively as shown in the figure. What will be the thickness $'t'$ and intrinsic impedance $'\eta_2'$ of the medium 2 for which the reflected wave having wavelength $'\lambda'$ is eliminated in medium 1 are

- thickness $'t'$
- intrinsic impedance $\eta_2$

(A) $\lambda/4$
(B) $\lambda/2$
(C) $\lambda/4$
(D) $\lambda/2$

**Statement for Linked Question 29 - 30 :**
A quarter wave dielectric of thickness $'t'$ and permittivity $'\varepsilon'$ eliminates reflections of uniform plane waves of frequency $2.5\ \text{GHz}$ incident normally from free space onto a dielectric of permittivity $16\varepsilon_0$. (Assume all media to have $\mu = \mu_0$)

**MCQ 8.2.29**
The permittivity of the dielectric coating equals to

(A) $\varepsilon_0/2$
(B) $\varepsilon_0/4$
(C) $4\varepsilon_0$
(D) $2\varepsilon_0$

**MCQ 8.2.30**
What is the thickness $'t'$ of the dielectric coating ?

(A) 25 cm
(B) 2.5 cm
(C) 1 cm
(D) 10 cm
MCQ 8.2.31 A transmission line has characteristics impedance \( 100 \, \Omega \) and standing wave ratio 3. The distance between the first voltage maximum and load is \( 0.125 \lambda \). Load impedance of the transmission line is
(A) \( (30 + j40) \, \Omega \)  
(B) \( (60 + j80) \, \Omega \)  
(C) \( (30 - j40) \, \Omega \)  
(D) \( (60 - j80) \, \Omega \)

MCQ 8.2.32 A \( 100 \, \Omega \) lossless transmission line with it’s parameter \( L' = 0.25 \, \mu \text{H/m} \) and \( C' = 100 \, \text{PF/m} \) is terminated by it’s characteristic impedance. A 15 V voltage source with internal resistance \( 50 \, \Omega \) is connected to the transmission line at \( t = 0 \). Plot of the voltage on the line at a distance \( 5 \, \text{m} \) from the source against time will be

MCQ 8.2.33 A lossless transmission line terminated by a load impedance \( Z_L \neq Z_0 \) is connected to a D.C. voltage source. The height of the first forward voltage pulse is \( V_1^+ \). If the voltage reflection coefficients at the load and source are respectively \( \Gamma_L \) and \( \Gamma_s \) then the steady state voltage across the load is
(A) \( V_1^+ \left( \frac{1 + \Gamma_L}{1 - \Gamma_L} \right) \)  
(B) \( V_1^+ \left( \frac{1 - \Gamma_s \Gamma_L}{1 + \Gamma_s} \right) \)  
(C) \( V_1^+ \left( \frac{1 - \Gamma_L}{1 + \Gamma_L} \right) \)  
(D) \( V_1^+ \left( \frac{1 + \Gamma_L}{1 - \Gamma_s} \right) \)

MCQ 8.2.34 A \( 60 \, \Omega \) transmission line, terminated by a load of \( 180 \, \Omega \) is connected to a 100 V DC source at \( t = 0 \). The internal resistance of the source is \( 120 \, \Omega \). The steady state voltage across the load will be
MCQ 8.2.35  At \( t = 0 \) a 50 Volt D.C. source with an internal resistance 30 \( \Omega \) is connected to a transmission line of 15 \( \Omega \) characteristic impedance having a load of 45 \( \Omega \). The steady state load current for the transmission line is

(A) 0.67 A  
(B) 1.5 A  
(C) 0.33 A  
(D) 1.3 A

Statement for Linked Question 36 - 37:

A transmission line of an unknown length terminated in a resistance is connected to a 5 V battery with zero internal resistance. The plot of input current to the line is shown in the figure below.

MCQ 8.2.36  The characteristic impedance of the transmission line will be

(A) 1.2 k\( \Omega \)  
(B) 80 \( \Omega \)  
(C) 8 \( \Omega \)  
(D) 12.5 \( \Omega \)

MCQ 8.2.37  The load resistance terminated to the transmission line will be

(A) 263 \( \Omega \)  
(B) 80 \( \Omega \)  
(C) 150 \( \Omega \)  
(D) 43 \( \Omega \)

***********
MCQ 8.3.1  A coaxial-cable with an inner diameter of 2 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi \times 10^{-7} \text{H/m}$, $\varepsilon_0 = \frac{10^{-9}}{30\pi} \text{F/m}$, the characteristic impedance of the cable is

(A) 330 $\Omega$  
(B) 100 $\Omega$  
(C) 143.3 $\Omega$  
(D) 43.4 $\Omega$

MCQ 8.3.2  A transmission line with a characteristic impedance of 100 $\Omega$ is used to match a 50 $\Omega$ section to a 200 $\Omega$ section. If the matching is to be done both at 429 MHz and 1 GHz, the length of the transmission line can be approximately

(A) 82.5 cm  
(b) 1.05 m  
(C) 1.58 cm  
(D) 1.75 m

MCQ 8.3.3  A transmission line of characteristic impedance 50 $\Omega$ is terminated by a 50 $\Omega$ load. When excited by a sinusoidal voltage source at 20 GHz, the phase difference between two points spaced 2 mm apart on the line is found to be $\pi/4$ radians. The phase velocity of the wave along the line is

(A) $0.8 \times 10^8 \text{m/s}$  
(B) $1.2 \times 10^8 \text{m/s}$  
(C) $1.6 \times 10^8 \text{m/s}$  
(D) $3 \times 10^8 \text{m/s}$

MCQ 8.3.4  A transmission line of characteristic impedance 50 $\Omega$ is terminated in a load impedance $Z_L$. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of $\lambda/4$ from the load. The value of $Z_L$ is

(A) 10 $\Omega$  
(B) 250 $\Omega$  
(C) $(19.23 + j46.15)\Omega$  
(D) $(19.23 - j46.15)\Omega$

MCQ 8.3.5  If the scattering matrix $[S]$ of a two port network is $[S] = \begin{bmatrix} 0.2/0^\circ & 0.9/90^\circ \\ 0.9/90^\circ & 0.1/90^\circ \end{bmatrix}$, then the network is

(A) lossless and reciprocal  
(B) lossless but not reciprocal  
(C) not lossless but reciprocal  
(D) neither lossless nor reciprocal
MCQ 8.3.6
A transmission line has a characteristic impedance of 50 Ω and a resistance of 0.1 Ω/m. If the line is distortion less, the attenuation constant (in Np/m) is
(A) 500
(B) 5
(C) 0.014
(D) 0.002

MCQ 8.3.7
In the circuit shown, all the transmission line sections are lossless. The Voltage Standing Wave Ratio (VSWR) on the 60 Ω line is

(A) 1.00
(B) 1.64
(C) 2.50
(D) 3.00

MCQ 8.3.8
A transmission line terminates in two branches, each of length \( \lambda /2 \), as shown. The branches are terminated by 50 Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance \( Z_i \) as seen by the source.

(A) 200 Ω
(B) 100 Ω
(C) 50 Ω
(D) 25 Ω

MCQ 8.3.9
One end of a loss-less transmission line having the characteristic impedance of 75 Ω and length of 2 cm is short-circuited. At 5 GHz, the input impedance at the other end of transmission line is
(A) 0
(B) Resistive
(C) Capacitive
(D) Inductive

MCQ 8.3.10
A load of 50 Ω is connected in shunt in a 2-wire transmission line of \( Z_0 = 50 Ω \) as shown in the figure. The 2-port scattering parameter matrix (S-matrix) of the shunt element is
MCQ 8.3.11

The parallel branches of a 2-wire transmission line are terminated in 50 Ω and 200 Ω resistors as shown in the figure. The characteristic impedance of the line is $Z_0 = 50 \, \Omega$ and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient $\Gamma$ at the input is

\[
\begin{bmatrix}
-\frac{1}{2} & \frac{j}{2} \\
\frac{j}{2} & -\frac{1}{2}
\end{bmatrix}
\]

(A) $-\frac{j}{2}$

(B) $\frac{5}{7}$

(C) $\frac{j}{7}$

(D) $\frac{5}{7}$

MCQ 8.3.12

A transmission line is feeding 1 watt of power to a horn antenna having a gain of 10 dB. The antenna is matched to the transmission line. The total power radiated by the horn antenna into the free space is

(A) 10 Watts

(B) 1 Watts

(C) 0.1 Watts

(D) 0.01 Watt

MCQ 8.3.13

Characteristic impedance of a transmission line is 50 Ω. Input impedance of the open circuited line is $Z_{oc} = 100 + j150 \, \Omega$. When the transmission line is short circuited, then value of the input impedance will be

(A) 50 Ω

(B) 100 + j150 Ω

(C) 7.69 + j11.54 Ω

(D) 7.69 − j11.54 Ω
Statement of Linked Questions 14 - 15:

Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 and a resistive load is shown in the figure.

MCQ 8.3.14
The value of the load resistance is
(A) 50 Ω (B) 200 Ω
(C) 12.5 Ω (D) 0

MCQ 8.3.15
The reflection coefficient is given by
(A) 0.6 (B) 1
(C) -0.6 (D) 0

MCQ 8.3.16
Many circles are drawn in a Smith Chart used for transmission line calculations. The circles shown in the figure represent
(A) Unit circles (B) Constant resistance circles
(C) Constant reactance circles (D) Constant reflection coefficient circles.

MCQ 8.3.17
Consider a 200 Ω, quarter - wave long (at 1 GHz) transmission line as shown in Fig. It is connected to a 20 V, 50 Ω source at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is

(A) 10 V (B) 5 V
(C) 60 V (D) 60/7 V
MCQ 8.3.18
Consider an impedance $Z = R + jX$ marked with point $P$ in an impedance Smith chart as shown in Fig. The movement from point $P$ along a constant resistance circle in the clockwise direction by an angle $45^\circ$ is equivalent to

(A) adding an inductance in series with $Z$
(B) adding a capacitance in series with $Z$
(C) adding an inductance in shunt across $Z$
(D) adding a capacitance in shunt across $Z$

MCQ 8.3.19
A lossless transmission line is terminated in a load which reflects a part of the incident power. The measured VSWR is 2. The percentage of the power that is reflected back is

(A) 57.73  (B) 33.33  
(C) 0.11    (D) 11.11

MCQ 8.3.20
A short-circuited stub is shunt connected to a transmission line as shown in fig. If $Z_0 = 50\,\Omega$, the admittance $Y$ seen at the junction of the stub and the transmission line is

(A) $(0.01 - j0.02)\,mho$  (B) $(0.02 - j0.01)\,mho$
(C) $(0.04 - j0.02)\,mho$  (D) $(0.02 + j0.01)\,mho$
### MCQ 8.3.21

The VSWR can have any value between
- (A) 0 and 1
- (B) $-1$ and $+1$
- (C) 0 and $\infty$
- (D) 1 and $\infty$

### MCQ 8.3.22

In an impedance Smith chart, a clockwise movement along a constant resistance circle gives rise to
- (A) a decrease in the value of reactance
- (B) an increase in the value of reactance
- (C) no change in the reactance value
- (D) no change in the impedance

### MCQ 8.3.23

A transmission line is distortionless if
- (A) $RL = \frac{1}{GC}$
- (B) $RL = GC$
- (C) $LG = RC$
- (D) $RG = LC$

### MCQ 8.3.24

The magnitudes of the open-circuit and short-circuit input impedances of a transmission line are $100 \Omega$ and $25 \Omega$ respectively. The characteristic impedance of the line is,
- (A) $25 \Omega$
- (B) $50 \Omega$
- (C) $75 \Omega$
- (D) $100 \Omega$

### MCQ 8.3.25

In a twin-wire transmission line in air, the adjacent voltage maxima are at 25 m and 12.4 m. The operating frequency is
- (A) 300 MHz
- (B) 1 GHz
- (C) 2 GHz
- (D) 6.28 GHz

### MCQ 8.3.26

In air, a lossless transmission line of length 50 cm with $L = 10 \mu H/m$, $C = 40 \text{ pF/m}$ is operated at 25 MHz. Its electrical path length is
- (A) 0.5 meters
- (B) $\lambda$ meters
- (C) $\pi/2$ radians
- (D) 180 degrees

### MCQ 8.3.27

A transmission line of $50 \Omega$ characteristic impedance is terminated with a $100 \Omega$ resistance. The minimum impedance measured on the line is equal to
- (A) 0 $\Omega$
- (B) 25 $\Omega$
- (C) 50 $\Omega$
- (D) 100 $\Omega$

### MCQ 8.3.28

A very lossy, $\lambda/4$ long, $50 \Omega$ transmission line is open circuited at the load end. The input impedance measured at the other end of the line is approximately
- (A) 0
- (B) 50 $\Omega$
- (C) $\infty$
- (D) None of the above
MCQ 8.3.29
A lossless transmission line having 50Ω characteristic impedance and length $\lambda/4$ is short circuited at one end and connected to an ideal voltage source of 1 V at the other end. The current drawn from the voltage source is
(A) 0  
(B) 0.02 A  
(C) $\infty$  
(D) none of these

MCQ 8.3.30
The capacitance per unit length and the characteristic impedance of a lossless transmission line are $C$ and $Z_0$ respectively. The velocity of a travelling wave on the transmission line is
(A) $Z_0C$  
(B) $\frac{1}{Z_0C}$  
(C) $\frac{Z_0}{C}$  
(D) $\frac{C}{Z_0}$

MCQ 8.3.31
A $\lambda/4$ line, shorted at one end, presents impedance at the other end equal to
(A) $Z_0$  
(B) $\sqrt{2}Z_0$  
(C) $\infty$  
(D) 0

where $Z_0$ is characteristic impedance of the line.

MCQ 8.3.32
A 100Ω transmission line is first short-terminated and the minima locations are noted. When the short is replaced by a resistive load $R_L$, the minima locations are not altered and the VSWR is measured to be 3. The value of $R_L$ is
(A) 25Ω  
(B) 50Ω  
(C) 225Ω  
(D) 250Ω

MCQ 8.3.33
If maximum and minimum voltage on a transmission line are 2 V and 5 V respectively, VSWR is
(A) 0.5  
(B) 2  
(C) 1  
(D) 8

MCQ 8.3.34
An ideal lossless transmission line of $Z_0 = 60\Omega$ is connected to unknown $Z_L$. If $SWR = 4$, find $Z_L$.
(A) 240Ω  
(B) 480Ω  
(C) 120Ω  
(D) 100Ω

MCQ 8.3.35
Loading of a cable is done to
1. Increase its inductance
2. Increase its leakage resistance
3. Decrease its leakage resistance
4. Achieve distortionless condition
(A) 1, 2, 3 and 4  
(B) 1 and 3 only  
(C) 2 and 3 only  
(D) 1 and 4 only
MCQ 8.3.36
IES EC 2010
Given a range of frequencies, which of the following systems is best for transmission line load matching?
(A) Single stub (B) Double stub
(C) Single stub with adjustable position (D) Quarter wave transformer

MCQ 8.3.37
IES EC 2010
A line of characteristic impedance 50Ω is terminated at one end by +j50Ω. The VSWR on the line is
(A) 1 (B) ∞
(C) 0 (D) j

MCQ 8.3.38
IES EC 2010
At UHF short-circuited lossless transmission lines can be used to provide appropriate values of impedance. Match List I with List II and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.  l &lt; λ/4</td>
<td>1. Capacitive</td>
</tr>
<tr>
<td>b.  λ/4 &lt; l &lt; λ/2</td>
<td>2. Inductive</td>
</tr>
<tr>
<td>c.  l = λ/4</td>
<td>3. 0</td>
</tr>
<tr>
<td>d.  l = λ/2</td>
<td>4. ∞</td>
</tr>
</tbody>
</table>

Codes:

(A) 2 1 4 3
(B) 3 1 4 2
(C) 2 4 1 3
(D) 3 4 1 2

MCQ 8.3.39
IES EC 2010
Consider the following statements regarding a transmission line:
1. Its attenuation is constant and is independent of frequency
2. Its attenuation varies linearly with frequency
3. Its phase shift varies linearly with frequency
4. Its phase shift is constant and is independent of frequency
Which of the above statements are correct for distortion less line?
(A) 1, 2, 3, and 4
(B) 2 and 3 only
(C) 1 and 3 only
(D) 3 and 4 only

MCQ 8.3.40
IES EC 2009
The reflection coefficient on a 200 m long transmission line has a phase angle of −150°. If the operating wavelength is 250 m, what will be the number of voltage maxima on the line?
(A) 0 (B) 3
(C) 6 (D) 7
With regard to a transmission line, which of the following statements is correct?

(A) Any impedance repeats itself every \( \lambda/4 \) on the Smith chart.

(B) The SWR = 2 circle and the magnitude of reflection coefficient = 0.5 circle coincide on the Smith chart.

(C) At any point on a transmission line, the current reflection coefficient is the reciprocal of the voltage reflection coefficient.

(D) Matching eliminates the reflected wave between the source and the matching device location.

It is required to match a 200 \( \Omega \) load to a 450 \( \Omega \) transmission line. To reduce the SWR along the line to 1, what must be the characteristic impedance of the quarter-wave transformer used for this purpose, if it is connected directly to the load?

(A) 90 k\( \Omega \)  
(B) 300 \( \Omega \)  
(C) 9 \( \Omega \)  
(D) 3 \( \Omega \)

The load end of a quarter wave transformer gets disconnected thereby causing an open-circuited load. What will be the input impedance of the transformer?

(A) Zero  
(B) Infinite  
(C) Finite and positive  
(D) Finite and negative

A lossless transmission line of characteristic impedance \( Z_0 \) and length \( l < \lambda/4 \) is terminated at the load end by an open circuit. What is its input impedance \( Z_{in} \)?

(A) \( Z_{in} = jZ_0 \tan \beta l \)  
(B) \( Z_{in} = jZ_0 \cot \beta l \)  
(C) \( Z_{in} = -jZ_0 \tan \beta l \)  
(D) \( Z_{in} = -jZ_0 \tan \beta l \)

Which one of the following statements for a short circuited loss free line is not correct?

(A) The line appears as a pure reactance when viewed from the sending end  
(B) It can be either inductive or capacitive  
(C) There are no reflections in the line  
(D) Standing waves of voltage and current are set up along length of the lines

Match List I (Load impedance) with List II (Value of Reflection Coefficient) and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Short Circuit</td>
<td>1. 0</td>
</tr>
<tr>
<td>b. Open Circuit</td>
<td>2. -1</td>
</tr>
<tr>
<td>c. Line characteristics impedance</td>
<td>3. +1</td>
</tr>
<tr>
<td>d. ( 2 \times ) line characteristic impedance</td>
<td>4. +1/3</td>
</tr>
</tbody>
</table>
MCQ 8.3.47
When the reflection coefficient equals \(1/0\) what is the VSWR?
(A) Zero (B) 1 (C) 3 (D) Infinite

MCQ 8.3.48
If the reflection coefficient is 1/5, what is the corresponding VSWR?
(A) 3/2 (B) 2/3 (C) 5/2 (D) 2/5

MCQ 8.3.49
Which one of the following is the characteristic impedance of lossless transmission line?
(A) \(\sqrt{R/G}\) (B) \(\sqrt{L/G}\) (C) \(\sqrt{L/C}\) (D) \(\sqrt{R/C}\)

MCQ 8.3.50
Match List I (Quantity) with List II (Range of Values) and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Input Impedance</td>
<td>1. -1 to +1</td>
</tr>
<tr>
<td>b. Reflection coefficient</td>
<td>2. 1 to (\infty)</td>
</tr>
<tr>
<td>c. VSWR</td>
<td>3. 0 to (\infty)</td>
</tr>
</tbody>
</table>

Codes:
(a) 2 3 1
(b) 3 2 1
(c) 3 1 2
(d) 2 1 3

MCQ 8.3.51
A quarter wave impedance transformer is terminated by a short circuit. What would its input impedance be equal to?
(A) The line characteristic impedance
(B) Zero
(C) Infinity
(D) Square root of the line characteristic impedance
MCQ 8.3.52
In a transmission line the reflection coefficient at the load end is given by $0.3 e^{-j30\pi}$. What is the reflection coefficient at a distance of 0.1 wavelength towards source?
(A) $0.3 e^{j30\pi}$
(B) $0.3 e^{j90\pi}$
(C) $0.3 e^{j258\pi}$
(D) $0.3 e^{i6\pi}$

MCQ 8.3.53
Scattering parameters are more suited than impedance parameters to describe a waveguide junction because
(A) the scattering parameters are frequency invariant whereas the impedance parameters are not so
(B) scattering matrix is always unitary
(C) impedance parameters vary over unacceptably wide ranges
(D) scattering parameters are directly measurable but impedance parameters are not so

MCQ 8.3.54
To couple a coaxial line to a parallel wire, it is best to use a:
(A) Balun
(B) Slotted line
(C) Directional coupler
(D) Quarter wave transformer

MCQ 8.3.55
A plane wave having $x$-directed electric field propagating in free space along the $z$-direction is incident on an infinite electrically conducting (perfect conductor) sheet at $z = 0$ plane. Which one of the following is correct?
(A) The sheet will absorb the wave
(B) There will be $x$-directed surface electric current on the sheet
(C) There will be $y$-directed surface electric current on the sheet
(D) There will be magnetic current in the sheet.

MCQ 8.3.56
For sea water with $\sigma = 5\, \text{mho/m}$ and $\varepsilon_r = 80$, what is the distance for which radio signal can be transmitted with 90% attenuation at 25 kHz?
(A) 0.322 m
(B) 3.22 m
(C) 32.2 m
(D) 322 m

MCQ 8.3.57
Consider the following statements regarding Smith charts:
1. A normalized Smith chart applies to a line of any characteristic resistance and serves as well for normalized admittance
2. A polar coordinate Smith chart contains circles of constant $|z|$ and circles of constant $/z$
3. In Smith chart, the distance towards the load is always measured in clockwise direction.
Which of the statements given above are correct?

(A) 1, 2 and 3  (B) 2 and 3  
(C) 1 and 3  (D) 1 and 2

MCQ 8.3.58
A \((100 - j75)\Omega\) load is connected to a co-axial cable of characteristic impedance 75 ohms at 12 GHz. In order to obtain the best matching, which one of the following will have to be connected?

(A) A short-circuited stub at load
(B) Inductance at load
(C) A capacitance at a specific distance at load
(D) A short-circuited stub at some specific distance from load

MCQ 8.3.59
In a line VSWR of a load is 6 dB. The reflection coefficient will be

(A) 0.033  (B) 0.33  
(C) 0.66  (D) 3.3

MCQ 8.3.60
\(Z_L = 200 \Omega\) and it is desired that \(Z_m = 50 \Omega\). The quarter wave transformer should have a characteristic impedance of

(A) 100 \(\Omega\)  (B) 40 \(\Omega\)  
(C) 10,000 \(\Omega\)  (D) 4 \(\Omega\)

MCQ 8.3.61
Consider the following:
For a lossless transmission line we can write:

1. \(Z_m = -jZ_0\) for a shorted line with \(l = \lambda/8\)
2. \(Z_m = \pm j\alpha\) for a shorted line with \(l = \lambda/4\)
3. \(Z_m = Z_0\) for a matched line of any length

Select the correct answer using the codes given below:

(A) 1 and 2  (B) 2 and 3  
(C) 1 and 3  (D) 2 and 4

MCQ 8.3.62
The input impedance of a short circuited quarter wave long transmission line is

(A) purely reactive  (B) purely resistive  
(C) dependent on the characteristic impedance of the line  (D) none of the above

MCQ 8.3.63
A transmission line of output impedance 400 \(\Omega\) is to be matched to a load of 25 \(\Omega\) through a quarter wavelength line. The quarter wave line characteristic impedance must be

(A) 40 \(\Omega\)  (B) 100 \(\Omega\)  
(C) 400 \(\Omega\)  (D) 425 \(\Omega\)
MCQ 8.3.64
IES EC 2001
The input impedance of \( \lambda/8 \) long short-circuited section of a lossless transmission line is

(A) zero  
(B) inductive  
(C) capacitive  
(D) infinite

MCQ 8.3.65
IES EC 2001
Match List I (Parameters) with List II (Values) for a transmission line with a series impedance \( Z = R' + j\omega L' \Omega/m \) and a shunt admittance \( Y = G' + j\omega C' \text{mho/m} \), and select the correct answer:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Characteristic impedance ( Z_0 )</td>
<td>1. ( \sqrt{ZY} )</td>
</tr>
<tr>
<td>b. Propagation constant ( \gamma )</td>
<td>2. ( \sqrt{Z/Y} )</td>
</tr>
<tr>
<td>c. The sending-end input impedance ( Z_{in} ) when the line is terminated in its characteristic impedance ( Z_0 )</td>
<td>3. ( \sqrt{Y/Z} )</td>
</tr>
</tbody>
</table>

Codes:

(A) 3 1 1  
(B) 2 3 3  
(C) 2 1 2  
(D) 1 2 2

MCQ 8.3.66
IES EC 2001
Which of the following conditions will not guarantee a distortionless transmission line?

(A) \( R = G = 0 \)  
(B) \( RC = GL \)  
(C) Very low frequency range \( (R \gg \omega L, G \gg \omega C) \)  
(D) Very high frequency range \( (R \ll \omega L, G \ll \omega C) \)

MCQ 8.3.67
IES EC 2001
In an air line, adjacent maxima are found at 12.5 cm and 37.5 cm. The operating frequency is

(A) 1.5 GHz  
(B) 600 MHz  
(C) 300 MHz  
(D) 1.2 GHz

MCQ 8.3.68
IES EC 2001
Fig. I shows an open circuited transmission line. The switch is closed at time \( t = 0 \) and after a time \( t \) the voltage distribution on the line reaches that shown in Fig. II. If \( c \) is the velocity in the line, then
MCQ 8.3.69
A 75Ω transmission line is first short-terminated and the minima locations are noted. When the short is replaced by a resistive load $R_L$, the minima locations are not altered and the VSWR is measured to be 3. The value of $R_L$ is
(A) 25Ω  
(B) 50Ω  
(C) 225Ω  
(D) 250Ω

MCQ 8.3.70
For a lossy transmission line, the characteristic impedance does not depend on
(A) the operating frequency of the line  
(B) the conductivity of the conductors  
(C) conductivity of the dielectric separating the conductors  
(D) length of the line

MCQ 8.3.71
If the maximum and minimum voltages on a transmission line are 4V and 2V, respectively for a typical load, VSWR is
(A) 1.0  
(B) 0.5  
(C) 2.0  
(D) 8.0

MCQ 8.3.72
A transmission line is distortionless if
(A) $RG = LC$  
(B) $RC = GL$  
(C) $\frac{R}{C} = \frac{G}{L}$  
(D) $R = G$

MCQ 8.3.73
If reflection coefficient for voltage be 0.6, the voltage standing wave ratio (VSWR) is
(A) 0.66  
(B) 4  
(C) 1.5  
(D) 2

MCQ 8.3.74
A signal of 10V is applied to a 80 ohm coaxial transmission line, terminated in a 100 ohm load. The voltage reflected coefficient is
(A) 1/4  
(B) 1/3  
(C) 1/2  
(D) 1
MCQ 8.3.75
A transmission line of characteristic impedance of 50 ohm is terminated by a load impedance of \((15 - j20)\) ohm. What is the normalized load impedance?

(A) \(0.6 - j0.8\)  
(B) \(0.3 - j0.6\)  
(C) \(0.3 - j0.4\)  
(D) \(0.3 + j0.4\)

MCQ 8.3.76
Two lossless resistive transmission lines each of characteristic impedance \(Z\) are connected as shown in the circuit below. If the maximum voltage on the two lines is the same and the power transmitted by line A is \(W_1\), then what is the power transmitted by the line B?

![Circuit Diagram]

(A) \(4W_1\)  
(B) \(3W_1\)  
(C) \(2W_1\)  
(D) \(W_1\)

MCQ 8.3.77
A transmission line section shows an input impedance of 36 Ω and 64 Ω respectively, when short circuited and open circuited. What is the characteristic impedance of the transmission line?

(A) 100 Ω  
(B) 50 Ω  
(C) 45 Ω  
(D) 48 Ω

MCQ 8.3.78
Consider the following statements for transmission lines:

1. When a transmission line is terminated by its characteristic impedance the line will not have any reflected wave.
2. For a finite line terminated by its characteristic impedance the velocity and current at all points on the line are exactly same.
3. For a lossless half wave transmission line the input impedance is not equal to load impedance.

Which of the statements given above are correct?

(A) 1 and 2  
(B) 2 and 3  
(C) 1 and 3  
(D) 1, 2 and 3

MCQ 8.3.79
What does the standing wave ratio (SWR) of unity imply?

(A) Transmission line is open circuited  
(B) Transmission line is short circuited  
(C) Transmission line’s characteristic impedance is equal to load impedance  
(D) Transmission line’s characteristic impedance is not equal to load impedance
MCQ 8.3.80  
IES EE 2007

\[ h = \text{half centre to centre spacing}, r = \text{conductor radius and } \varepsilon = \text{permittivity of the medium.} \]  
Which one of the following is equal to the capacitance per unit length of a two-wire transmission line?

\[
\text{(A)} \quad \frac{\pi \varepsilon}{\log_e \left( \frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1} \right)} \\
\text{(B)} \quad \frac{2\pi \varepsilon}{\log_e \left( \frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1} \right)} \\
\text{(C)} \quad \frac{3\pi \varepsilon}{\log_e \left( \frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1} \right)} \\
\text{(D)} \quad \frac{4\pi \varepsilon}{\log_e \left( \frac{h}{r} + \sqrt{\frac{h^2}{r^2} - 1} \right)}
\]

MCQ 8.3.81  
IES EE 2006

For a line of characteristic impedance \( Z_0 \) terminated in a load of \( Z_R \) such that \( Z_R = Z_0/3 \), what is the reflection coefficient \( \Gamma_L \)?

\[
\text{(A)} \quad 1/3 \\
\text{(B)} \quad 2/3 \\
\text{(C)} \quad -1/3 \\
\text{(D)} \quad -1/2
\]

MCQ 8.3.82  
IES EE 2006

A transmission line has \( R, L, G, C \) distributed parameters per unit length of line. If \( \gamma \) is the propagation constant of the line, which one of the following expressions represents the characteristics impedance of the line?

\[
\text{(A)} \quad \frac{\gamma}{R + j\omega L} \\
\text{(B)} \quad \frac{R + j\omega L}{\gamma} \\
\text{(C)} \quad \frac{G + j\omega C}{\gamma} \\
\text{(D)} \quad \sqrt{\frac{G + j\omega C}{R + j\omega L}}
\]

MCQ 8.3.83  
IES EE 2005

What is the value of standing wave ratio (SWR) in free space for reflection for reflection coefficient \( \Gamma = -1/3 \)?

\[
\text{(A)} \quad 2/3 \\
\text{(B)} \quad 0.5 \\
\text{(C)} \quad 4.0 \\
\text{(D)} \quad 2.0
\]

MCQ 8.3.84  
IES EE 2005

What is the attenuation constant \( \alpha \) for distortionless transmission line?

\[
\text{(A)} \quad \alpha = 0 \\
\text{(B)} \quad \alpha = R\sqrt{\frac{C}{L}} \\
\text{(C)} \quad \alpha = R\sqrt{\frac{L}{C}} \\
\text{(D)} \quad \alpha = \sqrt{\frac{RL}{C}}
\]

MCQ 8.3.85  
IES EE 2005

A 75 \( \Omega \) distortionless transmission line has a capacitance of \( 10^{-10} \) f/m. What is the inductance per meter?

\[
\text{(A)} \quad 0.25 \ \mu\text{H} \\
\text{(B)} \quad 500 \ \mu\text{H} \\
\text{(C)} \quad 5000 \ \mu\text{H} \\
\text{(D)} \quad 50 \ \mu\text{H}
\]

MCQ 8.3.86  
IES EE 2005

The open circuit and short circuit impedances of a line are 50 \( \Omega \) each. What is the characteristic impedance of the line?

\[
\text{(A)} \quad 100\sqrt{2} \ \Omega \\
\text{(B)} \quad 100 \ \Omega \\
\text{(C)} \quad 100/\sqrt{2} \ \Omega \\
\text{(D)} \quad 50 \ \Omega
\]
A load impedance of \((75 - j50)\) is connected to a transmission line of characteristic impedance \(Z_0 = 75\, \Omega\). The best method of matching comprises
(A) A short circuit stub at load
(B) A short circuit stub at some specific distance from load
(C) An open stub at load
(D) Two short circuited stubs at specific distances from load

When a lossless transmission line is terminated by a resistance equal to surge impedance, then what is value of the reflection coefficient ?
(A) 1
(B) \(-1\)
(C) 0
(D) 0.5

A lossless transmission line of length 50 cm with \(L = 10 \, \mu\text{H/m}\), \(C = 40\, \text{pF/m}\) is operated at 30 MHz. What is its electric length \(\beta l\) ?
(A) \(20\lambda\)
(B) \(0.2\lambda\)
(C) 108°
(D) \(40\pi\)

Which one of the following is the correct expression for the propagation constant in a transmission line ?
(A) \(\sqrt{(R + j\omega L)}/(G + j\omega C)\)
(B) \(\sqrt{(R - j\omega L)}/(G - j\omega C)\)
(C) \((R - j\omega L)(G - j\omega C)\)
(D) \((R + j\omega L)(G + j\omega C)\)

In a lossless transmission line the voltage and current distributions along the line are always constant.

The voltage and current distributions in an open line are such that at a distance \(\lambda/4\) from the load end, the line looks like a series resonant circuit.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

Consider the following statements :
1. \(\sqrt{(R + j\omega L)}/(G + j\omega C)\), (R, L, G and C are line constants)
2. \(\sqrt{Z_{oc}/Z_{sc}}\), (\(Z_{oc}\) and \(Z_{sc}\) are the open and short circuit impedances of the line)
3. \(V'/I'\), (\(V'\) and \(I'\) are the voltage and current of the wave travelling in the positive \(y\) direction)

Which of these are correct ?
(A) 1,2 and 3
(B) 1 and 2
(C) 2 and 3
(D) 1 and 3
MCQ 8.3.93
A loss-less transmission line of characteristic impedance $Z_0$ and $l < \lambda/4$ is terminated at the load end by a short circuit. Its input impedance $Z_i$ is

(A) $Z_i = -jZ_0 \tan \beta l$

(B) $Z_i = jZ_0 \cot \beta l$

(C) $Z_i = jZ_0 \tan \beta l$

(D) $Z_i = -jZ_0 \cot \beta l$

MCQ 8.3.94
A loss-less transmission line with characteristic impedance of 600 ohms is terminated in a purely resistive load of 900 ohms. The reflection coefficient is

(A) 0.2

(B) 0.5

(C) 0.667

(D) 1.5

MCQ 8.3.95
A transmission line has $R$, $L$, $G$ and $C$ distributed parameters per unit length of the line, $\gamma$ is the propagation constant of the lines. Which expression gives the characteristic impedance of the line?

(A) $R + \frac{j\omega L}{\gamma}$

(B) $G + \frac{j\omega C}{\gamma}$

(C) $G + \frac{j\omega C}{\gamma}$

(D) $R + \frac{j\omega L}{\gamma}$

MCQ 8.3.96
The open circuit impedance of a certain length of a loss-less line is 100 $\Omega$. The short circuit impedance of the same line is also 100 $\Omega$. The characteristic impedance of the line is

(A) $100 \sqrt{2}$ $\Omega$

(B) 50 $\Omega$

(C) 100 $\Omega$

(D) 100 $\Omega$

MCQ 8.3.97
In the relations $S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$, the values of $S$ and $\Gamma$ (where $S$ stands for wave ratio and $\Gamma$ is reflection coefficient), respectively, vary as

(A) 0 to 1 and $-1$ to 0

(B) 1 to $\infty$ and $-1$ to $+1$

(C) $-1$ to $+1$ and 1 to $\infty$

(D) $-1$ to 0 and 0 to 1

MCQ 8.3.98
Consider the following statements:
The characteristic impedance of a transmission line can increase with the increase in
1. resistance per unit length
2. conductance per unit length
3. capacitance per unit length
4. inductance per unit length
Which of these statements are correct?

(A) 1 and 2

(B) 2 and 3

(C) 1 and 4

(D) 3 and 4

**********
SOLUTIONS 8.1

SOL 8.1.1 Option (D) is correct.

Given

the input voltage,

\[ v_i = V_0 \cos(4 \times 10^4 \pi t) \]

and length of transmission line, \( l = 20 \text{ cm} = 20 \times 10^{-2} \text{ m} \)

So, the angular frequency of the applied voltage is

\[ \omega = 4 \times 10^4 \pi \]

and the wavelength of the voltage wave is

\[ \lambda = \frac{v_p}{f} = \frac{2\pi v_p}{\omega} \]

Therefore,

\[ \frac{l}{\lambda} = \frac{\omega(20 \times 10^{-2})}{2\pi v_p} \]

\[ = \frac{(4 \times 10^4 \pi) \times (20 \times 10^{-2})}{2\pi \times (3 \times 10^8)} \] (in free space \( v_p = 3 \times 10^8 \text{ m/s} \))

\[ = 1.33 \times 10^{-2} \]

Since,

\[ \frac{l}{\lambda} \leq 0.01 \]

So the effect of transmission line on the voltage wave is negligible i.e. the output voltage will be in the same phase to the input voltage.

Thus, A and R both are true and R is correct explanation of A.

SOL 8.1.2 Option (C) is correct.

Given,

Inner diameter of coaxial line, \( 2a = 1 \text{ cm} \Rightarrow a = 0.5 \times 10^{-2} \text{ m} \)

and outer diameter of coaxial line, \( 2b = 2 \text{ cm} \Rightarrow b = 10^{-2} \text{ m} \)

Permeability of conductor, \( \mu_c = 2\mu_0 \)

Conductivity of conductor, \( \sigma_c = 11.6 \times 10^7 \text{ S/m} \)

Operating frequency, \( f = 4 \text{ GHz} = 4 \times 10^9 \text{ Hz} \)

So, the resistance per unit length of transmission line is given as :

\[ R' = \frac{1}{2\pi} \sqrt{\frac{\frac{\pi f\mu_c}{\sigma_c}(\frac{1}{a} + \frac{1}{b})}{\frac{\pi \times (4 \times 10^9) \times (2 \times 4\pi \times 10^{-7})}{11.6 \times 10^7} \left( \frac{1}{0.5 \times 10^{-2}} + \frac{1}{10^{-2}} \right)}} \]

\[ = 0.788 \Omega/\text{m} \]

SOL 8.1.3 Option (D) is correct.

Inner diameter of coaxial line, \( 2a = 1.5 \text{ cm} \Rightarrow a = 0.75 \times 10^{-2} \text{ m} \)
Outer diameter of coaxial line, \(b = 3 \text{ cm} \Rightarrow b = 1.5 \times 10^{-2} \text{ m}\)
Permeability of the filled dielectric, \(\mu = 2\mu_0\)
So, it’s inductance per unit length is given as
\[
L' = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) = \frac{2 \times (4\pi \times 10^{-7})}{2\pi} \ln \left( \frac{1.5 \times 10^{-2}}{0.75 \times 10^{-2}} \right)
\]
\[
= 2.77 \times 10^{-7} \text{ H/m} = 277 \text{ nH/m}
\]

**SOL 8.1.4**
Option (D) is correct.
Given,
Inner radius of the coaxial line, \(a = 1/8 \text{ cm} = 1.25 \times 10^{-3} \text{ m}\)
Outer radius of the coaxial line, \(b = 1/2 \text{ cm} = 5 \times 10^{-3} \text{ m}\)
Conductivity of dielectric, \(\sigma = 2 \times 10^{-3} \text{ S/m}\)
So, the conductance per unit length of the transmission line is given as
\[
G' = \frac{2\pi \sigma}{\ln \left( \frac{b}{a} \right)} = \frac{2\pi \times (2 \times 10^{-3})}{\ln \left( \frac{5 \times 10^{-3}}{1.25 \times 10^{-3}} \right)}
\]
\[
= 3.1 \text{ mS/m}
\]

**SOL 8.1.5**
Option (D) is correct.
Given,
Inner diameter of coaxial line, \(2a = 1 \text{ cm} \Rightarrow a = 0.5 \times 10^{-2} \text{ m}\)
Outer diameter of coaxial line, \(2b = 4 \text{ cm} \Rightarrow b = 2 \times 10^{-2} \text{ m}\)
Permittivity of the dielectric, \(\varepsilon = 9\varepsilon_0\)
So, the capacitance per unit length of the line is given as
\[
C' = \frac{2\pi \varepsilon}{\ln \left( \frac{b}{a} \right)} = \frac{2\pi \times 9 \times 8.85 \times 10^{-12}}{\ln \left( \frac{2 \times 10^{-2}}{0.5 \times 10^{-2}} \right)}
\]
\[
= 3.61 \times 10^{-10} \text{ F/m} = 323 \text{ pF/m}
\]

**SOL 8.1.6**
Option (C) is correct.
Given,
Width of strips, \(w = 2.4 \times 10^{-2} \text{ m}\)
Conductivity of strips, \(\sigma = 1.16 \times 10^9 \text{ S/m}\)
Permeability of strips, \(\mu = \mu_0\)
Operating frequency, \(f = 4 \text{ GHz} = 4 \times 10^9 \text{ Hz}\)
So, the parameter \(R'\) is given as
\[
R' = \frac{2}{w} \sqrt{\frac{\pi \mu}{\sigma}} = \frac{2}{2.4 \times 10^{-2}} \sqrt{\frac{4 \times 10^9 	imes 4\pi \times 10^{-7}}{1.16 \times 10^8}}
\]
\[
= 0.9722 \Omega/\text{m}
\]

**SOL 8.1.7**
Option (D) is correct.
Strips width, \(w = 4.8 \text{ cm} = 4.8 \times 10^{-2} \text{ m}\)
Separation between the plates, \(d = 0.3 \text{ cm} = 0.3 \times 10^{-2} \text{ m}\)
Permittivity of dielectric, \(\mu = 2\mu_0\)
So, the inductance per unit length is given as
\[ L' = \frac{\mu d}{w} = 2 \times 4\pi \times 10^{-7} \times 0.3 \times 10^{-2} \]
\[ = 2.57 \times 10^5 \text{H/m} \]

SOL 8.1.8 Option (C) is correct.
The width of strips, \( w = 9.6 \text{ cm} = 9.6 \times 10^{-2} \text{ m} \)
Separation between the strips, \( d = 0.6 \text{ cm} = 0.6 \times 10^{-2} \text{ m} \)
Relative permittivity of dielectric, \( \varepsilon_r = 1.3 \)
Conductivity of dielectric, \( \sigma \approx 0 \)
So, the conductance per unit length of line is given as
\[ G' = \frac{\sigma w}{d} = 0 \quad \sigma \approx 0 \]
and the capacitance per unit length of the line is given as
\[ C' = \frac{\varepsilon_r w}{d} = \varepsilon_r \frac{w}{d} = (8.85 \times 10^{-12}) \times 1.3 \times \frac{9.6 \times 10^{-2}}{0.6 \times 10^{-2}} \]
\[ = 1.84 \times 10^{-10} \text{ F/m} = 0.28 \text{ nF/m} \]

SOL 8.1.9 Option (C) is correct.
Characteristic impedance of a transmission line is defined as
\[ Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \]
and the propagation constant of the transmission line is defined as
\[ \gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} \]
where, \( \alpha \) is attenuation constant
\( \beta \) is phase constant
\( R' \) is resistance per unit length of the line
\( G' \) is conductance per unit length of the line
\( L' \) is inductance per unit length of the line
\( C' \) is capacitance per unit length of the line
Now, for lossless line, \( R' = G' = 0 \)
So, the characteristic impedance of lossless transmission line is
\[ Z_0 = \sqrt{\frac{L'}{C'}} \]
and the propagation constant of lossless transmission line is
\[ \gamma = \alpha + j\beta = j\omega \sqrt{L'/C'} \]
or
\[ \alpha = 0 \]
Therefore, the attenuation constant of lossless line is always zero (real).
i.e. statement (A) is correct.
Again for distortionless line,
\[ \frac{R'}{L'} = \frac{G'}{C'} \]
So, the characteristic impedance of distortionless line is
\[ Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{R'}{G'}} \]

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
and the propagation constant of the distortionless line is
\[ \gamma = \alpha + j\beta = \sqrt{R'G' + j\omega L'C'} \]
or,
\[ \alpha = \sqrt{R'G'} \neq 0 \]
Therefore, the attenuation constant of distortionless line is not zero but it is real.
Thus, (A) and (B) is correct statement while (C) is not a correct statement.

SOL 8.1.10 Option (C) is correct.
Inductance per unit length, \[ L' = 250 \text{nH/m} = 250 \times 10^{-9} \text{H/m} \]
Capacitance per unit length \[ C' = 0.1 \text{nF/m} = 0.1 \times 10^{-9} \text{F/m} \]
So, the velocity of wave propagation along the lossless transmission line is given as
\[ v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(250 \times 10^{-9})(0.1 \times 10^{-9})}} \]
\[ = 4 \times 10^8 \text{m/s} \]
The characteristic impedance of the lossless transmission line is given as
\[ Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{250 \times 10^{-9}}{0.1 \times 10^{-9}}} \]
\[ = 50 \Omega \] (for lossless line, \( R' = G' = 0 \))

SOL 8.1.11 Option (A) is correct.
Operating frequency, \( f = 1 \text{GHz} = 10^9 \text{Hz} \)
Conductivity, \( \sigma = 6.4 \times 10^7 \text{S/m} \)
Permittivity, \( \varepsilon = 6\varepsilon_0 \)
Axial component of electric field = \( E_z \)
Transverse component of electric field = \( E_y \)
So, the ratio of the two components for the transmission line is
\[ \frac{E_z}{E_y} = \sqrt{\frac{\omega \varepsilon}{\sigma}} = \sqrt{\frac{2\pi \times 10^9 \times 6\varepsilon_0}{6.4 \times 10^7}} \]
\[ = 9.23 \times 10^{-4} \] \((\omega = 2\pi f)\)

SOL 8.1.12 Option (B) is correct.
Given the operating angular frequency of the transmission line is
\[ \omega = 6 \times 10^7 \text{rad/s} \]
and the parameters of transmission line are
\[ R' = 0.2 \text{kΩ/m} = 200 \text{Ω/m} \]
\[ L' = 4 \mu\text{H/m} = 4 \times 10^{-6} \text{H/m} \]
\[ G' = 8 \mu\text{S/m} = 8 \times 10^{-6} \text{S/m} \]
\[ C' = 4 \text{pF/m} = 4 \times 10^{-12} \text{F/m} \]
So, the propagation constant of the transmission line is given as
\[ \gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \]
\[ = \sqrt{[200 + j(6 \times 10^7)(4 \times 10^{-6})][(8 \times 10^{-6}) + j(6 \times 10^7)(4 \times 10^{-12})]} \]
\[ = \sqrt{[200 + j(24 \times 10^7)][(8 \times 10^{-6}) + j(24 \times 10^{-7})]} \]
\[ = (2.10 + j1.7) \text{ per meter} \]

SOL 8.1.13 Option (C) is correct.
Given the operating angular frequency of the transmission line.
\[ \omega = 1.2 \times 10^9 \text{ rad/s} \]

and the parameters of transmission line are

\[ R' = 10 \Omega/m \]
\[ L' = 0.4 \mu\text{H/m} = 0.1 \times 10^{-6} \text{ H/m} \]
\[ C' = 10 \text{pF/m} = 10 \times 10^{-12} \text{ F/m} \]
\[ G' = 40 \mu\text{S/m} = 40 \times 10^{-6} \text{ S/m} \]

So, the characteristic impedance of the line is given as

\[
Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{10 + j(1.2 \times 10^9)(0.1 \times 10^{-6})}{40 \times 10^{-6} + j(1.2 \times 10^9)(10 \times 10^{-12})}}
\]
\[ = 75 - j5 \Omega \]

**SOL 8.1.14** Option (B) is correct.

The amplitude of voltage wave after travelling a distance \( l \) along a transmission line is given as

\[ V_1 = V_0 e^{-\alpha l} \]

where \( V_0 \) is the amplitude of the source voltage wave.

Now, in the given problem, after travelling 20 m distance along the transmission line the voltage wave remains 13% of it’s source amplitude. So, we get

\[ V_1 = V_0 e^{-\alpha l} = 13\% \text{ of } V_0 \]

\[ e^{-\alpha(20)} = 0.13 \]

\[ \alpha = 0.10 \text{ NP/m} \]

\((l = 20 \text{ m})\)

**SOL 8.1.15** Option (B) is correct.

Given the propagation constant of the voltage wave

\[ \gamma = \alpha + j\beta = 0.5 + j2.4 \]

So, we get the attenuation constant of the wave

\[ \alpha = 0.5 \]

and phase constant of the wave along the transmission line is

\[ \beta = 2.4 \]

Since, the amplitude of voltage wave after travelling a distance \( l \) along a transmission line is given as

\[ V_1 = V_0 e^{-\alpha l} \]

where \( V_0 \) is the amplitude of the source voltage wave. Since the amplitude of a voltage wave after travelling a certain distance down a transmission line is reduced by 87% so, for the given transmission line we have

\[ V_1 = V_0 e^{-\alpha l} = \left(1 - \frac{87}{100}\right) V_0 \]

\[ e^{-\alpha l} = 0.13 \]

\[ l = \frac{1}{\alpha} \ln \left(\frac{1}{0.13}\right) = 4.08 \text{ m} \]

Therefore, the shift in phase angle for the travelled distance is given as

\[ \phi = \beta l \left(\frac{360^\circ}{2\pi}\right) = (2.4)(4.08)\left(\frac{360^\circ}{2\pi}\right) = 561^\circ \]
SOL 8.1.16 Option (C) is correct.

The width of strips = \( w \)
Separation between strips = \( d \)
So, the characteristic impedance of lossless transmission line is given as
\[
Z_0 = \frac{d}{w} \sqrt{\frac{\mu}{\varepsilon}}
\]

When \( d \) and \( W \) is doubled, the characteristic impedance of the transmission line will be given as
\[
Z_0' = \frac{2d}{2W} \sqrt{\frac{\mu}{\varepsilon}} = Z_0
\]

Therefore, the characteristic impedance will remain same.

SOL 8.1.17 Option (A) is correct.

Attenuation constant, \( \alpha = 10 \text{ mNP/m} = 10^{-2} \text{ NP/m} \)
Characteristic impedance, \( Z_0 = 0.1 \text{ k\Omega} = 100 \Omega \)
Phase velocity, \( v_p = 0.5 \times 10^8 \text{ m/s} \)
Since the transmission line is distortion less so, the resistance per unit length of the transmission line is given as
\[
R' = \alpha Z_0 = (10^{-2})(100) = 1 \Omega/\text{m}
\]
and the inductance per unit length of the lossless transmission line is given as
\[
L' = \frac{Z_0}{v_p} = \frac{100}{0.25 \times 10^8} = 4 \mu\text{H/m}
\]

SOL 8.1.18 Option (C) is correct.

Given the parameters of distortionless transmission line are
\[
R' = 4 \Omega/\text{m}
\] and
\[
G' = 4 \times 10^{-4} \text{ S/m}
\]
So, the attenuation constant of the distortion less transmission line is given as
\[
\alpha = \sqrt{R'G'} = \sqrt{4 \times 4 \times 10^{-4}} = 4 \times 10^{-2} \text{ NP/m}
\]
and the characteristic impedance of the distortionless transmission line is given as
\[
Z_0 = \sqrt{\frac{R'}{G'}} = \sqrt{\frac{4}{16 \times 10^{-4}}} = 50 \Omega
\]
distortionless line

SOL 8.1.19 Option (D) is correct.

Operating frequency, \( f = 5 \text{ GHz} = 5 \times 10^9 \text{ Hz} \)
Characteristic impedance, \( Z_0 = 80 \Omega \)
Phase constant, \( \beta = 1.5 \text{ rad/m} \)
So, the inductance per unit length of the transmission line is given as
\[
L' = \frac{\beta Z_0}{\omega} = \frac{1.5 \times 80}{2\pi \times 5 \times 10^9} \quad (\omega = 2\pi f)
\]
\[
= 4.88 \text{ nH/m}
\]

SOL 8.1.20 Option (A) is correct.

Load impedance, \( Z_L = 300 \Omega \)
Characteristic impedance, \( Z_0 = 150 \Omega \)
So, the reflection coefficient at the load terminal is given as
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 150}{300 + 150} = \frac{1}{3} \]
and the reflection coefficient at generator end is given as
\[ \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} \]
where \( Z_g \) is internal impedance of the generator. Since, it is given that the internal resistance of the generator is zero (i.e., \( Z_g = 0 \)) so, we get
\[ \Gamma_g = \frac{0 - 150}{0 + 150} = -1 \]

**SOL 8.1.21** Option (D) is correct.
The maximum magnitude of voltage wave, \( V_{\text{max}} = 6 \text{ volt} \)
The minimum magnitude of voltage wave, \( V_{\text{min}} = 2.4 \text{ volt} \)
So, the standing wave ratio on the transmission line is given as
\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{6}{2.4} = 2.5 \]
Therefore, the reflection coefficient of the transmission line is evaluated as
\[ \Gamma = S - 1 = \frac{2.5 - 1}{2.5 + 1} = 0.43 \]

**SOL 8.1.22** Option (B) is correct.
Characteristic impedance, \( Z_0 = 25 \Omega \)
Inner radius of the coaxial line, \( a = 0.6 \text{ mm} = 0.6 \times 10^{-3} \)
Permittivity of insulated material, \( \varepsilon = 9 \varepsilon_0 \Rightarrow \varepsilon_r = 9 \)
Now, the characteristic impedance of a lossless coaxial line is given as
\[ Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln \left( \frac{b}{a} \right) \]
where \( b \) is the outer radius of the coaxial line. So, we get
\[ 25 = \frac{60}{\sqrt{9}} \ln \left( \frac{b}{0.6 \times 10^{-3}} \right) \]
or,
\[ b = (0.6 \times 10^{-3}) e^{50.57} \approx 0.0021 \text{ m} = 3.1 \text{ mm} \]

**SOL 8.1.23** Option (B) is correct.
Load impedance, \( Z_L = (15 - j25) \Omega \)
Characteristic impedance \( Z_0 = 25 \Omega \)
So, the reflection coefficient of the transmission line is given as
\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(15 - j25) - 25}{(15 - j25) + 25} = 0.57 e^{-j0.8^\circ} \]
Therefore, the standing wave ratio of the transmission line is determined as
\[ S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65 \]

**SOL 8.1.24** Option (C) is correct.
Characteristic impedance, \( Z_0 = 50 \Omega \)
Voltage standing wave ratio, \( S = 3 \)
Since, the load connected to the lossless transmission line is purely resistive so, phase angle of the reflection coefficient of the line will be 
\[ \theta_r = 0 \text{ or } \pi \]

Now, the magnitude of the reflection coefficient is given as
\[ |\Gamma| = \frac{S - 1}{S + 1} = \frac{3 - 1}{3 + 1} = 0.5 \]

So, reflection coefficient of the transmission line is
\[ \Gamma = |\Gamma| e^{j\theta} \]
\[ = 0.5 e^0 \text{ or } 0.5 e^{j\pi} \]
\[ = 0.5 \text{ or } -0.5 \]

For \( \Gamma = 0.5 \) the load impedance of the transmission line is given as
\[ Z_L = Z_0 \left[ \frac{1 + \Gamma}{1 - \Gamma} \right] = 150 \left[ \frac{1 + 0.5}{1 - 0.5} \right] = 450 \Omega \]

and for \( \Gamma = -0.5 \) the load impedance of the transmission line is given as
\[ Z_L = Z_0 \left[ \frac{1 + \Gamma}{1 - \Gamma} \right] = 150 \left[ \frac{1 - 0.5}{1 + 0.5} \right] = 75 \Omega \]

Therefore, the possible values of load impedance connected to the transmission line are
\[ Z_L = 50 \Omega \text{ or } 450 \Omega \]

**SOL 8.1.25** Option (A) is correct.

Load impedance,
\[ Z_L = (200 - j200) \Omega \]

Characteristic impedance,
\[ Z_0 = 100 \Omega \]

Length of transmission line,
\[ l = 10 \text{ cm} = 10 \times 10^{-2} = 0.1 \text{ m} \]

Generator voltage,
\[ v_p(t) = 3\cos(\pi \times 10^9 t) \text{ volt} \]

So, we get the angular frequency
\[ \omega = \pi \times 10^9 \]

and the phase constant of the wave on the transmission line is
\[ \beta = \frac{\omega}{v_p} = \frac{\pi \times 10^9}{3 \times 10^8} = \frac{10\pi}{3} \text{ (in air } v_p = 3 \times 10^8 \text{ m/s)} \]

or
\[ \beta l = \frac{10\pi}{3} \times 0.1 = \frac{\pi}{3} \]

Therefore, the input impedance of the lossless transmission line is given as
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]
\[ = 100 \left( \frac{200 - j200 + j100\tan(\pi/3)}{100 + j(200 - j200)\tan(\pi/3)} \right) \]
\[ = (35 - j35.4) \Omega \]

**SOL 8.1.26** Option (A) is correct.

Load impedance, \( Z_L = 0 \) (Short circuit)

Input impedance, \( Z_{in} = \infty \) (Open circuit)

and, wave length \( = \lambda \)

Now, the input impedance of lossless transmission line is defined as
where, \( l \) is the length of the transmission line and \( \beta \) is the phase constant of the voltage wave along the transmission line. So, we get

\[
\tan \beta l = \frac{Z_0(0 + jZ_0\tan \beta l)}{(Z_0 + j0\tan \beta l)}
\]

or,

\[
\tan \beta l = \infty
\]

or, \( \beta l = \pi/2 \) (for minimum length)

Therefore, the minimum required length of the transmission line is

\[
l = \frac{\pi}{2 \times \frac{1}{\beta}} = \frac{\pi}{2} \times \frac{\lambda}{2\pi} = \lambda/2
\]

SOL 8.1.27 Option (A) is correct.

Given,

Operating frequency, \( f = 2 \text{ MHz} = 2 \times 10^6 \text{ Hz} \)

So, the angular frequency of voltage wave is

\( \omega = 2\pi f = 4\pi \times 10^6 \text{ rad/sec} \)

When the line is short circuited, input impedance is

\[
Z_{in}^{sc} = j\omega L = j(4\pi \times 10^6)(32 \times 10^{-3}) = 0.4 \Omega
\]

( equitable to 32 nH inductance)

When the line is open circuited, input impedance is

\[
Z_{in}^{oc} = \frac{1}{j\omega C} = \frac{1}{j(4\pi \times 10^6)(20 \times 10^{-12})} = -j3979.9 \Omega
\]

( equivalent to 20 pF capacitance)

Therefore, the characteristic impedance of the transmission line is given as

\[
Z_0 = \sqrt{Z_{in}^{sc}Z_{in}^{oc}} = \sqrt{j(0.4)(-j3978.9)} = 20 \Omega
\]

SOL 8.1.28 Option (D) is correct.

Given, the length of the transmission lines 1 and 2

\( l_1 = l_2 = \lambda/4 \)

So, the input impedance for line 1 is given as :

\[
Z_{in1} = \frac{Z_0^2}{Z_L} = \frac{(100)^2}{150} = \frac{200}{3} \Omega
\]

From the shown arrangement of the transmission line it is clear that the effective load for line 2 will be equal to the input impedance of line 1.

i.e.

\[
Z_L' = Z_{in1} = \frac{200}{3} \Omega
\]

Therefore, the input impedance for the whole combination is
SOL 8.1.29 Option (C) is correct.
Since the transmission line has one short circuited and one open circuited end so at the short circuit end voltage must be zero while at open circuit end voltage must be maximum. So the voltage standing wave pattern will be half sinusoids with zeros at short circuited end and maxima at the open circuited end.

\[
Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(200)^2}{(200)/3} = 300 \Omega
\]

SOL 8.1.30 Option (C) is correct.
The natural frequency of oscillation of a wave in a transmission line of length \( l \) which is open circuited at one end and short circuited at other end is given as

\[
f_n = \frac{(2n + 1)v_p}{4l}, \quad n = 1, 2, 3, ..., \infty
\]

where \( v_p \) is phase velocity of the wave.

SOL 8.1.31 Option (C) is correct.
The dimension of the transmission line will remain same at all frequencies i.e. \( l \) will be constant but as it is defined in terms of wavelength which changes with the frequency so, the expression for length will vary in terms of wavelength \( \lambda \). The wavelength of a wave is defined in terms of frequency \( f \) as

\[
\lambda = \frac{c}{f}
\]

where, \( c \) is the velocity of wave in free space so, at \( f = 500 \) Hz we have

\[
\lambda = \frac{c}{500}
\]

Therefore, the length of transmission line is

\[
l = \frac{\lambda}{4} = \frac{c}{2000} \quad (1)
\]

Now, the wavelength at frequency, \( f = 1 \) kHz = 1000 Hz is given as

\[
\lambda = \frac{c}{1000} \quad (2)
\]

Since, the length of the transmission line will be same as determined in equation (1). So, we get

\[
l = \frac{c}{2000} = \frac{(c/1000)}{2} = \frac{\lambda}{2}
\]

(from eq. (2))
SOL 8.1.32  
Option (D) is correct.  
Given, the transmission line is terminated in short circuit i.e., $Z_L = 0$ and line should be short circuited at its input terminal i.e. $Z_{in} = 0$.  
The input impedance of a lossless transmission line is defined as  
$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$  
So,  
$$0 = Z_0 \left( 0 + \frac{jZ_0 \tan \beta l}{Z_0 + 0} \right) \quad (Z_L = 0, Z_{in} = 0)$$  
$$j \tan \beta l = 0$$  
$$\beta l = 0, \pi, 2\pi, \ldots$$  
Since, length of transmission line can’t be zero i.e., $l \neq 0$, so, we get  
$$l = \frac{\pi}{\beta} \Rightarrow l = \frac{\pi}{(2\pi/\lambda)} \Rightarrow l = \frac{\lambda}{2}$$  

SOL 8.1.33  
Option (D) is correct.  
Given, the distance between successive maxima and minima is 10 cm. i.e.  
$$\frac{\lambda}{2} = 10 \text{ cm}$$  
$$\lambda = 20 \text{ cm}$$  
Now, the distance between first minima and load is  
$$l_{min} = 7.5 \text{ cm}$$  
$$l_{min} > \frac{\lambda}{4}$$  
So, the distance between first maxima and load will be  
$$l_{max} = l_{min} - \frac{\lambda}{4} = 7.5 - 7.5 \times \frac{\lambda}{20} - \frac{\lambda}{4} = \frac{\lambda}{2}$$  

***********
SOLUTIONS 8.2

SOL 8.2.1 Option (C) is correct.

Since, the TEM wave is z-polarized i.e. the electric field of the wave is directed along \( +a_z \).

i.e. \( a_E = a_z \)

and the direction of wave propagation is along \( a_x \).

i.e. \( a_k = a_x \)

So, the direction of magnetic field intensity will be

\[ a_H = a_x \times a_E = a_x \times a_z = -a_y \]

As \( E \) is in \( +a_z \) direction and \( H \) is in \( -a_y \) direction so, we can consider the two vectors as

\[ E = E_x a_z \] (1)

and

\[ H = -H_y a_y \] (2)

Now, from the Maxwell’s equation in phasor form we have

\[ \nabla \times E = j \omega E \]

(for perfect dielectric \( \sigma = 0 \))

\[ \begin{vmatrix} a_z & a_y & a_x \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -H_y & 0 \end{vmatrix} = j \omega E, a_z \]

using equation (1) and (2)

\[ \frac{\partial H_y}{\partial z} a_x - \frac{\partial H_x}{\partial x} a_z = j \omega E, a_x \]

It gives the result as

\[ \frac{\partial H_y}{\partial z} = 0 \]

Again from Maxwell’s equation we have

\[ \nabla \times H = -j \omega H \]

\[ \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_z \end{vmatrix} = -j \omega H, a_y \]

using equation (1) and (2)

\[ \frac{\partial E_y}{\partial y} a_x - \frac{\partial E_x}{\partial x} a_y = j \omega H, a_y \]

So, it gives the result as

\[ \frac{\partial E_y}{\partial y} = 0 \]

Thus, Both (A) and (B) are correct.
SOL 8.2.2 Option (C) is correct.
The voltage maximum exists at the point where the incident and the reflected voltage wave both are in same phase and the distance of voltage maximum from the load is given as

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} \]  

(1)

where \( \theta_r \) is phase angle of reflection coefficient, \( \lambda \) is the wavelength of the voltage wave and \( n = 0, 1, 2, \ldots \).

Now, the reflection coefficient of a transmission line is given as

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0.3 - j0.5 - 0.5}{0.3 - j0.5 + 0.5} \]

\[ = -0.2 - j0.5 \]

\[ = 0.57e^{-79.8^\circ} \]

i.e.

\[ \theta_r = -79.8^\circ \]

So, from equation (1) for \( n = 0 \) we have

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4\pi} = \frac{-79.8^\circ \times 4 \times 10^{-2}}{4\pi} \]

\[ = -0.44 \times 10^{-2} \text{ m} \]

which is negative (i.e. the point doesn’t exist). Therefore, the 1st maximum voltage will exist for \( n = 1 \) and the distance of the 1st maximum from the load is

i.e.

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} \]  \hspace{1cm} (n = 1)

\[ = -0.44 \times 10^{-2} + 2 \times 10^{-2} = 1.56 \times 10^{-2} \text{ m} \]

\[ = 2.56 \text{ cm} \]

SOL 8.2.3 Option (B) is correct.
In a lossless transmission line, the current maximum lies at the same point where the voltage minima lies and similarly, the current minima lies at the same point where the voltage maxima lies as shown in the figure below:

Now, it is clear from the figure that the distance between two adjacent maxima and minima is \( \lambda/4 \).
\[ l_{\text{max}} - l_{\text{min}} = \frac{\lambda}{4} \]

Since the maximum voltage wave lies at a distance
\[ l_{\text{max}} = 1.56 \text{ cm} \]

So, the distance of 1st voltage minimum (the distance of 1st current maxima) from the load will be
\[ l_{\text{min}} = l_{\text{max}} - \frac{\lambda}{4} = 1.56 - \frac{4}{4} = 0.24 \text{ cm} \]

Thus, the distance of 1st current maximum from the load is 0.56 cm.

**SOL 8.2.4**

Option (C) is correct.

Given,

The position of first voltage maximum, \( l_{\text{max}} = 4.5 \text{ cm} \)

Position of first current maximum(voltage minima), \( l_{\text{min}} = 1.5 \text{ cm} \)

Standing wave ratio, \( S = 3 \)

Characteristic impedance, \( Z_0 = 50 \Omega \)

Since, the distance between a maximum and an adjacent minimum is \( \lambda/4 \) as discussed in previous question.

\[ l_{\text{max}} - l_{\text{min}} = \frac{\lambda}{4} 
\]

\[ 4.5 - 1.5 = \frac{\lambda}{4} \]

\[ \lambda = 12 \text{ cm} \]

Again the distance of first voltage maximum from the load is given as

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4 \pi} + \frac{n \lambda}{2} \]

\[ 4.5 = \frac{\theta_r (12)}{4 \pi} + 0 \]

\[ \theta_r = \frac{3 \pi}{2} \]

Now, the magnitude of reflection coefficient is given as

\[ |\Gamma| = \left| \frac{S - 1}{S + 1} \right| = \frac{3 - 1}{3 + 1} = \frac{2}{4} = 0.5 \]

So, the reflection coefficient of the transmission line is

\[ \Gamma = |\Gamma| |\theta_r| = 0.5 \times \frac{3 \pi}{2} = 0.5 e^{3\pi/2} = -j0.5 \]

Therefore, the load impedance of the transmission line is given as

\[ Z_L = Z_0 \left[ \frac{1 + \Gamma}{1 - \Gamma} \right] = 50 \left[ \frac{1 - j0.5}{1 + j0.5} \right] \]

\[ = (30 - j10) \Omega \]

**SOL 8.2.5**

Option (A) is correct.

Characteristic impedance, \( Z_0 = 50 \Omega \)

Load impedance, \( Z_L = (30 + j15) \Omega \)

Length of transmission line, \( l = 7\lambda/20 \)

Since, the transmission line is lossless so, the attenuation constant is zero.
i.e. \( \alpha = 0 \)
or, \( \gamma = \alpha + j\beta = j\beta \)

Therefore, the input impedance of the lossless transmission line is given as

\[
Z_{in} = Z_0 \left( \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right) = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \quad (\gamma = j\beta)
\]

\[
= Z_0 \left( \frac{30 + j15 + j50 \tan \left( \frac{2\pi 20}{\lambda} \right)}{50 + j(30 + j15) \tan \left( \frac{2\pi 20}{\lambda} \right)} \right) \quad (\beta = \frac{2\pi}{\lambda})
\]

\[
= Z_0 \left( \frac{30 + j15 + j50 \tan \left( \frac{7\pi 10}{\lambda} \right)}{50 + j(30 + j15) \tan \left( \frac{7\pi 10}{\lambda} \right)} \right)
\]

\[
= (18.4 - j19.2) \Omega
\]

**SOL 8.2.6**
Option (B) is correct.

In the assertion (A) given,
Length of the transmission line, \( l = \lambda/4 \)
Load impedance, \( Z_L = 0 \)

So, we get
\[
\beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{4} - \frac{\pi}{2} \right)
\]

(\( \beta = \frac{2\pi}{\lambda} \))

The input impedance of the lossless transmission line is given as

\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)
\]

\[
= Z_0 \left( \frac{jZ_0 \tan \frac{\pi}{2}}{Z_0} \right) = j\infty
\]

Now, we consider the reason part,
Distance of the maxima from load is given as
\[
l_{\text{max}} = \left( \theta_r + 2n\pi \right) / 2\beta
\]

where,
\( \theta_r \) is the phase angle of reflection coefficient
\( \beta \) is the phase constant of the voltage wave

and
\( n = 0,1,2,.... \)

Therefore, the input impedance at the point of maxima is given as

\[
Z_{in} = Z_0 \left( \frac{1 + \Gamma e^{-j\theta_r}}{1 - \Gamma e^{-j\theta_r}} \right) = Z_0 \left( \frac{1 + \left| \Gamma \right| e^{j\theta_r} e^{-j\theta_r}}{1 - \left| \Gamma \right| e^{j\theta_r} e^{-j\theta_r}} \right) \quad (\Gamma = \left| \Gamma \right| e^{j\theta})
\]

So, \( Z_{in} \) is real if \( Z_0 \) is real and since, \( Z_0 \) is always real for a distortionless line. Thus, \( Z_{in} \) will be purely real at the position of voltage maxima in a distortionless line.

i.e. A and R both are true but R is not the explanation of A.

**SOL 8.2.7**
Option (A) is correct.

Length of transmission line, \( l = 6 \) m
Characteristic impedance, \( Z_0 = 30 \, \Omega \)
Relative permittivity, \( \varepsilon_r = 2.25 \)
Load impedance, \( Z_L = (30 - j10) \, \Omega \)

So, we get the angular frequency,
\[
\omega = 8\pi \times 10^7
\]
and the phase constant of the voltage wave along the transmission line is
\[
\beta = \frac{\omega}{v_p} = \frac{8\pi \times 10^7}{c/\sqrt{\varepsilon_r}} = \frac{8\pi \times 10^7 \times \sqrt{2.25}}{3 \times 10^8} = \frac{8\pi \times 10^7 \times 1.5}{3 \times 10^8} = \frac{2\pi}{5} \text{ rad/cm}
\]
or,
\[
\beta l = \frac{2\pi}{5} \times 6 = 2.4\pi \text{ rad}
\]
Therefore, the input impedance of the lossless transmission line is given as
\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = 30 \left( \frac{30 - j10 + j30 \tan(2.4\pi)}{30 + j(30 - j10)\tan(2.4\pi)} \right) = (12.14 + j5.48) \, \Omega
\]

**SOL 8.2.8**
Option (C) is correct.

Given the generator voltage to the transmission line,
\[
V_g(t) = 10\cos(8\pi \times 10^7 t - 30^\circ)
\]
So, in phasor form the generator voltage is
\[
V_g = 10e^{-j30^\circ}
\]
and as determined in previous question, the input impedance of transmission line is
\[
Z_{in} = (23.14 + j5.48) \, \Omega
\]
So, for determining the input voltage, we draw the equivalent circuit for the transmission line as shown in figure below:

Using voltage division, we get the input voltage to the transmission line as
\[
V_{in} = V_g \times \left( \frac{Z_{in}}{Z_{in} + Z_g} \right)
\]
or,
\[
V_{s,in} = V_g \left( \frac{Z_{in}}{Z_{in} + Z_g} \right) \quad \text{(in phasor form)}
\]
\[
= 10e^{-j30^\circ} \left( \frac{23.14 + j5.48}{30 + 23.14 + j5.48} \right) = (15e^{-j35^\circ})(0.44e^{j0.44^\circ})
\]
Thus, the instantaneous input voltage of the transmission line is
\[ v_{in}(t) = \Re\{V_{in} e^{j\omega t}\} \]
\[ = 4.4 \cos(8\pi \times 10^7 t - 22.56^\circ) \text{ volt} \]

**SOL 8.2.9** Option (C) is correct.

Given,

Load impedances to the line 1 and 2,
\[ Z_{L1} = Z_{L2} = 150 \Omega \]

Length of the transmission lines 1 and 2,
\[ l_1 = l_2 = \frac{\lambda}{5} \]

Now, we consider the input impedance of line 1 and line 2 be \( Z_{in1} \) and \( Z_{in2} \) respectively. Since, the transmission line are identical so, the input impedances of the transmission lines 1 and 2 will be equal and given as
\[
Z_{in1} = Z_{in2} = Z_0 \left( \frac{Z_{L1} + jZ_0 \tan \beta l_1}{Z_0 + jZ_{L1} \tan \beta l_1} \right) \quad \text{(lossless transmission line)}
\]
\[
= 100 \left( \frac{150 + j100 \tan \frac{2\pi \lambda}{5}}{100 + j150 \tan \frac{2\pi \lambda}{5}} \right) \quad (\beta = \frac{2\pi}{\lambda})
\]
\[
= 100 \left( \frac{150 + j100 \tan \frac{2\pi}{5}}{100 + j150 \tan \frac{2\pi}{5}} \right)
\]
\[
= (70.4 - j17.24) \Omega
\]

Therefore, the effective load impedance of the feedline will be equal to the equivalent input impedance of the parallel combination of the line 1 and 2.

i.e.
\[ Z'_{L} = \frac{Z_{in1} \parallel Z_{in2}}{2} \]
\[ = \frac{(70.4 - j17.24)}{2} \quad Z_{in1} = Z_{in2}
\]
\[ = (35.20 - j8.62) \Omega
\]

**SOL 8.2.10** Option (B) is correct.

Given the length of the feed line,
\[ l = 0.3\lambda \]

and as calculated in above question, the effective load impedance of the feedline is
\[ Z'_{L} = (35.20 - j8.62) \Omega \]

So,
\[ \beta l = \left( \frac{2\pi}{\lambda} \right) (0.3\lambda) = 0.6\lambda \]

Therefore, input impedance of the feedline (lossless transmission line) is given as
\[ Z_{in} = Z_0 \left( \frac{Z'_{L} + jZ_0 \tan \beta l}{Z_0 + jZ'_{L} \tan \beta l} \right) \]
\[ = 100 \left( \frac{35.20 - j8.62 + j100 \tan(0.6\pi)}{100 + j(35.20 - j8.62) \tan(0.6\pi)} \right)
\]
\[ = (215.14 - j113.4) \Omega
\]

**SOL 8.2.11** Option (B) is correct.
Operating frequency \( f = 0.3 \text{ GHz} = 0.3 \times 10^9 \text{ Hz} \)
Load impedance, \( Z_L = (100 - j100) \Omega \)
Characteristic impedance \( Z_0 = 100 \Omega \)
Generator voltage in phasor form, \( V_{sg} = 150 \text{ volt} \)
Internal resistance of generator \( Z_g = 100 \Omega \)
Length of the transmission line, \( l = 0.375 \lambda \)

So, the input impedance of the lossless transmission line is given as
\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)
\]
\[
= 100 \left( \frac{100 - j100 + j100 \tan \left( \frac{2\pi}{\lambda} \cdot 0.375 \lambda \right)}{100 + j(100 - j100) \tan \left( \frac{2\pi}{\lambda} \cdot 0.375 \lambda \right)} \right)
\]
\[
= (200 + j100) \Omega
\]

Now, for determining the load current, we draw the equivalent circuit for the transmission line as shown in the figure below:

![Equivalent Circuit](image)

Therefore, the voltage across the input terminal of the transmission line is given as
\[
V_{s, in} = V_{sg} \left( \frac{Z_{in}}{Z_g + Z_{in}} \right)
\]
\[
= 150 \left( \frac{200 + j100}{100 + 200 + j100} \right) = 106.1 e^{8.13^\circ}
\]

Since, at any point, on the transmission line voltage is given as
\[
V(z) = V_0^+ (e^{\beta l} + \Gamma e^{\beta z})
\]
where \( V_0^+ \) is the voltage due to incident wave, \( \Gamma \) is the reflection coefficient of the transmission line at load terminal and \( z \) is the distance of the point from load as shown in figure. So, for \( z = -l \)
\[
V_{in} = V_0^+ (e^{\beta l} + \Gamma e^{-j\beta l})
\]

Now, the reflection coefficient of the transmission line at load terminal is
\[
\Gamma = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{100 - j100 - 100}{100 - j100 + 100} = 0.45e^{-j0.35}\, \text{rad} \, \text{s}^{-1}
\]

Putting the value of \(\Gamma\) and \(V_{s,m}\) in equation (1), we get
\[
106.1e^{j0.35} = V_0^+ \left( e^{j(\frac{2\pi}{\lambda})0.375\lambda} + 0.45e^{-j0.35}\right)
\]
\[
V_0^+ = \frac{106.1e^{j0.35}}{e^{j0.35} + 0.45e^{-j0.35}}
\]
\[
= 75e^{-j0.35}\, \text{rad} \, \text{s}^{-1}
\]

The current at any point on the transmission line is given as
\[
I(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z})
\]

So, the current flowing in the load (at \(z = 0\)) is
\[
I_L = \frac{V_0^+}{Z_0}(1 - \Gamma) = \frac{75e^{-j0.35}}{100}(1 - 0.45e^{-j0.35})
\]
\[
= 0.67e^{-j0.4}\, \text{rad} \, \text{s}^{-1}
\]

Therefore, the instantaneous current at the load terminal will be
\[
\text{\(i_L(t) = Re\{I_L e^{j\omega t}\} = 0.67\cos(2\pi \times 0.3 \times 10^2 \, t - 108.4^\circ)\)}
\]
\[
= 0.75\cos(3\pi \times 10^2 \, t - 108.4^\circ)
\]

**SOL 8.2.12** Option (C) is correct.

Generator voltage in phasor form, \(V_g = 150 \, \text{V}\)

Internal impedance of generator, \(Z_g = 100 \, \Omega\)

Load impedance \(Z_L = 150 \, \Omega\)

Length of transmission line, \(l = 0.15\lambda\)

Characteristic impedance \(Z_0 = 100 \, \Omega\)

So, the input impedance of the lossless transmission line is given as
\[
Z_{in} = Z_0 \left( \frac{Z_L + jZ_0\tan \beta l}{Z_0 + jZ_L\tan \beta l} \right)
\]
\[
= 100 \left( \frac{150 + j100\tan \left( \frac{2\pi}{\lambda} \times 150 \right)}{100 + j150\tan \left( \frac{2\pi}{\lambda} \times 0.15 \right)} \right)
\]
\[
= 100 \left( \frac{150 + j100\tan 54^\circ}{100 + j50\tan 54^\circ} \right) = (80.5 - j32.7) \, \Omega
\]

Now, for determining the power delivered, we draw the equivalent circuit for the transmission line as shown in figure below:

Using voltage division, we get the input voltage as
\[ V_{s,\text{in}} = V_s \left( \frac{Z_m}{Z_m + Z_g} \right) = 150 \left( \frac{82.5 - 32.7}{82.5 - 32.7 + 100} \right) = 71.8 e^{-j1.46^\circ} \]

So, the current at the input current is
\[ I_{s,\text{in}} = \frac{V_{s,\text{in}}}{Z_m} = 71.8 e^{-j1.46^\circ} = 0.81 e^{j0.16^\circ} \]

Therefore, the average input power delivered to the transmission line is given as
\[ P_{\text{in}} = \frac{1}{2} \text{Re}[V_{s,\text{in}} I_{s,\text{in}}^*] = \frac{1}{2} \text{Re}[(71.8 e^{-j1.46^\circ})(0.81 e^{j0.16^\circ})] \]
\[ = 27 \text{ Watt} \]

SOL 8.2.13 Option (C) is correct.

Since, the lengths of line 1 and line 2 are
\[ l_1 = l_2 = \lambda/2 \]

So, the input impedance of the line 1 is given as
\[ Z_{\text{in}1} = Z_0 \left( \frac{Z_{l1} + jZ_0 \tan \beta l}{Z_0 + jZ_{l1} \tan \beta l} \right) \]
\[ = Z_0 \left( \frac{Z_{l1} + jZ_0 \tan \left( \frac{2\pi \lambda}{2} \right)}{Z_0 + jZ_{l1} \tan \left( \frac{2\pi \lambda}{2} \right)} \right) \]
\[ = Z_0 \left( \frac{Z_{l1} + 0}{Z_0 + 0} \right) = Z_{l1} \]
\[ = 50 \Omega \]

Similarly, the input impedance of line 2 is given as
\[ Z_{\text{in}2} = Z_{l2} = 150 \Omega \]

The effective load for line 3 will be equal to the equivalent impedance of the parallel combination of input impedances of line 1 and line 2.

\[ Z_L' = Z_{\text{in}1} || Z_{\text{in}2} \]
\[ = \frac{150}{2} = 75 \Omega \]

So, the input impedance for line 3 is given as
\[ Z_{\text{in}} = Z_L' = 75 \Omega \]

(length of line 3, \( l = \lambda/2 \))

Therefore, the input voltage of line 3 is
\[ V_{s,\text{in}} = V_s \left( \frac{Z_{\text{in}}}{Z_{\text{in}} + Z_g} \right) = 500 \left( \frac{75}{75 + 100} \right) \]
\[ = 214.28 \text{ volt} \]

and so the current at the input terminal of line 3 is
\[ I_{s,\text{in}} = \frac{V_{s,\text{in}}}{Z_{\text{in}}} = 2.86 \text{ A} \]

Thus, the average power delivered to the lossless transmission line 3 is given as
\[ P_{\text{in}} = \text{Re}[V_{s,\text{in}} I_{s,\text{in}}^*] \]
\[ = \frac{1}{2} \times (214.28) \times (2.86) = 306.11 \text{ Watt} \]

Since, the transmission line is lossless so, the power delivered to each load will be same and given as
SOL 8.2.14  Option (D) is correct.
Given, transmission line is of infinite length i.e. \( l = \infty \).
and input impedance of the transmission line is equal to its characteristic impedance
i.e. \( Z_{in} = Z_0 \)
Since, the input impedance of a transmission line is defined as
\[
Z_{in} = Z_0 \left( \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right)
\]
So,
\[
Z_0 = Z_0 \left( \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right)
\]
Solving the equation, we get
\[
\tanh \gamma l = 1
\]
\[
e^{\gamma l} - e^{-\gamma l} = 1
\]
\[
e^{\gamma l} + e^{-\gamma l} = 0
\]
Since, \( l = \infty \). So for satisfying the above condition propagation constant \( \gamma \) must have a real part.
i.e. real part of \( \gamma \neq 0 \)
or, \( \alpha \neq 0 \)
(\( \gamma = \alpha + j\beta \))
As the attenuation constant of the voltage wave along the transmission line is not equal to zero therefore, it is a lossy transmission line.

SOL 8.2.15  Option (C) is correct.
As discussed in previous question the input impedance of infinitely long lossy transmission line is equal to it’s characteristic impedance. So, the input impedance to line 1 will be
\[
Z_{in1} = Z_{in2} = 200 \Omega
\]
From the shown arrangement of the transmission line it is clear that the effective load impedance for line 2 will be equal to the input impedance of line 1.
i.e. \( Z_{L2} = Z_{m1} = 200 \Omega \)
Since the length of the line 2 is \( \lambda/2 \) so, the input impedance of line 2 will be equal to its load
i.e. \( Z_{n2} = Z_{L2} = 200 \Omega \)
\( (l = \lambda/2) \)
Therefore, the reflection coefficient at the load terminal of line 2 is given as
\[
\Gamma = \frac{Z_{L2} - Z_{n2}}{Z_{L2} + Z_{n2}} = \frac{200 - 100}{200 + 100} = \frac{1}{3}
\]
Now, the input voltage of line 2 is determined by using voltage division rule as
\[
V_{in} = V_{\Lambda} \left( \frac{Z_{n2}}{Z_{n2} + Z_{g}} \right)
\]
\[
= 4 \left( \frac{200}{200 + 100} \right) = \frac{8}{3} \text{ volt}
\]
Again, the voltage at any point on line 2 is given as
where $V_0^+$ is voltage of incident wave $\beta$ is phase constant of the voltage wave and $z$ is distance from load. So, for $z = -\lambda/2$

$$V_s(z) = V_0^+ \left( e^{\frac{2\pi}{\lambda} \left( \frac{z}{\lambda} \right)} + e^{-\frac{2\pi}{\lambda} \left( \frac{z}{\lambda} \right)} \right)$$

$$\frac{8}{3} = V_0^+ (e^{-\beta x} + Ge^{\beta x}) \quad (V_s(z) = V_s, in at z = -\lambda/2)$$

$$V_0^+ = \frac{8}{3} \times \left( \frac{1}{-1 - \frac{1}{3}} \right) = -2 \text{ volt}$$

Therefore, the incident average power to the line 2 is given as

$$P_{av}^i = \left| \frac{V_0^+}{2Z_{02}} \right|^2 = \frac{4}{2 \times 100} = 20 \text{ mWatt}$$

So, the reflected average power at the input terminal of line 1 (load terminal of line 2) is

$$P_{av}^r = \left| \Gamma \right|^2 P_{av}^i = \left( \frac{1}{3} \right)^2 \times 20 = 2.2 \text{ mWatt}$$

Thus, we get the transmitted power to the line 1 as

$$P_{av}^t = P_{av}^i - P_{av}^r = 20 - 2.2 = 17.8 \text{ mWatt}$$

SOL 8.2.16 Option (B) is correct.

Consider the length of the transmission line is $l$ as shown in figure below

The generator voltage is applied to the transmission line at time $t = 0$ for which the voltage at the sending end is

$$v(0) = 10 \text{ volt} \quad \text{(at } t = 0)$$

After time $\Delta t = 4 \mu s$ the voltage $v(t)$ at the sending end changes to 6 V. This change in the voltage will be caused only if the reflected voltage wave from the load comes to the sending end. So, the time duration for the change in voltage at sending end can be given as

$$\Delta t = (\text{time taken by incident wave to reach the load})$$

$$+ \text{ (time taken by reflected wave to reach sending end from the load)}$$

$$\text{or}, \quad \Delta t = \frac{l}{v_p} + \frac{l}{v_p} = \frac{2l}{v_p} \quad (1)$$

where $l$ is the length of the transmission line (distance between load and sending terminal) and $v_p$ is phase velocity of the wave along the transmission line. Since,
the line is air spaced so,

\[ v_p = c = 3 \times 10^8 \text{ m/s} \]

Putting it in equation (1) we get

\[ 4 \mu s = \frac{2l}{3 \times 10^5} \quad (\Delta t = 4 \mu s) \]

Thus, length of the transmission line is

\[ l = \frac{3 \times 10^5 \times 4 \times 10^{-6}}{2} = 240 \text{ m} \]

**SOL 8.2.17** Option (C) is correct.

Let the load impedance connected to the transmission line is \( Z_L \) so the equivalent circuit for the transmission line will be as shown in figure below:

![Equivalent circuit diagram](image)

Since, the internal resistance of the generator is equal to the characteristic impedance of the line

i.e.

\[ R_g = Z_0 = 100 \Omega \]

So, the reflection coefficient due to source resistance will be zero and therefore, the change in voltage at sending will be caused only due to the reflection coefficient at load terminal given as

\[ \Delta v(t) = \Gamma V_0^+ \]

where, \( V_0^+ \) is amplitude of the incident voltage wave and \( \Gamma \) is the reflection coefficient at the load terminal. Since, the change in voltage at \( t = 4 \mu s \) is

\[ \Delta v(t) = 6 - 10 = -4 \]

So, we get

\[ -4 = 10 \Gamma \]

\[ \Gamma = -\frac{4}{10} = -0.4 \]

\[ \left( \frac{Z_L - Z_0}{Z_L + Z_0} \right) = -0.4 \]

\[ \frac{Z_L - 100}{Z_L + 100} = -0.4 \]

\[ Z_L = 29.86 \Omega \]

\[ Z_0 = 100 \Omega \]

**SOL 8.2.18** Option (B) is correct.

Observing the waveform we conclude that at the sending end voltage changes at \( t = t_1 \). The changed voltage at the sending is given as

\[ v(t_1) = V_0^+ + \Gamma_i V_0^+ + \Gamma_i \Gamma L V_0^+ \]

\[ (1) \]
where $V_0^+$ is voltage at sending end at $t = 0$, $\Gamma_L$ and $\Gamma_g$ are the reflection coefficients at the load terminal and the source terminal respectively. So, we get

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 \quad (Z_L = 0)$$

and

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \quad (Z_g = R_g)$$

Putting these values in equation (1), we get

$$v(t_1) = V_1^+ - V_1^+ - \Gamma_g V_1^+$$
$$v(t_1) = -\Gamma_g V_1^+$$

(2)

From the shown wave form of the voltage at sending end, we have

$$v(t_1) = 6 \text{ volt}$$
$$V_0^+ = 24 \text{ volt}$$

Putting these values in equation (2), we get

$$6 = -\Gamma_g(24)$$

or,

$$R_g - Z_0$$
$$\frac{R_g + Z_0}{R_g} = -4 \quad (Z_0 = 100 \Omega)$$

At $t = 0$ as the voltage just applied to transmission line, the input impedance is independent of $Z_L$ and equals to $Z_0$ (i.e. $Z_{in} = Z_0$ at $t = 0$). Therefore, using voltage division the voltage at the sending end is given as

$$V_0^+ = V_g \left(\frac{Z_0}{R_g + Z_0}\right)$$
$$24 = V_g \left(\frac{100}{60 + 100}\right)$$

$$V_g = \frac{24 \times 160}{100} = 38.4 \text{ volt}$$

SOL 8.2.19

Option (D) is correct.

Length of the transmission line, $l = 1.5 \text{ m}$

Internal resistance of generator, $R_g = 200 \Omega$

Characteristic impedance, $Z_0 = 100 \Omega$

Generator voltage, $V_g = 30 \text{ volt}$

Load impedance, $Z_L = 50 \Omega$

So, the reflection coefficient at the load terminal is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$

and the reflection coefficient at the source terminal is

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{200 - 100}{200 + 100} = \frac{1}{3}$$

Again as discussed in previous question at time, $t = 0$ as the voltage is just applied to the transmission line, the input impedance is independent of $Z_L$ and equals to $Z_0$ (i.e. $Z_{in} = Z_0$ at $t = 0$). Therefore, using voltage division the input voltage at the sending end is given as
\[ V_0^+ = V_0 \left( \frac{Z_a}{R_s + Z_0} \right) = 30 \times \left( \frac{100}{200 + 100} \right) = 10 \text{ volt} \]

Now, the time taken by the wave to travel from source terminal to the load terminal (or load terminal to source terminal) is given as
\[ T = \frac{l}{c} \]

where, \( l \) is length of transmission line and \( c \) is the velocity of the voltage wave along the transmission line. So, we get
\[ T = \frac{1.5}{3 \times 10^8} = 5 \text{ ns} \]

Therefore, for the interval \( 0 \leq t < 5 \text{ ns} \), the incident wave will be travelling from source to load and will have the voltage
\[ V_1^+ = 10 \text{ volt} \]

For the interval \( 5 \text{ ns} \leq t < 10 \text{ ns} \) an additional reflected wave will be travelling from load to source and will have the voltage
\[ V_1^- = \Gamma_s V_1^+ = -\frac{10}{3} = -3.33 \text{ volt} \]

For \( 10 \text{ ns} \leq t \leq 15 \text{ ns} \) the wave reflected by source resistance travelling from source to load will be added to that has the voltage
\[ V_2^+ = \Gamma_s V_1^- = -\frac{3.33}{3} = -1.11 \text{ volt} \]

For \( 15 \text{ ns} \leq t \leq 20 \text{ ns} \) again the wave reflected by load travelling from load to source will be added that has the voltage
\[ V_2^- = \Gamma_l V_2^+ = \frac{-1.11}{3} = 0.37 \text{ volt} \]

This will be continuous and the bounce diagram obtained between source (at \( z = 0 \)) and load (at \( z = 1.5 \text{ m} \)) will be as shown in figure below:

\[ \text{SOL 8.2.20} \]
Option (C) is correct.

From the bounce diagram that obtained between source terminal (\( z = 0 \)) and load terminal (\( z = 1.5 \text{ m} \)) in previous question, we can determine the voltage \( v(t) \) at any
instant by just summing all the voltage waves existing at any time $t$.

Since, for interval $0 \leq t \leq 10 \text{ ns}$ only a single voltage wave with $V_1^+ = 10 \text{ volt}$ exists at sending end so, the voltage at the sending end ($z = 0$) for the interval is

$$v(t) = V_1^+ = 10 \text{ volt}$$

for $0 \leq t < 10 \text{ ns}$

again for the interval $10 \text{ ns} \leq t < 20 \text{ ns}$, three voltage waves with $V_1^+ = 10 \text{ volt}$, $V_1^- = -3.33 \text{ volt}$ and $V_2^- = -1.11 \text{ volt}$ exists at the sending end so, the voltage at the sending end for the interval is

$$v(t) = V_1^+ + V_1^- + V_2^+ = 10 - 3.33 - 1.11 = 5.6 \text{ volt}$$

for $10 \text{ ns} \leq t < 20 \text{ ns}$

Thus, the obtained voltage wave form is plotted in the figure below.

![Voltage Waveform](image-url)

SOL 8.2.21 Option (A) is correct.
As shown in the smith chart, SWR circle meets the $I_r$ axis (real part of reflection coefficient) at $L_1$ and $L_2$ respectively. So, We have the two possible values of normalised impedance (real values of $z_L$).

$z_{L1} = 2.5$ at $L_1$

$z_{L2} = 0.4$ at $L_2$

Since, the normalised impedance is defined as

\[
\frac{z_L}{Z_0} = \frac{\text{Load impedance}}{\text{Characteristic impedance}}
\]

So, we have

$z_{L1} = \frac{Z_L}{Z_{01}} = 2.5$

or,

$Z_{01} = \frac{Z_L}{2.5} = \frac{50}{2.5} = 20 \ \Omega$

Similarly,

$z_{L2} = \frac{Z_L}{Z_{02}} = 0.4$

or,

$Z_{02} = \frac{Z_L}{0.4} = \frac{50}{0.4} = 125 \ \Omega$

Therefore, the two possible values of the characteristic impedance of the lossless transmission line are $20 \ \Omega$ and $125 \ \Omega$.

**SOL 8.2.22**

Option (B) is correct.

We can determine the reflection coefficient of the transmission line using smith
chart as explained below:

1) First we determine the normalized load impedance of the transmission line as
   \[ z_L = \frac{Z_L}{Z_0} = \frac{100 + 50}{100} = 1 + j0.5 \]

2) Comparing the normalized impedance to its general form
   \[ z_L = r + jx \]
   where \( r \) is the normalized resistance (real component) and \( x \) is the normalized reactance (imaginary component). we get
   \[ r = 1 \quad \text{and} \quad x = 0.5 \]

3) Now, we determine the intersection point of \( r = 1 \) circle and \( x = 0.5 \) circle on the smith charge and denote it by point \( P \) as shown in the smith chart. It gives the position of normalized load impedance.

4) We join the point \( P \) and the centre \( O \) to form the line \( OP \)

5) Extend the line \( OP \) to meet the \( r = 0 \) circle at \( Q \). The magnitude of the reflection coefficient of the transmission line is given as
   \[ |\Gamma| = \frac{OP}{OQ} = \frac{2.1 \, \text{cm}}{9.4 \, \text{cm}} = 0.22 \]

6) Angle of the reflection coefficient in degrees is read out from the scale at point \( Q \) as
   \[ \theta_r = 76.0^\circ \]

7) Thus, we get the reflection coefficient of the transmission line as
   \[ \Gamma = |\Gamma| e^{j\theta_r} = 0.22 e^{j76} \]
Alternate Method:
Reflection coefficient of the transmission line is defined as
\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 100}{100 + j50 + 100} = \frac{0.24}{\sqrt{76^2}} = 0.24e^{j\phi} \]
which is same as calculated from smith chart.

**SOL 8.2.23**
Option (C) is correct.
As shown in the smith chart in previous question normalized load impedance is located at point \(P\). So, for determining the input impedance at a distance of \(0.35\lambda\) from the load we follow the steps as explained below:

1. First we draw a SWR circle (circle centered at origin with radius \(OP\)).
2. For finding input impedance at a distance of \(0.35\lambda\) from load we move a distance of \(0.35\lambda\) on WTG scale (wave length toward generator) along the SWR circle.
3. Since, the line \(OP\) corresponds to the reading of \(0.144\lambda\) on WTG scale so, after moving a distance of \(0.35\lambda\) on WTG scale we reach at \(0.144\lambda + 0.35\lambda = 0.494\lambda\) on WTG scale. The reading corresponds to the point \(A\) on the SWR circle.
4. Taking the values of \(r\) and \(x\)-circle at point \(A\) we find out normalized input impedance as
Therefore, the input impedance at a distance of 0.35λ from load is given as

$$Z_{in} = z_{in}Z_0 = 100(0.61 - j0.022) = (61 - j2.2)\,\Omega$$

Alternate Method:

We can conclude the input impedance at $l = 0.35\lambda$ directly by using formula

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$

$$= \frac{100 + j50 + j100 \tan \left( \frac{2\pi}{\lambda} 0.35\lambda \right)}{100 + j(100 + j50) \tan \left( \frac{2\pi}{\lambda} 0.35\lambda \right)}$$

$$= (87 - j2.2)\,\Omega$$

as calculated above using with chart.

SOL 8.2.24

Option (C) is correct.

For determining the shortest length of the transmission line for which the input impedance appears to be purely resistive, we follow the steps as explained below:

1. First we determine the WTG reading of the point denoting the normalized load impedance on the smith chart. From the above question, we have the reading of point $P$ as 0.144 on WTG circle.

2. Since, the resistive load lies on the real axis of reflection coefficient ($\Gamma_r$-axis). So, we move along the SWR circle to reach the $\Gamma_r$-axis and denote the points as $A$ and $B$.

4. Since, point $B$ is nearer to the point $P$ so, it will give the shortest length of the transmission line for which the input impedance appears to be purely resistive.

5. Now, we have the reading of point $B$ on WTG scale as 0.25λ. So, the shortest length for the input impedance to be purely resistive is given as the difference between the readings at point $B$ and $P$. i.e.,

$$l = 0.25\lambda - 0.144\lambda$$

$$= 0.106\lambda$$

SOL 8.2.25

Option (A) correct.

The voltage maximum occurs at the point where the SWR circle intersects the positive $\Gamma_r$ axis on smith chart. The SWR circle of the load impedance intersects the positive $\Gamma_r$ axis at point $B$ as shown in the Smith chart. So, the point $B$ gives the position of first voltage maxima.

As calculated in previous question the distance between point $B$ and point $A$ on the WTG scale is 0.106λ. Therefore, the 1st voltage maximum occurs at a distance of 0.106λ from load.

SOL 8.2.26

Option (B) is correct.

At any time $t$, the currents of positive and negative waves are respectively $I^+$ and $I^-$ and the voltages of positive and negative waves are respectively $V^+$ and $V^-$ as shown in the figure.
\[ Z_0 \]

\[ I^+ = \frac{V^+}{Z_0} = \frac{1}{Z_0} \]

\[ V^+ = 1 \text{ Volt} \]

and

\[ I^- = -\frac{V^-}{Z_0} \]

Now, the voltage and current across an inductor are related as

\[ v = L \frac{di}{dt} \]

\[ V^+ + V^- = 2 \frac{d}{dt} (I^+ + I^-) \]

\[ 1 + V^- = 2 \frac{d}{dt} \left[ 1 - \frac{V^-}{50} \right] \]

\[ 1 + V^- = -\frac{1}{25} \frac{dV^-}{dt} \]

\[ -25 dt = \frac{dV^-}{1 + V^-} \]

Taking integration both sides we get

\[ \ln (1 + V^-) = -25t + C_1 \]

where \( C_1 \) is a constant

\[ (1 + V^-) = A e^{-25t} \] (1)

Since, the voltage \( (V^+) \) wave is incident at \( t = 0 \) so, at \( t = 0^+ \) the current through inductor is zero and therefore, from the property of an inductor at \( t = 0^+ \) the current through inductor will be also zero.

i.e. \( (I^+ + I^-)_{t=0^+} = 0 \)

\[ \left[ \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \right]_{t=0^+} = 0 \]

\[ \left[ \frac{1}{Z_0} - \frac{V_0}{Z_0} \right]_{t=0^+} = 0 \]

\[ (V^+ = 1 \text{ Volt}) \]

So at \( t = 0^+ \), \( V_0 = 1 \) volt

Putting it in equation (1), we get

\[ (1 + 1) = A \]

\[ A = 2 \]

Thus, the voltage of the reflected wave is

\[ V^- = (2e^{-25t} - 1) \text{ Volt} \]

SOL 8.2.27 Option (D) is correct.

The voltage of positive wave in transmission line is \( V_0^+ \). So, at the voltage maxima, magnitude of the voltage is given as

\[ |V_t|_{\text{max}} = |V_0^+| \left[ 1 + I^+ \right] \]
and at the point of voltage maxima the current will be minimum and given as

\[ I_{\text{min}} = \left| \frac{V_0}{Z_0} \right| (1 - \Gamma) \]

So, the line impedance at the point of voltage maxima will be

\[ Z_{\text{max}} = \left| \frac{V_0}{I_{\text{min}}} \right|_V = Z_0 \left( \frac{1 + \Gamma}{1 - \Gamma} \right) \]

= \left| Z_0 S \left( S = \frac{1 + \Gamma}{1 - \Gamma} \right) \right| \]

Now, at the voltage minimum the voltage magnitude is

\[ V_{\text{min}} = \left| V_0 \right| (1 - \Gamma) \]

and at the point of voltage minimum current will be maximum and given as,

\[ I_{\text{max}} = \left| \frac{V_0}{Z_0} \right| (1 + \Gamma) \]

and the line impedance at the point will be

\[ Z_{\text{min}} = \left| \frac{V_0}{I_{\text{max}}} \right|_V = Z_0 \left( \frac{1 - \Gamma}{1 + \Gamma} \right) = \frac{Z_0}{S} \left( S = \frac{1 + \Gamma}{1 - \Gamma} \right) \]

SOL 8.2.28

Option (A) is correct.

To determine the required quantity, we note that for a particular line of characteristic impedance \( Z_0 \), the product of the line impedances at two positions (two values of \( d \)) separated by an odd multiple of \( \lambda/4 \) is given by

\[
\{Z[d]\} \{Z[d + (2n - 1)\lambda/4]\} = \left[ Z_0 \left( \frac{1 + \Gamma(d)}{1 - \Gamma(d)} \right) \right] \left[ Z_0 \left( \frac{1 - \Gamma(d + (2n - 1)\lambda/4)}{1 + \Gamma(d + (2n - 1)\lambda/4)} \right) \right]
\]

\[
= Z_0^2 \left[ \frac{1 + \Gamma(d)}{1 - \Gamma(d)} \right] \left[ \frac{1 + \Gamma(d)e^{-j2\pi(2n - 1)\lambda/4}}{1 + \Gamma(d)e^{j2\pi(2n - 1)\lambda/4}} \right]
\]

\[
= Z_0^2 \left[ \frac{1 + \Gamma(d)}{1 - \Gamma(d)} \right] \left[ \frac{1 + \Gamma(d)e^{-j\lambda(2n - 1)\sigma}}{1 - \Gamma(d)e^{j\lambda(2n - 1)\sigma}} \right]
\]

\[
= Z_0^2 \left[ \frac{1 + \Gamma(d)}{1 - \Gamma(d)} \right] \left[ \frac{1 + \Gamma(d)e^{-j\lambda(2n - 1)\sigma}}{1 - \Gamma(d)e^{j\lambda(2n - 1)\sigma}} \right]
\]

As the intrinsic impedance of medium 1 is \( \eta_1 \) and that of medium 3 is \( \eta_3 \) so, for required match, thickness \( t \) is \( \lambda/4 \) and the intrinsic impedance \( (\eta_2) \) of the medium 2 is given as

\[ \eta_1 \eta_3 = \eta_2^2 \quad \text{or} \quad \eta_2 = \sqrt{\eta_1 \eta_3} \]

SOL 8.2.29

Option (C) is correct.

As determined in previous question, for a wave travelling through the three mediums of intrinsic impedances \( \eta_1, \eta_2 \) and \( \eta_3 \), the condition for matching dielectric (the intrinsic impedance of medium 2 that eliminates the reflected wave in medium 1) is

\[ \eta_2 = \sqrt{\eta_1 \eta_3} \]

Since, all the media have \( \mu = \mu_0 \) so, for the dielectrics \( (\sigma = 0) \) the above equation can be rewritten as
\[ \sqrt{\frac{\mu_0}{\varepsilon_2}} = \sqrt{\left( \sqrt{\frac{\mu_0}{\varepsilon_0}} \right) \left( \frac{\mu_0}{16\varepsilon_0} \right)} \quad \left( \eta = \sqrt{\frac{\mu}{\varepsilon}} \right) \]

where \( \varepsilon_2 \) is the permittivity of the medium 2.

\[ \varepsilon = 4\varepsilon_0 \]

**SOL 8.2.30**  
Option (B) is correct.  
The thickness \( t \) of the dielectric coating for the perfect matching (the condition for eliminating reflection) is given as

\[ t = \frac{\lambda}{4} \]

(quarter wave)

where \( \lambda \) is the wavelength of plane wave. The wavelength in terms of frequency is

\[ \lambda = \frac{v_p}{f} \]

where \( v_p \) is the phase velocity of the wave in the propagation medium which is given as

\[ v_p = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 4\varepsilon_0}} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8 \]

So, at frequency \( f = 1.5 \text{ GHz} \) the thickness of the dielectric coating is given as

So,

\[ t = \frac{v_p}{4f} = \frac{1.5 \times 10^8}{4(1.5 \times 10^9)} = 0.25 \text{ m} = 2.5 \text{ cm} \]

**SOL 8.2.31**  
Option (B) is correct.  
Distance between load and first voltage maxima, \( l_{\text{max}} = 0.125 \lambda \)

Characteristics impedance, \( Z_0 = 100 \Omega \)

Standing wave ratio, \( S = 3 \)

Position of voltage maxima \( (l_{\text{max}}) \) in terms of reflection coefficient \(|\Gamma|/\theta_r\) is

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} \]

where \( n = 0, 1, 2, \ldots \)

So, for 1st voltage maxima we have \( n = 0 \) and so, we get the position of first voltage maxima as

\[ l_{\text{max}} = \frac{\theta_r \lambda}{4\pi} \]

\[ 0.125 \lambda = \frac{\theta_r \lambda}{4\pi} \quad \Rightarrow \quad \theta_r = \frac{\pi}{2} \]

The magnitude of reflection coefficient is defined in terms of SWR as

\[ |\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{1}{2} \]

So, the reflection coefficient of the transmission line is

\[ \Gamma = |\Gamma|/\theta_r = \frac{1}{2} e^{j\pi/2} = j \]

Therefore, the load impedance of the transmission line is given as

\[ Z_L = Z_0 \left( \frac{1 + \Gamma}{1 - \Gamma} \right) = 100 \left( \frac{1 + \frac{j}{2}}{1 - \frac{j}{2}} \right) = (60 + j80) \Omega \]

**SOL 8.2.32**  
Option (A) is correct.
Given, the transmission line is terminated by its characteristic impedance i.e.,
\[ Z_L = Z_0 \]
So, there will be no reflected wave and therefore, the height of the voltage pulse will be given as
\[ V_i^+ = \frac{Z_0 V_g}{Z_0 + Z_0} = \frac{100 \times 15}{100 + 50} = 10 \text{ Volt} \]
As the wave travels in the \( +Z \) direction along transmission line at velocity
\[ v_p = \frac{1}{\sqrt{LC'}} = \frac{1}{\sqrt{(0.25 \times 10^{-6}) \times (100 \times 10^{-12})}} = 2 \times 10^8 \text{ m/s} \]
So, the voltage pulse will reach at \( l = 5 \text{ m} \) at time,
\[ t_0 = \frac{5}{2 \times 10^8} = 25 \text{ ns} \]
So, at \( l = 5 \text{ m} \) for \( 0 < t < 25 \text{ ns} \),
\[ V = 0 \]
and for \( t \geq 25 \text{ ns} \)
\[ V = V_i^+ = 10 \text{ Volt} \]
Therefore the plot of voltage against time at a distance 5 m from the source is as shown in graph below.

SOL 8.2.33 Option (A) is correct.
As the first forward voltage pulse is \( V_i^+ \) so, the first reflected pulse voltage is
\[ V_i^- = \Gamma_L V_i^+ \]
The 2\(^{nd}\) forward pulse voltage is given as
\[ V_2^+ = \Gamma_g V_i^- = \Gamma_g \Gamma_L V_i^+ \]
The 2\(^{nd}\) reflected pulse voltage is given as
\[ V_2^- = \Gamma_L V_2^+ = \Gamma_g \Gamma_L^2 V_i^+ \]
So, summing up all the pulses at load end for steady state \( (t \rightarrow \infty) \) we get the load voltage as
\[ V_L = V_i^+ + V_i^- + V_2^+ + V_2^- + ... \]
\[ = V_i^+ \left[ 1 + \Gamma_L + \Gamma_g \Gamma_L + \Gamma_g \Gamma_L^2 + ... \right] \]
\[ = V_i^+ \left[ 1 + \Gamma_g \Gamma_L + \Gamma_g^2 \Gamma_L^2 + ... \right] + \Gamma_L \left[ 1 + \Gamma_g \Gamma_L + ... \right] \]
\[ = V_i^+ \left[ \frac{1}{1 - \Gamma_g \Gamma_L} \right] + \left( \frac{1}{1 - \Gamma_L} \right) = V_i^+ \left( \frac{1 + \Gamma_L}{1 - \Gamma_L} \right) \]
SOL 8.2.34 Option (A) is correct.

Characteristic impedance, \( Z_0 = 60 \Omega \)
Load impedance, \( Z_L = 180 \Omega \)
Voltage generator, \( V_g = 100 \text{ V} \)
Internal resistance, \( Z_g = 120 \Omega \)

So, the first forward voltage pulse will be
\[
V_{1}^+ = \left( \frac{Z_0}{Z_0 + Z_g} \right) V_g = \left( \frac{60}{60+120} \right) 100 = \frac{100}{3} \text{ Volt}
\]

The reflection coefficient at load terminal is given as
\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{180 - 60}{180 + 60} = \frac{1}{2}
\]

The reflection coefficient at source terminal is given as
\[
\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{120 - 60}{120 + 60} = \frac{1}{3}
\]

Therefore, the voltage across the load at steady state is given by the expression as determined in previous question
\[
V_L = V_1^+ \left( 1 + \frac{\Gamma_g}{1 - \Gamma_g \Gamma_L} \right) = \frac{100}{3} \left( 1 + \frac{1/2 \Gamma_g}{1 - \left( \frac{1}{3} \right) \left( \frac{1}{2} \right)} \right)
\]
\[
= \frac{100}{3} \times \frac{3}{2} \times \frac{6}{5} = 75 \text{ Volt}
\]

SOL 8.2.35 Option (D) is correct.

Voltage generator, \( V_g = 50 \text{ Volt} \)
Internal impedance, \( Z_g = 30 \Omega \)
Characteristic impedance, \( Z_0 = 15 \Omega \)
Load impedance, \( Z_L = 45 \Omega \)

So, first forward voltage pulse is
\[
V_{1}^+ = \left( \frac{Z_0}{Z_0 + Z_g} \right) V_g = \left( \frac{15}{15+30} \right) 50 = \frac{50}{3}
\]

Now, the reflection coefficient at source terminal is
\[
\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{30 - 15}{30 + 15} = \frac{1}{3}
\]

and the reflection coefficient at load terminal is
\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{45 - 15}{45 + 15} = \frac{1}{2}
\]

So, at steady state (\( t = \infty \)) voltage across load is given as
\[
V_L = V_{1}^+ \left( \frac{1 + \Gamma_g}{1 - \Gamma_g \Gamma_L} \right) = \frac{50}{3} \left( \frac{1 - \frac{1}{2}}{1 - \left( \frac{1}{3} \right) \left( \frac{1}{2} \right)} \right)
\]
\[
= \frac{50}{3} \times \frac{6}{5} \times \frac{3}{2} = 30 \text{ Volt}
\]

Therefore, the current through load at steady state is given as
\[
I_L = \frac{V_L}{Z_L} = \frac{30}{45} = \frac{2}{3} \text{ A}
\]
Option (B) is correct.

Since, the internal resistance of the battery is zero so, the 1st forward voltage pulse is

\[ V_1^+ = V_g = 6 \text{ Volt} \]

and from the plot we get the first forward pulse current as

\[ I_1^+ = 75 \text{ mA} \]

Therefore, the characteristic impedance of the transmission line is given as

\[ Z_0 = \frac{V_1^+}{I_1^+} = \frac{6}{\frac{75}{10^{-3}}} = 80 \Omega \]

Option (D) is correct.

Reflection coefficient at source and load end are given as

\[ \Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} = -1 \]

and

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \]

Now, from the plot of input current (current at generator end) we get,

\[ V_1^+ = 75 \text{ mA} \quad (1) \]

and

\[ V_1^+ - V_1^- + V_2^+ = -5 \text{ mA} \quad (2) \]

where, \( V_1^+ \) is the first forward voltage pulse, \( V_1^- \) is the first reflected voltage pulse and \( V_2^+ \) is the second forward voltage pulse. So, putting the values of these voltages in terms of reflection coefficients we get

\[ V_1^+ - \Gamma_L V_1^+ + \Gamma_L \Gamma_s V_1^+ = -5 \text{ mA} \]

\[ V_1^+(1 - \Gamma_L - \Gamma_s) = -5 \text{ mA} \quad (\Gamma_s = -1) \]

\[ 1 - 2\Gamma_L = -\frac{5}{75} \quad (V_1^+ = 75 \text{ mA}) \]

or,

\[ \Gamma_L = \frac{8}{15} \]

For determining load resistance of the line the reflection coefficient is written in the terms of impedances as

\[ \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{8}{15} \]

\[ \frac{Z_0 - 80}{Z_L + 80} = \frac{8}{15} \quad (Z_0 = 80 \Omega \text{ as calculated in previous question}) \]

\[ Z_L(15 - 8) = 80 \times 8 + 15 \times 80 \]

Thus,

\[ Z_L = 262.85 \Omega \]

***********
SOLUTIONS 8.3

SOL 8.3.1  Option (B) is correct.
Characteristic impedance of a coaxial cable is defined as
\[ Z_0 = \sqrt{\frac{\mu}{\varepsilon}} \ln \left( \frac{b}{a} \right) \]
where, 
- \( b \rightarrow \) outer cross sectional diameter 
- \( a \rightarrow \) inner cross sectional diameter
So,
\[ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \ln \left( \frac{b}{a} \right) \]
\[ = \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi}{10^{-9} \times 10.89 \ln \left( \frac{2.4}{1} \right)}} \]
\[ = 50 \, \Omega \]

SOL 8.3.2  Option (C) is correct.
Since,
\[ Z_0 = \sqrt{Z_1 Z_2} \]
\[ = \frac{100}{\sqrt{50 \times 340}} \]
As this is quarter wave matching so, the length of the transmission line would be odd multiple of \( \lambda/4 \).
Now,
\[ l = (2m + 1) \frac{\lambda}{4} \]
For \( f_1 = 429 \, \text{MHz} \),
\[ l_1 = \frac{c}{f_1 \times 4} = \frac{3 \times 10^8}{429 \times 10^6 \times 4} = 0.174 \, \text{m} \]
For \( f_2 = 1 \, \text{GHz} \),
\[ l_2 = \frac{c}{f_2 \times 4} = \frac{3 \times 10^8}{1 \times 10^9 \times 4} = 0.075 \, \text{m} \]
Now, only the length of the line given in option (C) is the odd multiple of both \( l_1 \) and \( l_2 \) as :
\[ (2m + 1) = \frac{1.58}{l_1} = 9 \]
\[ (2m + 1) = \frac{1.58}{l_2} \approx 34 \]
Therefore, the length of the line can be approximately 1.58 cm.

SOL 8.3.3  Option (C) is correct.
Length on the transmission line, \( d = 2 \, \text{mm} \)
Operating frequency, \( f = 10 \, \text{GHz} \)
Phase difference, \( \theta = \pi/4 \)
Since the phase difference between the two points on the line is defined as
\[ \theta = \frac{2\pi}{\lambda} d \]

where \( \lambda \) is operating wavelength and \( d \) is the distance between the two points. So, we get

\[ \frac{\pi}{4} = \frac{2\pi}{\lambda} d \]

or

\[ \lambda = 8d = 8 \times 2 \text{ mm} = 16 \text{ mm} \]

Therefore, the phase velocity of the wave is given as

\[ v_p = f \lambda = 10 \times 10^9 \times 16 \times 10^{-3} = 2.6 \times 10^8 \text{ m/sec} \]

**SOL 8.3.4**

Option (D) is correct.

Since, voltage maxima is observed at a distance of \( \lambda/4 \) from the load and we know that the separation between one maxima and minima equals to \( \lambda/4 \) so voltage minima will be observed at the load.

Now, the input impedance at the point of voltage minima on the line is defined as

\[ [Z_{in}]_{\min} = \frac{Z_0}{S} \]

where, \( Z_0 \) is characteristic impedance and \( S \) is the standing wave ratio on the line. Therefore, the load impedance of the transmission line (equal to the input impedance at load) is given as

\[ Z_L = [Z_{in}]_{\min} = \frac{Z_0}{S} = \frac{50}{5} = 10 \Omega \quad (Z_0 = 50 \Omega, \ S = 5) \]

**SOL 8.3.5**

Option (C) is correct.

For a lossless network,

\[ |S_{11}|^2 + |S_{22}|^2 = 1 \]

Since, from the given scattering matrix we have

\[ S_{11} = 0.2/0^\circ, \ S_{12} = 0.9/90^\circ, \ S_{21} = 0.9/90^\circ, \ S_{22} = 0.1/90^\circ \]

So, we get

\[ (0.2)^2 + (0.9)^2 \neq 1 \]

Therefore, the two port is not lossless.

Now, for a reciprocal network, \( S_{12} = S_{21} \)

As for the given scattering matrix we have

\[ S_{12} = S_{21} = 0.9/90^\circ \]

Therefore , the two port is reciprocal.

**SOL 8.3.6**

Option (A) is correct.

For a distortion less transmission line characteristics impedance

\[ Z_0 = \sqrt{\frac{R}{G}} \quad (1) \]

Attenuation constant for distortionless line is

\[ \alpha = \sqrt{RG} \quad (2) \]

So, using equation (1) and (2) we get

\[ \alpha = \frac{R}{Z_0} = \frac{0.1}{50} = 0.002 \]
Option (B) is correct.

For a lossless transmission line, the input impedance is defined as

\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]

Now, for the quarter wave (\( \lambda/4 \)) line we have

Load impedance, \( Z_L = 30 \Omega \)

Characteristic impedance, \( Z_0 = 30 \Omega \)

Length of the line, \( l = \frac{\lambda}{4} \)

So, \( \tan \beta l = \tan \left( \frac{2\pi \lambda}{4} \right) = \infty \)

Therefore, the input impedance of the quarter wave line is

\[ Z_{in1} = \frac{Z_0}{\tan \beta l + jZ_o} = \frac{Z_0^2}{Z_L} = 60 \Omega \]

Now, for \( \lambda/8 \) transmission line we have

Load impedance, \( Z_L = 0 \Omega \) (short circuit)

Characteristic impedance, \( Z_0 = 30 \Omega \)

Length of the line, \( l = \frac{\lambda}{8} \)

So, we get

\[ \tan \beta l = \tan \left( \frac{2\pi \lambda}{8} \right) = 1 \]

Therefore, the input impedance of the \( \lambda/8 \) transmission line is given as

\[ Z_{in2} = jZ_0 \tan \beta l = j30 \]

The equivalent circuit is shown below:

![Equivalent Circuit](image)

The effective load impedance of the 60 \( \Omega \) transmission line is

\( Z_L = 60 + j30 \)

So, the reflection coefficient at the load terminal is

\[ \Gamma = \left| \frac{Z_L - Z_o}{Z_L + Z_o} \right| = \left| \frac{60 + j3 - 60}{60 + j3 + 60} \right| = \frac{1}{\sqrt{17}} \]

Therefore, the voltage standing wave ratio of the line is given as

\[ S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \sqrt{17}}{1 - \sqrt{17}} = 1.64 \]
Option (A) is correct.

The transmission line are as shown below. Length of all line is $\frac{\lambda}{4}$.

The input impedance of a quarter wave ($\lambda/4$) lossless transmission line is defined as

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

where, $Z_0$ is the characteristic impedance of the line and $Z_L$ is the load impedance of the line. So, for line 1 we have the input impedance as

$$Z_{i1} = \frac{Z_0^2}{Z_{L1}} = \frac{100^2}{50} = 200 \ \Omega$$

Similarly, for line 2, the input impedance is

$$Z_{i2} = \frac{Z_0^2}{Z_{L2}} = \frac{100^2}{50} = 200 \ \Omega$$

So, the effective load impedance of line 3 is given as

$$Z_{L3} = Z_{i1} \parallel Z_{i2} = 200 \ \Omega \parallel 200 \ \Omega = 100 \ \Omega$$

Therefore, the input impedance of line 3 is

$$Z_{i3} = \frac{Z_0^2}{Z_{L3}} = \frac{60^2}{100} = 45 \ \Omega$$

Option (A) is correct.

The input impedance of the lossless transmission line is defined as

$$Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan (\beta l)}{Z_0 + jZ_L \tan (\beta l)} \right)$$

Since, the given transmission line of characteristic impedance $Z_0 = 75 \ \Omega$ is short circuited ($Z_L = 0$) at its one end. Therefore, the input impedance of the line is

$$Z_{in} = jZ_0 \tan (\beta l)$$

Now, the operating wavelength of the line is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \ m \ or \ 10 \ cm \ \ \ \ \ \ \ \ \ (f = 3 \ GHz)$$

So,

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{10} \times 1 = \frac{\pi}{5} \ \ \ \ \ \ \ \ \ (l = 1 \ cm)$$

Therefore,

$$Z_{in} = jZ_0 \tan \left( \frac{\pi}{5} \right)$$

Since, $Z_0 \tan (\pi/5)$ is positive so, $Z_{in}$ is inductive.
Option (C) is correct.

The 2-port scattering parameter matrix is

\[
S = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\]

\[
S_{11} = \frac{(Z_L || Z_0) - Z_o}{(Z_L || Z_0) + Z_o} = \frac{(50 || 50) - 50}{(50 || 50) + 50} = -\frac{1}{3}
\]

\[
S_{12} = S_{21} = \frac{2(Z_L || Z_o)}{2(50 || 50) + 50} = \frac{2}{3}
\]

\[
S_{22} = \frac{(Z_L || Z_o) - Z_o}{(Z_L || Z_o) + Z_o} = \frac{(50 || 50) - 50}{(50 || 50) + 50} = -\frac{2}{3}
\]

Option (A) is correct.

The input impedance of a quarter wave \((l = \lambda/4)\) lossless transmission line is defined as

\[
Z_{in} = \frac{Z_0^2}{Z_L}
\]

where, \(Z_0\) is characteristic impedance and \(Z_L\) is the load impedance of the line. So, we have the input impedance of line 1 as

\[
Z_{in1} = \frac{Z_0^2}{Z_L} = \frac{50^2}{100} = 25
\]

Similarly, the input impedance of line 2 is

\[
Z_{in2} = \frac{Z_0^2}{Z_L} = \frac{50^2}{200} = 12.5
\]

The effective load impedance of the line 3 is given as

\[
Z_L = Z_{in1} || Z_{in2}
\]

\[
= 25 || 12.5 = 25 \frac{25}{3}
\]

So, the input impedance of the 50 \(\Omega\) transmission line is

\[
Z_S = \frac{(50)^2}{25/3} = 300
\]

Therefore, the reflection coefficient at the input terminal is given as

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 - 50}{300 + 50} = \frac{2}{7}
\]

Option (D) is correct.

We have \(10\log G_p = 10\) dB

or \(G_p = 10\)

The power gain of the antenna is defined as

\[
G_p = \frac{P_{rad}}{P_{in}}
\]

where \(P_{rad}\) is the radiated power of the antenna and \(P_{in}\) is the input power feed to the antenna. So, putting all the values we get

\[
10 = \frac{P_{rad}}{1W}
\]

or \(P_{rad} = 10\) Watts
SOL 8.3.13 Option (A) is correct.

The characteristic impedance of a transmission line is defined as

\[ Z_0 = \frac{Z_{oc} \cdot Z_{sc}}{Z_{oc} + Z_{sc}} \]

where \( Z_{oc} \) and \( Z_{sc} \) are input impedance of the open circuited and is short circuited line. So, we get

\[ Z_{sc} = \frac{Z_0^2}{Z_{oc}} = \frac{50 \times 50}{100 + j150} = \frac{50}{2 + 3j} \]

\[ = \frac{50(2 - 3j)}{13} = 3.77 - 4.28j \]

SOL 8.3.14 Option (C) is correct.

From the diagram, VSWR is given as

\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{4}{1} = 4 \]

Since, voltage minima is located at the load terminal so, the load impedance of the transmission line is given as

\[ Z_L = \left[ Z_{\text{in}} \right]_{\text{min}} = \frac{Z_0}{S} = \frac{50}{4} = 12.5 \Omega \]

\( (Z_0 = 50 \Omega, S = 4) \)

SOL 8.3.15 Option (D) is correct.

The reflection coefficient at the load terminal is given as

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6 \]

SOL 8.3.16 Option (C) is correct.

The given circles represent constant reactance circle.

SOL 8.3.17 Option (C) is correct.

The ratio of the load impedance to the input impedance of the transmission line is given as

\[ \frac{V_L}{V_{\text{in}}} = \frac{Z_0}{Z_{\text{in}}} \]

or

\[ V_L = \frac{Z_0}{Z_{\text{in}}} V_{\text{in}} = \frac{10 \times 300}{90} = 60 \text{ V} \]

SOL 8.3.18 Option (D) is correct.

Suppose at point \( P \) impedance is

\[ Z = r + j(-1) \]

If we move in constant resistance circle from point \( P \) in clockwise direction by an angle 45°, the reactance magnitude increase. Let us consider a point \( Q \) at 45° from point \( P \) in clockwise direction. It’s impedance is

\[ Z_1 = r - 0.5j \]

or

\[ Z_1 = Z + 0.5j \]

Thus movement on constant \( r \) - circle by an /45° in CW direction is the addition of inductance in series with \( Z \).

SOL 8.3.19 Option (A) is correct.

The VSWR of a transmission line is defined as

\[ S = \frac{1 - |\Gamma|}{1 + |\Gamma|} \]
where $\Gamma$ is the reflection coefficient of the transmission line. So, we get

$$2 = \frac{1 - |\Gamma|}{1 + |\Gamma|} \quad (S = 2)$$

or

$$|\Gamma| = \frac{1}{3}$$

Thus, the ratio of the reflected and incident wave is given as

$$\frac{P_r}{P_i} = |\Gamma|^2 = \frac{1}{9}$$

or

$$P_r = \frac{P_i}{9}$$

i.e. 11.11% of incident power is reflected.

**SOL 8.3.20**

Option (D) is correct.

The input impedance of a lossless transmission line is defined as

$$Z_{in} = Z_0 \left[ \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right]$$

Now, for $\lambda/2$ transmission line we have

$$l = \lambda/2$$

and

$$Z_{L1} = 100 \, \Omega$$

So, the input impedance of the $\lambda/2$ transmission line is

$$Z_{in1} = Z_0 \left[ \frac{Z_{L1} + jZ_o \tan \frac{\pi}{2}}{Z_o + jZ_{L1} \tan \frac{\pi}{2}} \right] = Z_{L1} = 100 \, \Omega \quad (\beta = \frac{2\pi}{\lambda})$$

For $\lambda/8$ transmission line, we have

$$l = \lambda/8$$

and

$$Z_{L2} = 0$$

(short circuit)

So, the input impedance of $\lambda/8$ line is

$$Z_{in2} = Z_0 \left[ \frac{0 + jZ_o \tan \frac{\pi}{4}}{Z_o + 0} \right] = jZ_o = j50 \, \Omega \quad (\beta = \frac{2\pi}{\lambda})$$

Thus, the net admittance at the junction of the stub is given as

$$Y = \frac{1}{Z_{in1}} + \frac{1}{Z_{in2}} = \frac{2}{100} + \frac{1}{j50} = 0.02 - j0.42$$
SOL 8.3.21 Option (A) is correct.
VSWR (voltage standing wave ratio) of a transmission line is defined as
\[ S = \frac{1 + \Gamma}{1 - \Gamma} \]
where \( \Gamma \) is the reflection coefficient of the transmission line that varies from 0 to 1. Therefore, \( S \) varies from 1 to \( \infty \).

SOL 8.3.22 Option (B) is correct.
Reactance increases, if we move along clockwise direction in the constant resistance circle.

SOL 8.3.23 Option (C) is correct.
A transmission line is distortion less if \( L_G = R_C \)

SOL 8.3.24 Option (B) is correct.
\[ Z_0 = \sqrt{Z_{OC}Z_{SC}} = \sqrt{100 \times 25} = 10 \times 5 = 50 \Omega \]

SOL 8.3.25 Option (B) is correct.
We know that distance between two adjacent voltage maxima is equal to \( \lambda/2 \), where \( \lambda \) is wavelength. So, we get
\[ \frac{\lambda}{2} = 27.5 - 12.5 \]
or,
\[ \lambda = 2 \times 15 = 30 \text{ cm} \]
Therefore, the operating frequency of the transmission line is
\[ f = \frac{c}{\lambda} = \frac{3 \times 10^8}{30} = 1 \text{ GHz} \quad (c = 3 \times 10^8 \text{ cm/s}) \]

SOL 8.3.26 Option (C) is correct.
Electrical path length = \( \beta l \)
where
\[ \beta = \frac{2\pi}{\lambda}, \quad l = 50 \text{ cm} \]
Now, the operating wavelength \( \lambda \) of the transmission line is given as
\[ \lambda = \frac{v}{f} = \frac{1}{f} \times \frac{1}{\sqrt{LC}} \]
\[ = \frac{1}{25 \times 10^6} \times \frac{1}{\sqrt{10 \times 10^{-6} \times 40 \times 10^{-12}}} = \frac{5 \times 10^7}{25 \times 10^6} = 2 \text{ m} \]
So, the electric path length is
\[ \beta l = \frac{2\pi}{2} \times 50 \times 10^{-2} = \frac{\pi}{2} \text{ radian} \]

SOL 8.3.27 Option (B) is correct.
The input impedance at the voltage minima on the transmission line is defined as
\[ [Z_{in}]_{min} = \frac{Z_0}{S} \]
where \( S \) is standing wave ratio along the transmission line. Since, the reflection coefficient \( \Gamma_L \) of the transmission line is given as
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3} \]
So, the standing wave ratio of the line is

\[ S = \frac{1 + |T_1|}{1 - |T_1|} = \frac{1 + \frac{\beta}{\lambda}}{1 - \frac{\beta}{\lambda}} = 2 \]

Therefore, the minimum input impedance measured on the line is equal to

\[ |Z_{in}|_{min} = \frac{50}{2} = 25 \, \Omega \]

**SOL 8.3.28** Option (D) is correct.

For a lossy transmission line the input impedance is given as

\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tanh \frac{\gamma l}{Z_0}}{Z_0 + jZ_L \tanh \frac{\gamma l}{Z_L}} \right] \]

Load impedance, \( Z_L = \infty \)  
Length of line, \( l = \lambda/4 \)

So,

\[
Z_{in} = Z_0 \lim_{Z_L \to \infty} \frac{1 + jZ_0 \tanh \frac{\gamma l}{Z_0}}{Z_0 + jZ_L \tanh \frac{\gamma l}{Z_L}} = \frac{Z_0}{j \tanh \frac{\gamma l}{Z_0}} = 0 \quad (\tanh \frac{\gamma \pi}{4} \to \infty)
\]

**SOL 8.3.29** Option (D) is correct.

Input impedance of a lossless transmission line is given by

\[ Z_{in} = Z_0 \left[ \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] \]

where \( Z_0 \to \) Characteristic impedance of line  
\( Z_L \to \) Load impedance  
\( l \to \) Length of transmission line  
\( \beta = 2\pi/\lambda \)

So, we have

\[ \beta l = \frac{2\pi \lambda}{4} = \frac{\pi}{2} \]

(Short circuited at load)

\( Z_L = 0 \)

\( Z_0 = 50 \, \Omega \)

Therefore, the input impedance of the transmission line is

\[ Z_{in} = 50 \left[ \frac{0 + j50 \tan \frac{\pi}{2}}{50 + j0 \tan \frac{\pi}{2}} \right] = \infty \]

i.e. infinite input impedance and thus, the current drawn from the voltage source will be zero.

**SOL 8.3.30** Option (B) is correct.

For lossless transmission line, the phase velocity is defined as

\[ v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad \ldots(1) \]

Characteristics impedance for a lossless transmission line is given as

\[ Z_0 = \sqrt{\frac{L}{C}} \quad \ldots(2) \]
So, from equation (1) and (2) we get

\[ v_p = \frac{1}{\sqrt{C (Z_0 \sqrt{C})}} = \frac{1}{Z_0 C} \]

**SOL 8.3.31** Option (C) is correct.

Input impedance of a \( \frac{\lambda}{4} \) transmission line is defined as

\[ Z_{in} = \frac{Z_0^2}{Z_L} \]

where \( Z_0 \) is characteristic impedance of the line and \( Z_L \) is load impedance of the line. Since, the \( \frac{\lambda}{4} \) line is shorted at one end (i.e. \( Z_L = 0 \)) So, we get,

\[ Z_{in} = \lim_{Z_L \to 0} \frac{Z_0^2}{Z_L} = \infty \]

**SOL 8.3.32** Option (D) is correct.

Voltage minima of a short circuited transmission line is located at its load. As the location of minima is same for the load \( R_L \) (i.e. the minima located at \( R_L \)) so, the first voltage maxima will be located at \( \frac{\lambda}{4} \) distance from load.

Now,

\[ l_{\text{max}} = \frac{\theta_T \lambda}{4\pi} \quad \text{(1)} \]

where \( l_{\text{max}} \) is the distance of point of maxima from the load, \( \theta_T \) is phase angle of reflection coefficient and \( \lambda \) is operating wavelength of line. So, putting the value of \( l_{\text{max}} \) is equation (1), we get

\[ \frac{\lambda}{4} = \frac{\theta_T \lambda}{4\pi} \]

\[ \theta_T = \pi \]

Now, the standing wave ratio of the line is given as

\[ S = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \]

or,

\[ 3 = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \]

\[ |\Gamma_L| = \frac{1}{2} \]

i.e.

\[ \Gamma_L = \frac{|\Gamma_L|}{\theta_T} = \frac{1}{2} / \pi = -\frac{1}{2} \]

The reflection coefficient at the load terminal is given as

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1}{2} - \frac{R_L - 75}{R_L + 75} \]

\[ (Z_L = R_L), (Z_0 = 75 \Omega) \]

\[ -R_L - 75 = 2R_L - 150 \]

\[ 3R_L = 75 \quad \Rightarrow \quad R_L = 25 \Omega \]

**SOL 8.3.33** Option (B) is correct.

The VSWR (voltage standing wave ratio) in terms of maxima and minima voltage is defined as

\[ S = \frac{|V_{\text{max}}|}{|V_{\text{min}}|} = \frac{4}{2} = 2 \]
SOL 8.3.34 Option (D) is correct. 
Characteristic impedance, \( Z_0 = 60 \, \Omega \)

SWR \( S = 4 \)

So, we have \( \frac{1 + \Gamma_L}{1 - \Gamma_L} = S = 4 \)

\[ \Gamma_L = \frac{3}{5} = 0.6 \]

The reflection coefficient at load is defined as

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \]

So,

\[ 0.6 = \frac{Z_L - 60}{Z_L + 60} \]

\[ Z_L = \frac{1.6}{0.4} \times 60 = 120 \, \Omega \]

SOL 8.3.35 Option (A) is correct.

Loading of a cable is done to increase the inductance as well as to achieve the distortionless condition.

i.e. statement (1) and (4) are correct.

SOL 8.3.36 Option (C) is correct.

Single stub with adjustable position is the best method for transmission line load matching for a given frequency range.

SOL 8.3.37 Option (B) is correct.

The reflection coefficient at load terminal is defined as

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ = \frac{Z_L - 60}{Z_L + 60} \]

Therefore, the standing wave ratio is

\[ VSWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 1}{1 - 1} = \infty \]

SOL 8.3.38 Option (D) is correct.

Given, the load impedance is short circuit

i.e. \( Z_L = 0 \)

So, input impedance for lossless line is given as

\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = jZ_0 \tan \beta l \]

Now, for \( l < \lambda/4 \Rightarrow \beta l < \frac{\pi}{2} \)

So, \( \tan \beta l \) is positive and therefore, \( Z_{in} \) is inductive  \( a \rightarrow 2 \)

For \( \frac{\lambda}{4} < l < \frac{\lambda}{2} \Rightarrow \frac{\pi}{2} < \beta l < \pi \)

\( \tan \beta l \) is \( - \)ve and therefore, \( Z_{in} \) is capacitive  \( b \rightarrow 1 \)

For \( l = \frac{\lambda}{4} \Rightarrow \beta l = \frac{\pi}{2} \)
For View Only

For distortionless transmission line,
\[ \alpha = \sqrt{RG}, \quad \beta = \omega \sqrt{LC} \]
and for lossless transmission line,
\[ \alpha = 0, \quad \beta = \omega \sqrt{LC} \]
So, for both the type of transmission line attenuation is constant and is independent of frequency. Where as the phase shift \( \beta \) varies linearly with frequency \( \omega \).
i.e. statement 1 and 3 are correct.

**SOL 8.3.40** Option (A) is correct.

Given,
Length of transmission line, \( l = 500 \text{ m} \)
Phase angle, \( \phi_{r} = -150^\circ \)
Operating wavelength, \( \lambda = 150 \text{ m} \)

Consider the reflected voltage wave for the lossless transmission line terminated in resistive load as shown in figure.

Since, the reflection coefficient has a phase angle \( -150^\circ \) So, the wave lags by \( 150^\circ \) angle.

The voltage wave has the successive maxima at each \( \lambda/2 \) distance,
So, the total no. of maxima \( = \frac{\text{Total length}}{\text{Distance between two maxima}} = \frac{500}{(150/2)} = 6\frac{2}{3} \)
i.e. 6 maxima and remaining phase angle $= \frac{2}{3} \times 360^\circ = 240^\circ$

From the wave pattern shown above we conclude that the remaining phase ($240^\circ$) will include one more maxima and therefore the total no. of maxima is 7.

**SOL 8.3.41** Option (A) is correct.
Reflection coefficient at load terminal is defined as

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For a matched transmission line we have

$$Z_L = Z_0$$

So,

$$\Gamma_L = 0$$

i.e. matching eliminated the reflected wave between the source and the matching device location.

**SOL 8.3.42** Consider the quarter wave transformer connected to load has the characteristic impedance $Z'_0$ as shown in the figure.

So, we have the input impedance,

$$Z_{in} = Z'_0 \frac{Z_L + jZ'_0 \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)} = \frac{(Z'_0)^2}{Z_L}$$

this will be the load to 450 $\Omega$ transmission line

i.e.

$$Z'_L = \frac{(Z'_0)^2}{Z_L} = \frac{(Z'_0)^2}{200}$$

and for matching, $Z_0 = Z'_L$

$$450 = \frac{(Z'_0)^2}{200}$$

$$Z'_0 = \sqrt{(450)(200)} = 300 \Omega$$

**SOL 8.3.43** Option (D) is correct.
Given $Z_L = \infty$ (open circuit)
and $l = \frac{\lambda}{4}$ (quarter wave)

$$Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right) = -jZ_0 \cot \beta l \quad (Z_L \rightarrow \infty)$$
\[ = - jZ_0 \cot \left( \frac{2\pi}{\lambda} \right) \]

SOL 8.3.44 Option (A) is correct.
Length of transmission line \( l < \lambda/4 \) (open circuit)
Load impedance, \( Z_L = \infty \)
So, the input impedance of the transmission line is given as
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \] (lossless line)
\[ = Z_0 \left( \frac{1}{j\tan \beta l} \right) \] (\( Z_L = \infty \))
\[ = - jZ_0 \cot \beta l \]

SOL 8.3.45 Option (C) is correct.
Given,
Load impedance, \( Z_L = 0 \) (short circuit)
Line parameters, \( R = G = 0 \) (loss free line)
Attenuation constant, \( \alpha = 0 \) (loss free line)
So, the input impedance of the line is given as
\[ Z_{in} = jZ_0 \tan \beta l \]
i.e. pure reactance
Statement (A) is correct.
Since \( \tan \beta l \) can be either positive or negative \( Z_{in} \) can be either capacitive or inductive.
Statement (B) is correct.
The reflection coefficient at load is
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = - 1 \neq 0 \]
So, the reflection exists.
Statement (C) is incorrect.
and since the standing waves of voltage and current are set up along length of the lines so, statement (D) is also correct.

SOL 8.3.46 Option (C) is correct.
(a) short circuit \( (Z_L = 0) \)
So
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = - 1 \] \( (a \rightarrow 2) \)
(b) Open circuit \( (Z_L = \infty) \)
So,
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 \] \( (b \rightarrow 3) \)
(c) Line characteristic impedance \( (Z_L = Z_0) \)
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \] \( (c \rightarrow 1) \)
(d) \( 2 \times \) line characteristic impedance \( (Z_L = 2Z_0) \)
\[ \Gamma_L = \frac{2Z_L - Z_0}{2Z_L + Z_0} = \frac{1}{3} \] \( (d \rightarrow 4) \)
SOL 8.3.47  
Option (A) is correct. 
Given, reflection coefficient, 
\[ \Gamma_L = 1/e^{0^\circ} \]
So, 
\[ \text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1}{1 - 1} = \infty \]

SOL 8.3.48  
Option (D) is correct. 
Given, reflection coefficient, \( \Gamma = \frac{1}{5} \)
So, 
\[ \text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{6}{4} = \frac{3}{2} \]

SOL 8.3.49  
Option (C) is correct. 
Characteristic impedance of transmission line is defined as 
\[ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]
So, for lossless transmission line \( (R = G = 0) \) 
\[ Z_0 = \sqrt{\frac{L}{C}} \]

SOL 8.3.50  
Option (C) is correct. 
Input impedance has the range from 0 to \( \infty \). 
VSWR has the range from 1 to \( \infty \) 
Reflection coefficient \( (\Gamma) \) ranges from \(-1\) to \(1\). 
\( (a \to 3) \)
\( (c \to 2) \)
\( (b \to 1) \)

SOL 8.3.51  
Option (C) is correct. 
Input impedance of a quarter wave transformer is defined as 
\[ Z_{in} = \frac{Z_0^2}{Z_L} \]
where \( Z_0 \) is the characteristic impedance of the line and \( Z_L \) is the load impedance. 
Since the quarter wave transformer is terminated by a short circuit \( (Z_L = 0) \) so, we get the input impedance of the transformer as 
\[ Z_{in} = \infty \]

SOL 8.3.52  
Option (D) is correct. 
The scattering parameters linearly relate the reflected wave to incident wave and it is frequency invariant so the scattering parameters are more suited than impedance parameters.

SOL 8.3.53  
Option (C) is correct. 
Given, the reflection coefficient as 
\[ \Gamma_L = 0.3e^{-j0^\circ} \]
At any point on the transmission line the reflection coefficient is defined as 
\[ \Gamma(z) = \Gamma_L e^{-2\zeta} \]
where \( z \) is the distance of point from load. 
\[ z = 0.1\lambda \]  
\( \text{(Given)} \)
So, 
\[ \Gamma(z) = \Gamma_L e^{-2\zeta(0.1\lambda)} = (0.3e^{-j0^\circ})(e^{-2j(0.1\lambda)}) \]
\( (\text{Assume } \alpha = 0) \)
SOL 8.3.54 Option (D) is correct.
Balun is used to couple a coaxial line to a parallel wire.

SOL 8.3.55 Option (B) is correct.
The reflection coefficient of the conducting sheet is $\Gamma = -1$ where as the transmission coefficient is $\Gamma = 0$. So, there will be $x$-directed surface current on the sheet.

SOL 8.3.56 Option (B) is correct.
Given,
Operating frequency, $f = 25 \text{ kHz}$
Conductivity, $\sigma = 5 \text{ mho/m}$
Relative permittivity, $\varepsilon_r = 80$
The attenuation constant for the medium is defined as
\[
\alpha = \sqrt{\frac{\omega \mu \sigma}{2}} \quad (\sigma \gg \omega \varepsilon)
\]
\[
= \sqrt{\frac{(2\pi \times 25 \times 10^1) \times (4\pi \times 10^{-7})(5)}{2}} \quad (\mu = \mu_0)
\]
\[= 0.7025\]
The attenuated voltage at any point is given as
\[
V = V_0 e^{-\alpha l}
\]
where $V_0$ is source voltage and $l$ is the distance travelled by wave
Since, the radio signal is to be transmitted with 90% attenuation so, the voltage of the signal after 90% attenuation is
\[
V = V_0 - 90\% \text{ of } V_0
\]
\[= 0.1 V_0\]
Comparing it with equation (1) we get
\[
(0.1) = e^{-\alpha l}
\]
or,
\[
l = -\frac{\ln(0.1)}{0.7025} = 3.27 \text{ m}
\]
SOL 8.3.57 Option (A) is correct.
In Smith chart, the distance towards the load is always measured in anticlockwise direction. So, statement 3 is incorrect while statement 1 and 2 are correct.

SOL 8.3.58 Option (A) is correct.
Given,
Characteristic impedance, $Z_0 = 75 \Omega$
Load impedance, $Z_L = (100 - j75) \Omega$
The condition for matching is
\[
Z'_L = Z_0
\]
where $Z'_L$ is the equivalent load impedance of the transmission line after connecting an additional circuit. So, the best matching will be obtained by a short circuited stub at some specific distance from load.
SOL 8.3.59  
Option (B) is correct.  
Given, the voltage standing wave ratio in decibels is  
\[ VSWR \text{ indecibels} = 6 \text{ dB} \]
or,  
\[ 20 \log_{10} S = 6 \]
\[ S = (10)^{6/20} = 2 \]
So, the reflection coefficient at the load terminal is given as  
\[ \Gamma = \frac{S-1}{S+1} = \frac{2-1}{2+1} = 0.33 \]

SOL 8.3.60  
Option (D) is correct.  
Input impedance of a quarter wave transformer (lossless transmission line) is defined as  
\[ Z_{in} = \frac{Z_0^2}{Z_L} \]
where \( Z_0 \) is the characteristic impedance of the line and \( Z_L \) is the load impedance of the line. So, we get  
\[ Z_0 = \sqrt{Z_{in}Z_L} = \sqrt{(50)(200)} = 100 \Omega \]

SOL 8.3.61  
Option (A) is correct.  
(1) Given,  
Length of line, \( l = \lambda/8 \)  
Load impedance, \( Z_L = 0 \)  
So,  
\[ \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{8} \right) = \frac{\pi}{4} \]
Therefore the input impedance of the lossless transmission line is  
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = Z_0 \left( \frac{jZ_0 \tan \frac{\pi}{4}}{Z_0} \right) \]
\[ = jZ_0 \] (i.e., incorrect statement)

(2) Given,  
Length of line, \( l = \lambda/4 \)  
Load impedance, \( Z_L = 0 \)  
So,  
\[ \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{4} \right) = \frac{\pi}{2} \]
Therefore the input impedance of the lossless transmission line is  
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]
\[ = Z_0 \left( \frac{jZ_0 \tan \frac{\pi}{2}}{Z_0} \right) = j\infty \] (i.e., correct statement)

(3) Given,  
Length of line, \( l = \lambda/2 \)  
Load impedance, \( Z_L = \infty \)  
So,  
\[ \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{2} \right) = \pi \]
Therefore, the input impedance of the lossless transmission line is
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]
\[ = Z_0 \left( \frac{1}{j \tan \frac{\pi}{4}} \right) = -j\infty \]  
(i.e., incorrect statement)

(4) Matched line have the load impedance equal to its characteristic impedance
i.e. \( Z_L = Z_0 \)
So, for the matched line the input impedance is
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = Z_0 \]  
(i.e., correct statement)

**SOL 8.3.62**
Option (C) is correct.

Given,
- Length of line, \( l = \lambda/8 \)
- Load impedance, \( Z_L = 0 \)

So,
\[ \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{8} \right) = \frac{\pi}{4} \]

Therefore, the input impedance of the transmission line is
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tanh \gamma l}{Z_0 + jZ_L \tanh \gamma l} \right) = Z_0 \tanh \gamma l \]

If the line is distortion less (i.e. \( \alpha = 0 \)) then, the input impedance of the line is
\[ Z_{in} = jZ_0 \tan \beta l = jZ_0 \]

So, it will depend on characteristic impedance as the line is resistive or reactive.

**SOL 8.3.63**
Option (B) is correct.

Since, a transmission line of output impedance 400 \( \Omega \) is to be matched to a load of 25 \( \Omega \) through a quarter wavelength line. So, for the quarter wave line we have
- Input impedance, \( Z_m = 400 \Omega \) (same as the o/p impedance of the matched line)
- Load impedance, \( Z_L = 25 \Omega \)
- Length of line, \( l = \lambda/4 \)

The characteristic impedance of quarter wave transmission line is \( Z_0 \) that connected between the load and the transmission line of output impedance 400 \( \Omega \) as shown in figure.

![Diagram of transmission line](image)

So, the input impedance at \( AB \) is given as
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \frac{\pi}{2}}{Z_0 + jZ_L \tan \frac{\pi}{2}} \right) = \left( \frac{Z_0}{Z_L} \right) \]
Therefore, 

\[ Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{800 \times 50} = 200 \Omega \]

**SOL 8.3.64**

Option (B) is correct.

Given,

Length of transmission line, \( l = \lambda/8 \)
Load impedance, \( Z_L = 0 \) (Short circuited line)

So, we get \( \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{8} \right) = \frac{\pi}{4} \)

Therefore, the input impedance of lossless transmission line is given as

\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) = Z_0 \left( \frac{jZ_0}{Z_0} \right) = jZ_0 \]

which is inductive.

So, the input impedance of \( \lambda/8 \) long short-circuited section of a lossless transmission line is inductive.

**SOL 8.3.65**

Option (C) is correct.

In list I

(a) Characteristic impedance of a transmission line is defined as

\[ Z_0 = \sqrt{R + j\omega L} = \sqrt{Z} \]

(b) Propagation constant of the line is given as

\[ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY} \]

(c) Sending end input impedance is

\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \gamma l}{Z_0 + jZ_L \tan \gamma l} \right) \]

Given, \( Z_L = Z_0 \) (terminated in characteristic impedance, \( Z_0 \))

So, we get the input impedance as

\[ Z_{in} = Z_0 = \sqrt{\frac{Z}{Y}} \]

**SOL 8.3.66**

Option (C) is correct.

For a distortionless transmission line, the attenuation constant \( (\alpha) \) must be independent of frequency \( (\omega) \) and the phase constant \( (\beta) \) should be linear function of \( \omega \).

(a) \( R = G = 0 \)

For this condition propagation constant is given as

\[ \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \]

i.e.

\[ \alpha = 0 \text{ and } \beta = \omega \sqrt{LC} \]

As the attenuation constant is independent of frequency and the phase constant is linear function of \( \omega \) so, it is a distortionless transmission line.

(b) \( RC = GL \)

\[ \frac{R}{L} = \frac{G}{C} \]

This is the general condition for distortionless line for which

\[ \alpha = \sqrt{RG} \text{ and } \beta = \omega \sqrt{LC} \]

(c) \( R >> \omega L, G >> \omega C \)
\[ \gamma = \alpha + j\beta = \sqrt{RG} \]

i.e. \[ \alpha = \sqrt{RG}, \text{ and } \beta = 0 \]

Since, \( \beta \) is not function of \( \omega \) so, it is not the distortionless line.

(d) \( R << \omega L, \ G << \omega C \)

\[ \gamma = \alpha + j\beta = \sqrt{(j\omega L)(j\omega C)} \]

i.e. \[ \alpha = 0 \text{ and } \beta = \omega \sqrt{LC} \]

**SOL 8.3.67**

Option (B) is correct.

Distance between adjacent maxima of an EM wave propagating along a transmission line is \( \lambda/2 \). So, we get

\[ \frac{\lambda}{2} = (37.5 - 12.5) \]

\[ \frac{\lambda}{2} = 25 \text{ cm} \]

i.e. \[ \lambda = 50 \text{ cm} \]

Therefore, the operating frequency of the line is

\[ f = \frac{c}{\lambda} = \frac{3 \times 10^8}{50 \times 10^{-2}} \]

\[ f = 300 \text{ MHz} \]

**SOL 8.3.68**

Option (C) is correct.

Forward voltage wave along the transmission line is given as

\[ V_0^+ = \frac{Z_0}{Z_0 + Z_L} E = \frac{E}{2} \]

As the transmission line is open circuited at its load terminal \( (Z_L = \infty) \) so, the reflection coefficient at the load terminal is

\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 \]

Therefore, the voltage travelling in reverse direction is

\[ V_0^- = \Gamma_L V_0^+ = \frac{E}{2} \]

The time taken by the wave to travel the distance between source and load terminal is given as

\[ t_i = \frac{l}{c} \]

where \( l \) is the length of transmission line and \( c \) is velocity of propagating wave.

Now, from the plot we observe that at \( z = 0 \), voltage of the line is \( E/2 \) where as at \( z = l \), voltage is \( E \) therefore, it is clear that the voltage wave has been reflected
from the load but not reached yet to the generator.

\[ \frac{l}{c} < t < \frac{2l}{c} \]

**SOL 8.3.69** Option (A) is correct.

**SOL 8.3.70** Option (A) is correct.

The characteristic impedance for a lossy transmission line does not depend on the length of the line.

**SOL 8.3.71** Option (D) is correct.

**SOL 8.3.72** Option (B) is correct.

A distortionless transmission line has its parameters related as

\[ \frac{R}{L} = \frac{G}{C} \quad \text{or} \quad RC = GL \]

**SOL 8.3.73** Option (B) is correct.

Given the reflection coefficient of the line is \( \Gamma = 0.6 \)

So, the voltage standing wave ratio is defined as

\[ \text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.6}{1 - 0.6} = 4 \]

**SOL 8.3.74** Option (B) is correct.

Characteristic impedance, \( Z_0 = 50 \Omega \)

Load impedance, \( Z_L = 100 \Omega \)

Forward voltage \( V_m = 10 V \)

So, the reflection coefficient of the line is given as

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3} \]

**SOL 8.3.75** Option (C) is correct.

Characteristic impedance, \( Z_0 = 50 \Omega \)

Load impedance, \( Z_L = 15 - j20 \Omega \)

So, the normalized load impedance is given as

\[ z_L = \frac{Z_L}{Z_0} = \frac{15 - j20}{50} = 0.3 - j0.3 \]

**SOL 8.3.76** Option (D) is correct.

Since both the transmission lines are identical except that the loads connected to them are \( 2Z \) and \( Z/2 \) respectively. Let the maximum voltage across the loads be \( V_m \) So, the power transmitted to the loads are

\[ P_A = \frac{V_m^2}{2Z} \]

and

\[ P_B = \frac{V_m^2}{Z/2} \]

Given,

\[ P_A = W_1 \]
So, \[ V_m^2 = (2Z)W_l \]
and \[ P_B = \frac{V_m^2}{(Z/2)} = \frac{2ZW_l}{Z/2} = 2W_l \]

**SOL 8.3.77** Option (A) is correct.

Given, the short circuited and open circuited input impedance as

\[ Z_{sc} = 36 \, \Omega, \quad Z_{oc} = 64 \, \Omega \]

So, the characteristic impedance of the transmission line is defined as

\[ Z_0 = \sqrt{Z_{oc}Z_{sc}} = \sqrt{36 \times 64} = 48 \, \Omega \]

**SOL 8.3.78** Option (D) is correct.

1. \[ Z_L = Z_0 \] (line terminated by its characteristic impedance)

So, reflection coefficient

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \]

i.e. no any reflected wave.

2. \[ Z_L = Z_0 \]
\[ \Gamma = 0 \]
and so, there will be no reflected wave and the wave will have only forward voltage and current wave which will be equal at all the points on the line.

3. For a lossless half wave transmission line

\[ Z_{in} = Z_L \]

So, statement 3 is incorrect while statements 1 and 2 are correct.

**SOL 8.3.79** Option (C) is correct.

Since, the standing wave ratio of the wave is 1.

\[ SWR = 1 \]

i.e.

So, expressing it in terms of reflection coefficient, we get

\[ \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1 \]
\[ |\Gamma| = 0 \]
\[ \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \]
\[ Z_L = Z_0 \]

i.e. characteristic impedance is equal to load impedance.

**SOL 8.3.80** Option (D) is correct.
Given, the two wire transmission line has

Half center to center spacing \( h = \frac{d}{2} \)

Conductor radius \( r \)

So, the capacitance per unit length of the line is defined as

\[
C = \frac{\pi \varepsilon}{\log_e \left( \frac{h}{r} + \sqrt{\left( \frac{h}{r} \right)^2 - 1} \right)}
\]

**SOL 8.3.81** Option (A) is correct.

Reflection coefficient,

\[
\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}
\]

\[
= \frac{Z_0^2 - Z_0}{Z_0^2 + Z_0} = -5
\]

**SOL 8.3.82** Option (B) is correct.

Propagation constant,

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

The characteristic impedance of the transmission line is given as

\[
Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}
\]

\[
Z_0 = \frac{R + j\omega L}{\gamma}
\]

**SOL 8.3.83** Option (A) is correct.

Given the reflection coefficient,

\[
\Gamma = -\frac{1}{3}
\]

So, the standing wave ratio

\[
S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{4}{2} = 2
\]

**SOL 8.3.84** Option (B) is correct.

For distortionless transmission line

\[
\frac{R}{G} = \frac{L}{C}
\]

and so, the attenuation constant,

\[
\alpha = \sqrt{RG} = \sqrt{R\left(\frac{RC}{L}\right)} = R\sqrt{\frac{C}{L}}
\]

**SOL 8.3.85** Option (D) is correct.

Capacitance per unit length, \( C = 10^{-10} \text{ F/m} \)

Characteristic impedance, \( Z_0 = 50 \Omega \)

Now, for distortionless line the characteristic impedance is given as
For View Only

\[ Z_0 = \sqrt{\frac{L}{C}} \]

\[ 50 = \sqrt{\frac{L}{10^{-10}}} \]

So, the inductance per unit length is

\[ L = (50)^2 \times (10^{-10}) = 0.25 \mu H/m \]

**SOL 8.3.86** Option (B) is correct.

The characteristic impedance \( Z_0 \) in terms of open circuit impedance \( Z_{oc} \) and short circuit impedance \( Z_{sc} \) is defined as

\[ Z_0 = \sqrt{Z_{oc} Z_{sc}} = \sqrt{(100)(100)} = 50 \Omega \]

(Given \( Z_{oc} = Z_{sc} = 100 \))

**SOL 8.3.87** Option (D) is correct.

Given,

Load impedance, \( Z_L = (75 - j50) \Omega \)

Characteristic impedance, \( Z_0 = 75 \Omega \)

Since, for matching the load impedance is equal to the characteristic impedance (i.e., \( Z_L = Z_0 \)) so, we have to produce an additional impedance of \( +j50 \) at load to match it with transmission line. Therefore, for matching the transmission line a short circuit stub is connected at some specific distance from load.

**SOL 8.3.88** Option (C) is correct.

Given,

The load impedance = Surge impedance
i.e. \( Z_L = Z_0 \)

So, reflection coefficient of the line is given as

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \]

**SOL 8.3.89** Option (C) is correct.

Given,

Length of transmission line, \( l = 50 \text{ cm} = 0.5 \text{ m} \)

Operating frequency, \( f = 30 \text{ MHz} = 30 \times 10^6 \text{ Hz} \)

Line parameters, \( L = 10 \mu H/m = 10 \times 10^{-6} \text{ H/m} \)

and \( C = 40 \text{ pF/m} = 40 \times 10^{-12} \text{ F/m} \)

So, the phase constant of the wave along the transmission line is

\[ \beta = \omega \sqrt{LC} \]

\[ = 2\pi \times 30 \times 10^6 \sqrt{(10 \times 10^{-6})(40 \times 10^{-12})} \]

\[ = \frac{6\pi}{5} \]

Therefore,

\[ \beta l = \frac{6\pi}{5} \times 0.5 = 0.6\pi = 108^\circ \]
SOL 8.3.90 Option (A) is correct.
Propagation constant in a transmission line is defined as
\[ \gamma = \sqrt{(L + j\omega R)(R + j\omega C)} \]

SOL 8.3.91 Option (A) is correct.
For a series resonant circuit the required conditions are
1. The angular frequency is
   \[ \omega = \frac{1}{\sqrt{LC}} \]
2. The total equivalent impedance is pure resistive
   i.e. \( Z = R \)

Now, the input impedance at a distance \( \lambda/4 \) from the load is defined as
\[ Z_{in} = \frac{Z^2}{Z_L} \]
And since the transmission line is open \( (Z_L = \infty) \)
So, \( Z_{in} = 0 \) which is purely resistive
i.e. R is correct statement.
In a lossless line voltage or current along the line are not constant.
\( \text{i.e. A is not a correct statement.} \)

SOL 8.3.92 Option (D) is correct.
The characteristic impedance of a transmission line can be defined as below.
\[ Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]
\[ Z_0 = \sqrt{Z_{in} Z_L} \]
\[ Z_0 = \frac{V^+}{I^+} \]

So, all the three statements are correct.

SOL 8.3.93 Option (C) is correct.
Given, load impedance of the transmission line is
\( Z_L = 0 \)
\( \text{(Short circuit)} \)
So, the input impedance of the lossless line is given as
\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \]
\[ = Z_0 \left( \frac{jZ_0 \tan \beta l}{Z_0} \right) \]
\[ = 2jZ_0 \tan \beta l \]

SOL 8.3.94 Option (D) is correct.
Given,
Characteristic impedance, \( Z_0 = 600 \Omega \)
Load impedance, \( Z_L = 900 \Omega \)
So, the reflection coefficient of the transmission line is given as
\[ \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \]
EXERCISE 9.1

MCQ 9.1.1 An electromagnetic wave propagating in an airfilled $10 \times 8 \text{ cm}$ waveguide has its electric field in phasor form given as
$$E_x = 5 \sin(20\pi x)\sin(25\pi y)e^{-32t} \text{ V/m}$$
What is the mode of propagation of the EM wave?
(A) TM$_{21}$
(B) TM$_{12}$
(C) TE$_{21}$
(D) TE$_{12}$

MCQ 9.1.2 Assertion (A) : In a waveguide operating below cutoff frequency there is no net average power flow down the waveguide.
Reason (R) : Propagation of energy requires a propagating mode.
(A) A and R both are true and R is correct explanation of A.
(B) A and R both are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

MCQ 9.1.3 An airfilled rectangular waveguide is operating in TM mode at a frequency twice the cutoff frequency. What will be the intrinsic wave impedance?
(A) 289 $\Omega$
(B) 327 $\Omega$
(C) 211 $\Omega$
(D) 377 $\Omega$

MCQ 9.1.4 An EM wave is propagating in TM$_{21}$ mode in an air filled $10 \times 4 \text{ cm}$ waveguide at a frequency of 2.5 GHz. What will be the phase constant of the EM wave?
(A) 1.865 rad/m
(B) 1.207 rad/m
(C) 186.5 rad/m
(D) 120.7 rad/m

MCQ 9.1.5 The electric field component of an electromagnetic wave propagating in a rectangular waveguide is given in phasor form as
$$E_x = E_0\sin(50\pi x)\sin(40\pi y)e^{-\tau t} \text{ V/m}$$
The ratio of field components $E_x/E_y$ will be equal to
(A) $\cot(50\pi x)\tan(40\pi y)$
(B) 0.8 $\cot(50\pi x)\tan(40\pi y)$
(C) 1.25 $\tan(40\pi x)\cot(50\pi y)$
(D) 0.8 $\tan(40\pi x)\cot(50\pi y)$
MCQ 9.1.6
An EM wave is propagating in TEM mode in a parallel plate waveguide filled of a dielectric \((\varepsilon_r = 4.25, \mu_r = 1)\). If the waveguide operating at 20 GHz then the phase constant and the group velocity of the wave will be respectively
(A) \(4.5 \times 10^8 \text{ rad/s} , 139.6 \text{ m/s}\)
(B) 139.6 rad/s, \(4.5 \times 10^8 \text{ m/s}\)
(C) \(2 \times 10^8 \text{ rad/m} , 314.2 \text{ m/s}\)
(D) 314.2 rad/m, \(2 \times 10^8 \text{ m/s}\)

MCQ 9.1.7
An \(a < b\) rectangular waveguide is operating in four different modes as TM_{11}, TM_{12}, TE_{10} and TE_{20}. If \(a = 2b\) then the ascending order of the operating modes for their cut-off frequencies will be
(A) TE_{10} < TE_{20} < TM_{11} < TM_{12}
(B) TE_{10} < TM_{11} < TE_{20} < TM_{12}
(C) TM_{12} < TM_{11} < TE_{20} < TE_{10}
(D) TE_{10} < TE_{20} < TM_{12} < TM_{11}

MCQ 9.1.8
An airfilled \(4 \times 2 \text{ cm}\) rectangular waveguide is operating at TE_{10} mode at frequency of 3.75 GHz. What will be the group velocity of the propagating wave in the waveguide ?
(A) \(1.8 \times 10^8 \text{ m/s}\)
(B) \(2.4 \times 10^8 \text{ m/s}\)
(C) \(2.4 \times 10^8 \text{ m/s}\)
(D) \(1.8 \times 10^8 \text{ m/s}\)

MCQ 9.1.9
A rectangular waveguide with the dimensions \(a = 2.5 \text{ cm}, b = 5 \text{ cm}\) is operating at a frequency \(f = 15 \text{ GHz}\). If the wave guide is filled with a lossless dielectric with \(\mu_r = 1, \varepsilon_r = 2\) then the wave impedance of propagating TE_{20} mode in the waveguide will be
(A) 377 \(\Omega\)
(B) 323 \(\Omega\)
(C) 457 \(\Omega\)
(D) 470 \(\Omega\)

MCQ 9.1.10
Cutoff wavelength of a parallel plate waveguide for TM_{2} mode is \(3 \text{ mm}\). If the guide is operated at a wavelength \(\lambda = 0.1 \text{ cm}\) then the no. of possible modes that can propagate in the waveguide is
(A) 4
(B) 5
(C) 8
(D) 9

MCQ 9.1.11
A lossless parallel plate waveguide is operating in TM_{3} mode at frequencies as low as 15 GHz. What will be the dielectric constant of the medium between plates if the plate separation is 20 mm ?
(A) 1.73
(B) 3
(C) 6
(D) 9
**MCQ 9.1.12**
A parallel plate wave guide has the plate separation \( b = 20 \text{ mm} \) made with glass \((\varepsilon_r = 2.1)\) between it’s plates. If the guide is operating at a frequency \( f = 20 \text{ GHz} \) then which of the following modes will propagate?

(A) TM\(_1\)  
(B) TM\(_3\)  
(C) TE\(_2\)  
(D) all the three

**MCQ 9.1.13**
The cutoff frequency of TM\(_1\) mode in an air filled parallel plate wave guide is 2.5 GHz. If the guide is operating at wavelength \( \lambda = 3 \text{ cm} \) then what will be the group velocity of TE\(_3\) mode?

(A) \( 9.9 \times 10^7 \text{ m/s} \)  
(B) \( 2 \times 10^8 \text{ m/s} \)  
(C) \( 3.97 \times 10^8 \text{ m/s} \)  
(D) \( 1.5 \times 10^8 \text{ m/s} \)

**MCQ 9.1.14**
A symmetric slab waveguide has a slab thickness \( d = 15 \mu \text{m} \) with refractive indices \( n_1 = 3, \ n_2 = 2.5 \) as shown in figure. The phase velocity of the TE\(_1\) mode at cutoff will be

(A) \( 2.5 \times 10^8 \text{ m/s} \)  
(B) \( 1.2 \times 10^8 \text{ m/s} \)  
(C) \( 7.5 \times 10^8 \text{ m/s} \)  
(D) \( 2.07 \times 10^8 \text{ m/s} \)

**MCQ 9.1.15**
In a symmetrical slab waveguide, the phase velocity of TE\(_1\) mode at cutoff is \( v_{pl} \). So, the phase velocity of TM\(_2\) mode at cutoff will be

(A) \( v_{pl} \)  
(B) \( 2v_{pl} \)  
(C) \( \frac{v_{pl}}{2} \)  
(D) \( \sqrt{2} v_{pl} \)

**Common Data for Question 16 - 17:**
A strip line transmission line has the ground plane separation, \( b = 0.632 \text{ cm} \) and filled of a material with \( \varepsilon_r = 4.8 \).

**MCQ 9.1.16**
If characteristic impedance of the transmission line is 55 then what will be the width of conducting strip?

(A) 0.47 cm  
(B) 0.30 cm  
(C) 0.15 cm  
(D) 0.62 cm
MCQ 9.1.17 If the transmission line is operating at a frequency $f = 3 \text{GHz}$ then what will be it’s guide wavelength?

(A) 29.7 cm  
(B) 3.37 cm  
(C) 33.7 cm  
(D) 6.74 cm

MCQ 9.1.18 Consider the following statements

1. TEM mode can not exist within a hollow waveguide.
2. Any of the TM mode can’t be the dominant mode of propagation in rectangular waveguide.

The correct statement is

(A) only 1  
(B) only 2  
(C) 1 and 2 both  
(D) None of these

MCQ 9.1.19 The lowest order TM mode that can exist in a cavity resonator is

(A) $TM_{111}$  
(B) $TM_{110}$  
(C) $TM_{011}$  
(D) $TM_{101}$

MCQ 9.1.20 If the dimensions of cavity resonator are equal (i.e., $a = b$) then the lowest order TE mode will be

(A) $TE_{011}$  
(B) $TE_{100}$  
(C) $TE_{101}$  
(D) (A) and (C) both

MCQ 9.1.21 An air filled cavity resonator has the dimensions $a > b > c$. Which of the following modes are arranged in ascending order with respect to their resonant frequencies?

(A) $TM_{110}$, $TE_{011}$, $TE_{101}$  
(B) $TE_{011}$, $TM_{110}$, $TE_{101}$  
(C) $TM_{100}$, $TM_{101}$, $TM_{111}$  
(D) $TM_{110}$, $TE_{101}$, $TE_{011}$

MCQ 9.1.22 An airfilled, lossless cavity resonator has dimensions $a = 40 \text{cm}$, $b = 25 \text{cm}$ and $c = 20 \text{cm}$. What is the resonant frequency of $TE_{101}$ mode?

(A) 781 MHz  
(B) 901.4 MHz  
(C) 450.7 MHz  
(D) 960.4 MHz

MCQ 9.1.23 An airfilled cubic cavity resonator ($a = b = c$) has dominant resonant frequency of 15 GHz. The dimension of the cavity resonator is

(A) 1.41 cm  
(B) 70 cm  
(C) 3 cm  
(D) 3.8 cm
EXERCISE 9.2

MCQ 9.2.1 An electromagnetic wave is propagating in a parallel plate waveguide operating at TM\(_1\) mode. The magnetic field lines in the \(yz\)-plane will be (Assume the positive \(x\)-axis directs into the paper)

![Diagram A](image)

![Diagram B](image)

![Diagram C](image)

![Diagram D](image)

MCQ 9.2.2 An EM wave is propagating at a frequency \(f\) in an air filled rectangular waveguide having the cutoff frequency \(f_c\). Consider the phase velocity of the EM wave in the waveguide is \(v_p\). The plot of \((c/v_p)\) versus \((f_c/f)\) will be.

\((c)\) is the velocity of wave in air)

![Plot A](image)

![Plot B](image)
MCQ 9.2.3

An electromagnetic wave propagating at a frequency \( f \) in free space has the wavelength \( \lambda \). At the same frequency it's wavelength in an air-filled waveguide is \( \lambda_0 \). If the cutoff frequency of the waveguide is \( f_c \) then the plot of \( \frac{\lambda}{\lambda_0} \) versus \( \frac{f}{f_c} \) will be

Statement for Linked Question 4 - 5 :

A parallel plate waveguide is separated by a dielectric medium of thickness \( b \) with the constitutive parameters \( \varepsilon \) and \( \mu \)

MCQ 9.2.4

If the cut-off frequencies of the waveguide for the modes TE\(_1\), TE\(_2\) and TE\(_3\) are respectively \( \omega_1 \), \( \omega_2 \), \( \omega_3 \) then which of the following represents the correct relation between the cutoff frequencies ?

(A) \( \omega_1 = \omega_2 = \omega_3 \)  
(B) \( \omega_1 < \omega_2 < \omega_3 \)  
(C) \( \omega_1 > \omega_2 > \omega_3 \)  
(D) \( \sqrt{\omega_1 \omega_2} = \omega_3 \)
The $f$-$\beta$ curve (graph of frequency $f$ versus phase constant $\beta$) of the waveguide for the modes TM$_2$, TM$_3$ and TM$_4$ will be

MCQ 9.2.6 A parallel plate waveguide operating at a frequency of 5 GHz is formed of two perfectly conducting infinite plates spaced 8 cm apart in air. The maximum time average power that can be propagated per unit width of the guide for TM$_1$ mode without any voltage breakdown will be

(A) 828 MW/m 
(B) 414 MW/m 
(C) 104 MW/m 
(D) 207 MW/m

MCQ 9.2.7 An air filled parallel plate wave guide has the separation of 12 cm between its plates. The guide is operating at a frequency of 2.5 GHz. What is the maximum average power per unit width of the guide that can be propagated without a voltage breakdown for TEM mode?

(A) 358 MW/m 
(B) 143.2 MW/m 
(C) 716 MW/m 
(D) 1.432 GW/m

MCQ 9.2.8 A parallel plate waveguide filled of a dielectric ($\varepsilon_r = 8.4$) is constructed for operation in TEM mode only over the frequency range 0 < $f$ < 1.5 GHz. The maximum allowable separation between the plates will be

(A) 6.90 cm 
(B) 29 cm 
(C) 3.45 cm 
(D) 1.20 cm
Statement for Linked Question 9 - 10:
A parallel plate waveguide having plate separation \( b = 18.1 \text{ mm} \) is partially filled with two lossless dielectric with permittivity \( \varepsilon_1 = 2 \) and \( \varepsilon_2 = 1.05 \).

**MCQ 9.2.9** The frequency ‘\( f_0 \)’ at which the TM\(_1\) mode propagates through the guide without suffering any reflective loss is
(A) 12.8 GHz  
(B) 16.2 GHz  
(C) 9.28 GHz  
(D) 8.44 GHz

**MCQ 9.2.10** How many modes that can propagate in the guide at the frequency \( f_0 \)?
(A) one  
(B) two  
(C) three  
(D) four

**MCQ 9.2.11** An \( a \times b \) airfilled rectangular waveguide is operating at a frequency, \( f = 5 \text{ GHz} \). What will be it’s dimensions if the design frequency is 10\% larger than the cutoff frequency of dominant mode while being 15\% lower than the cutoff frequency for the next higher order mode?
(A) \( a = 3.3 \text{ cm}, b = 2.7 \text{ cm} \)  
(B) \( a = 1.1 \text{ cm}, b = 0.9 \text{ cm} \)  
(C) \( a = 0.37 \text{ cm}, b = 0.3 \text{ cm} \)  
(D) \( a = 0.5 \text{ cm}, b = 0.4 \text{ cm} \)

**MCQ 9.2.12** A symmetric dielectric slab waveguide with it’s permittivities \( \varepsilon_1 = 2.2 \) and \( \varepsilon_2 = 2.1 \) is operating at wavelength, \( \lambda = 2.6 \mu\text{m} \). If the slab thickness is \( d = 20 \mu\text{m} \) then how many modes can propagate in the slab?
(A) 8  
(B) 25  
(C) 15  
(D) 24

**MCQ 9.2.13** A rectangular waveguide operating in TE\(_{10}\) mode has the phase constant \( \beta \). If the average power density of the guide in this mode is \( P_{av} \) then what will be the relation between \( P_{av} \) and \( \beta \)?
(A) \( P_{av} \propto \beta \)  
(B) \( P_{av} \propto \beta^2 \)  
(C) \( P_{av} \propto \frac{1}{\beta} \)  
(D) \( P_{av} \) is independent of \( \beta \)

**MCQ 9.2.14** A symmetric dielectric slab waveguide with it’s refractive indices \( n_1 \) and \( n_2 \) is operating at wavelength, \( \lambda = 3.1 \mu\text{m} \). If the slab thickness is \( 10 \mu\text{m} \) and \( n_2 = 3.3 \) then what will be the maximum value of \( n_1 \) for which it supports only a single pair of TE and TM mode?
(A) 3.304  
(B) 3.20  
(C) 2.40  
(D) 3.42
Common Data for Question 15 - 16:

An asymmetric slab waveguide has the different mediums above and below the slab as shown in figure. The regions above and below the slab have refractive indices $n_2$ and $n_3$ respectively while the slab has refractive index $n_1$.

| $n_2$ | $n_1$ | Slab | $n_3$ |

**MCQ 9.2.15**  
If $n_1 = 2.8$, $n_2 = 1.7$, $n_3 =$  then the minimum possible wave angle for the wave propagation will be

- (A) 48.6°
- (B) 37.4°
- (C) 41.4°
- (D) 54.1°

**MCQ 9.2.16**  
If the refractive indices of the mediums are related as $n_1 > n_2 > n_3$ the maximum phase velocity of a guided mode will be

- (A) $n_3c$
- (B) $c/n_1$
- (C) $c/n_3$
- (D) $c/n_2$  

(c is velocity of wave in free space)

**MCQ 9.2.17**  
An air filled waveguide is of square cross-section of 4.5 cm on each side. The waveguide propagates energy in the TE$_{22}$ mode at 6 GHz. The wavelength of the TE$_{22}$ mode wave in the guide is

- (A) 4.2 cm
- (B) 2.72 cm
- (C) 5.3 cm
- (D) 1.18 cm

**MCQ 9.2.18**  
An attenuator is formed by using a section of waveguide of length ‘$l$’ operating below cutoff frequency. The operating frequency is 6 GHz and the dimension of the guide is $a = 4.572$ cm as shown in figure. What will be the required length ‘$l$’ to achieve attenuation of 100 dB between $I/P$ and $O/P$ guides?
**MCQ 9.2.19**
A rectangular waveguide with the dimension, \( a = 1.07 \text{ cm} \) is operating in \( \text{TE}_{10} \) mode at a frequency, \( f = 10 \text{ GHz} \). If the waveguide is filled with a dielectric material having \( \varepsilon_r = 8.8 \) and \( \tan \delta = 0.002 \) then the attenuation constant due to dielectric loss will be

- (A) 12.23 dB/m
- (B) 0.705 dB/m
- (C) 6.12 dB/m
- (D) 3.03 dB/m

**MCQ 9.2.20**
The first four propagating modes of a circular waveguide are respectively

- (A) \( \text{TM}_{01} \), \( \text{TE}_{21} \), \( \text{TE}_{01} \), \( \text{TM}_{11} \)
- (B) \( \text{TM}_{11} \), \( \text{TM}_{21} \), \( \text{TM}_{02} \), \( \text{TM}_{12} \)
- (C) \( \text{TE}_{11} \), \( \text{TE}_{21} \), \( \text{TM}_{11} \), \( \text{TE}_{01} \)
- (D) \( \text{TE}_{11} \), \( \text{TM}_{01} \), \( \text{TE}_{21} \), \( \text{TE}_{01} \)

**Statement for Linked Question 21 - 22**
A microstrip line has the substrate thickness \( d = 0.616 \text{ cm} \) with \( \varepsilon_r = 2.2 \)

**MCQ 9.2.21**
If the characteristic impedance of the guide is 100 \( \text{Ω} \) then what will be the width of microstrip.

- (A) \( \square \) cm
- (B) 0.28 cm
- (C) 0.36 cm
- (D) 0.14 cm

**MCQ 9.2.22**
If the transmission line is operating at a frequency, \( f = 8 \text{ GHz} \) then the effective permittivity \( \varepsilon_r \) and guide wavelength \( \lambda_g \) will be

- (A) \( \varepsilon_r = 1.76 \), \( \lambda_g = 2.83 \text{ cm} \)
- (B) \( \varepsilon_r = 2.83 \), \( \lambda_g = 1.76 \text{ cm} \)
- (C) \( \varepsilon_r = 0.158 \), \( \lambda_g = 18.87 \text{ cm} \)
- (D) \( \varepsilon_r = 18.87 \), \( \lambda_g = 0.158 \text{ cm} \)

**Common Data for Question 23 - 24**
A rectangular cavity resonator with dimensions \( a = 2.5 \text{ cm} \), \( b = 2 \text{ cm} \) and \( c = 5 \text{ cm} \) is filled with a lossless material (\( \mu = \mu_0 \), \( \varepsilon = 3\varepsilon_0 \)).

**MCQ 9.2.23**
The resonant frequency of the cavity resonator for \( \text{TE}_{101} \) mode will be

- (A) 6.7 GHz
- (B) 6.34 GHz
- (C) 7.74 GHz
- (D) 3.87 GHz
MCQ 9.2.24 If the resonator is made of copper then the quality factor for $\text{TE}_{101}$ mode is (Conductivity of copper $\sigma = 5.8 \times 10^7 \text{ S/m}$)

(A) 7733  
(B) 14358  
(C) 6625  
(D) 11075

MCQ 9.2.25 An air filled circular waveguide has it’s inner radius 1 cm. The cutoff frequency for $\text{TE}_{11}$ mode will be ($p'_{11} = 1.841$)

(A) 0.55 GHz  
(B) 49.3 GHz  
(C) 8.79 GHz  
(D) 4.71 GHz

MCQ 9.2.26 A cylindrical cavity shown in the figure below is operating at a wavelength of 2 cm in the dominant mode.

The radius $a$ of the cylindrical cavity will be ($p_{01} = 2.405$)

(A) 0.77 cm  
(B) 0.38 cm  
(C) 1.53 cm  
(D) 0.19 cm

************
EXERCISE 9.3

MCQ 9.3.1

The magnetic field among the propagation direction inside a rectangular waveguide with the cross-section shown in the figure is

\[ H_z = 3 \cos(2.094 \times 10^2 x) \cos(2.618 \times 10^2 y) \cos(6.283 \times 10^{10} t - \beta z) \]

The phase velocity \( v_p \) of the wave inside the waveguide satisfies

(A) \( v_p > c \)
(B) \( v_p = c \)
(C) \( 0 < v_p < c \)
(D) \( v_p = 0 \)

MCQ 9.3.2

The modes in a rectangular waveguide are denoted by \( \text{TE}_{mn} \) or \( \text{TM}_{mn} \) where \( m \) and \( n \) are the eigen numbers along the larger and smaller dimensions of the waveguide respectively. Which one of the following statements is TRUE?

(A) The \( \text{TM}_{10} \) mode of the waveguide does not exist
(B) The \( \text{TE}_{10} \) mode of the waveguide does not exist
(C) The \( \text{TM}_{10} \) and the \( \text{TE}_{10} \) modes both exist and have the same cut-off frequencies
(D) The \( \text{TM}_{10} \) and the \( \text{TM}_{01} \) modes both exist and have the same cut-off frequencies

MCQ 9.3.3

The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity \( \varepsilon_r \) and relative permeability \( \mu_r = 1 \) are given by \( \mathbf{E} = E_x e^{(\omega - 28000) t} \mathbf{a}_x \text{ V/m} \) and \( \mathbf{H} = 3 e^{(\omega - 28000) t} \mathbf{a}_y \text{ A/m} \). Assuming the speed of light in free space to be \( 3 \times 10^8 \text{ m/s} \), the intrinsic impedance of free space to be \( 120\pi \), the relative permittivity \( \varepsilon_r \) of the medium and the electric field amplitude \( E_x \) are

(A) \( \varepsilon_r = 3, E_x = 120\pi \)
(B) \( \varepsilon_r = 3, E_x = 360\pi \)
(C) \( \varepsilon_r = 9, E_x = 360\pi \)
(D) \( \varepsilon_r = 9, E_x = 120\pi \)
MCQ 9.3.4  GATE 2009

Which of the following statements is true regarding the fundamental mode of the metallic waveguides shown?

(A) Only $P$ has no cutoff-frequency
(B) Only $Q$ has no cutoff-frequency
(C) Only $R$ has no cutoff-frequency
(D) All three have cutoff-frequencies

MCQ 9.3.5  GATE 2008

A rectangular waveguide of internal dimensions $(a = 4$ cm and $b = 3$ cm) is to be operated in TE$_{11}$ mode. The minimum operating frequency is

(A) 6.25 GHz  
(B) 6.0 GHz  
(C) 5.0 GHz  
(D) 3.75 GHz

MCQ 9.3.6  GATE 2007

The $E$ field in a rectangular waveguide of inner dimension $a \times b$ is given by

$$E = \frac{\omega \mu}{h^2} \left( \frac{\pi}{a} \right) H_0 \sin \left( \frac{2\pi x}{a} \right) \sin (\omega t - \beta z) a_y$$

Where $H_0$ is a constant, and $a$ and $b$ are the dimensions along the $x$-axis and the $y$-axis respectively. The mode of propagation in the waveguide is

(A) TE$_{20}$
(B) TM$_{11}$
(C) TM$_{20}$
(D) TE$_{10}$

MCQ 9.3.7  GATE 2007

An air-filled rectangular waveguide has inner dimensions of 3 cm $\times$ 2 cm. The wave impedance of the TE$_{20}$ mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377 \, \Omega$)

(A) 308 $\Omega$  
(B) 355 $\Omega$  
(C) 400 $\Omega$  
(D) 461 $\Omega$

MCQ 9.3.8  GATE 2006

A rectangular wave guide having TE$_{10}$ mode as dominant mode is having a cut off frequency 18 GHz for the mode TE$_{30}$. The inner broad - wall dimension of the rectangular wave guide is

(A) $\frac{5}{3}$ cm  
(B) 5 cm  
(C) $\frac{5}{2}$ cm  
(D) 10 cm

MCQ 9.3.9  GATE 2005

Which one of the following does represent the electric field lines for the mode in the cross-section of a hollow rectangular metallic waveguide?
MCQ 9.3.10
The phase velocity of an electromagnetic wave propagating in a hollow metallic rectangular waveguide in the TE\textsubscript{10} mode is
(A) equal to its group velocity
(B) less than the velocity of light in free space
(C) equal to the velocity of light in free space
(D) greater than the velocity of light in free space

MCQ 9.3.11
In a microwave test bench, why is the microwave signal amplitude modulated at 1 kHz
(A) To increase the sensitivity of measurement
(B) To transmit the signal to a far-off place
(C) To study amplitude modulations
(D) Because crystal detector fails at microwave frequencies

MCQ 9.3.12
A rectangular metal wave guide filled with a dielectric material of relative permittivity \(\varepsilon_r = 4\) has the inside dimensions 3.0 cm \(\times\) 1.2 cm. The cut-off frequency for the dominant mode is
(A) 2.5 GHz
(B) 5.0 GHz
(C) 10.0 GHz
(D) 12.5 GHz

MCQ 9.3.13
The phase velocity for the TE\textsubscript{10} mode in an air-filled rectangular waveguide is \(c\) (is the velocity of plane waves in free space)
(A) less than \(c\)
(B) equal to \(c\)
(C) greater than \(c\)
(D) none of these

MCQ 9.3.14
The phase velocity of wave propagating in a hollow metal waveguide is
(A) greater than the velocity of light in free space
(B) less than the velocity of light in free space
(C) equal to the velocity of light free space
(D) equal to the velocity of light in free space
### MCQ 9.3.15
#### GATE 2001
The dominant mode in a rectangular waveguide is TE_{10}, because this mode has:

(A) the highest cut-off wavelength  
(B) no cut-off  
(C) no magnetic field component  
(D) no attenuation

### MCQ 9.3.16
#### GATE 2000
A TEM wave is incident normally upon a perfect conductor. The \(E\) and \(H\) field at the boundary will be respectively:

(A) minimum and minimum  
(B) maximum and maximum  
(C) minimum and maximum  
(D) maximum and minimum

### MCQ 9.3.17
#### GATE 2000
A rectangular waveguide has dimensions 1 cm \(\times\) 0.5 cm. Its cut-off frequency is:

(A) 5 GHz  
(B) 10 GHz  
(C) 15 GHz  
(D) 12 GHz

### MCQ 9.3.18
#### GATE 1999
Assuming perfect conductors of a transmission line, pure TEM propagation is NOT possible in:

(A) coaxial cable  
(B) air-filled cylindrical waveguide  
(C) parallel twin-wire line in air  
(D) semi-infinite parallel plate wave guide

### MCQ 9.3.19
#### GATE 1999
Indicate which one of the following will NOT exist in a rectangular resonant cavity.

(A) TE_{110}  
(B) TE_{011}  
(C) TM_{110}  
(D) TM_{111}

### MCQ 9.3.20
#### IES EC 2012
The ratio of the transverse electric field to the transverse magnetic field is called as:

(A) wave guide impedance  
(B) wave guide wavelength  
(C) phase velocity  
(D) Poynting vector

### MCQ 9.3.21
#### IES EC 2012
Consider a rectangular waveguide of internal dimensions 8 cm \(\times\) 4 cm. Assuming an \(H_{10}\) mode of propagation, the critical wavelength would be:

(A) 8 cm  
(B) 16 cm  
(C) 4 cm  
(D) 32 cm

### MCQ 9.3.22
#### IES EC 2012
\[ \gamma = \sqrt{\left( \frac{n \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 - \frac{\omega^2}{\mu \varepsilon}} \]
represents the propagation constant in a rectangular waveguide for:

(A) TE waves only  
(B) TM waves only  
(C) TEM waves  
(D) TE and TM waves
With the symbols having their standard meaning, cut-off frequency (frequency below which wave propagation will not occur) for a rectangular waveguide is

\[
\begin{align*}
\text{(A)} & \quad \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \\
\text{(B)} & \quad \frac{1}{2\pi \sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \\
\text{(C)} & \quad \frac{1}{2\pi \sqrt{\mu \varepsilon}} \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \\
\text{(D)} & \quad \frac{1}{\sqrt{\mu \varepsilon}} \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2
\end{align*}
\]

A standard air filled waveguide WR-187 has inside wall dimensions of \(a = 4.755\) cm and \(b = 2.215\) cm. At 12 GHz, it will support

\[
\begin{align*}
\text{(A)} & \quad \text{TE}_{10} \text{ mode only} \\
\text{(B)} & \quad \text{TE}_{10} \text{ and TE}_{20} \text{ modes only} \\
\text{(C)} & \quad \text{TE}_{10}, \text{TE}_{20} \text{ and TE}_{01} \text{ modes only} \\
\text{(D)} & \quad \text{TE}_{10}, \text{TE}_{20}, \text{TE}_{01} \text{ and TE}_{11} \text{ modes}
\end{align*}
\]

Consider the following statements relating to the cavity resonator:

1. The cavity resonator does not possess as many modes as the corresponding waveguides does.
2. The resonant frequencies of cavities are very closely spaced.
3. The resonant frequency of a cavity resonator can be changed by altering its dimensions.

Which of the above statements is/are correct?

\[
\begin{align*}
\text{(A)} & \quad 2 \text{ and } 3 \text{ only} \\
\text{(B)} & \quad 2 \text{ only} \\
\text{(C)} & \quad 3 \text{ only} \\
\text{(D)} & \quad 1, 2 \text{ and } 3
\end{align*}
\]

The correct statement is

\[
\begin{align*}
\text{(A)} & \quad \text{Microstrip lines can support pure TEM mode of propagation but shielded coaxial lines cannot} \\
\text{(B)} & \quad \text{Microstrip lines cannot support pure TEM mode of propagation but shielded coaxial lines can} \\
\text{(C)} & \quad \text{Both microstrip lines and shielded coaxial lines can support pure TEM mode of propagation} \\
\text{(D)} & \quad \text{Neither microstrip lines nor shielded coaxial lines can support pure TEM mode of propagation.}
\end{align*}
\]

An air-filled rectangular waveguide has dimensions of \(a = 6\) cm and \(b = 4\) cm. The signal frequency is 3 GHz. Match List I with List II and select the correct answer using the code given below the lists:

\[
\begin{align*}
\text{List I} & \quad \text{List II} \\
a. & \quad \text{TE}_{10} \quad 1. \quad 2.5 \text{ GHz} \\
b. & \quad \text{TE}_{01} \quad 2. \quad 3.75 \text{ GHz} \\
c. & \quad \text{TE}_{11} \quad 3. \quad 4.506 \text{ GHz} \\
d. & \quad \text{TM}_{11} \quad 4. \quad 4.506 \text{ GHz}
\end{align*}
\]
Codes:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(B)</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>(C)</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(D)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**MCQ 9.3.28**

**Assertion (A):** TEM (Transverse Electromagnetic) waves cannot propagate within a hollow waveguide of any shape.

**Reason (R):** For a TEM wave to exist within the waveguide, lines of $H$ field must be closed loops which requires an axial component of $E$ which is not present in a TEM wave.

(A) Both A and R are individually true and R is the correct explanation of A
(B) Both A and R are individually true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

**MCQ 9.3.29**

For plane wave propagating in free space or two conductor transmission line, what must be the relationship between the phase velocity $v_p$, the group velocity $v_g$ and speed of light $c$?

(A) $v_p > c > v_g$
(B) $v_p < c < v_g$
(C) $v_p = c = v_g$
(D) $v_p < v_g < c$

**MCQ 9.3.30**

Consider the following statements:

1. Wavelength $\lambda = \frac{\lambda_0}{\varepsilon_e}$, where $\varepsilon_e$ is the effective dielectric constant and $\lambda_0$ is the free space wavelength.
2. Electromagnetic fields exist partly in the air above the dielectric substrate and partly within the substrate itself.
3. The effective dielectric constant is greater than the dielectric constant of the air.
4. Conductor losses increase with decreasing characteristic impedance.

Which of the above statements is/are correct?

(A) 1, 2 and 3
(B) 1 and 2
(C) 2, 3 and 4
(D) 4 only
MCQ 9.3.31  Match List I with List II and select the correct answer using the codes given below the lists.

List I (Type of transmission structure)  List II (Modes of propagation)

a. Strip line 1. Quassi TEM
b. Hollow rectangular waveguide 2. Pure TEM
c. Microguide 3. TE/TEM
d. Corrugated waveguide 4. Hybrid

Codes

a b c d
(A) 2 1 3 4
(B) 4 1 3 2
(C) 2 3 1 4
(D) 4 3 1 2

MCQ 9.3.32  A standard waveguide WR90 has inside wall dimensions of \( a = 2.286 \text{ cm} \) and \( b = 1.016 \text{ cm} \). What is the cut-off waveguide for TE\(_{01}\) mode ?

(A) 4.572 cm  (B) 2.286 cm
(C) 2.032 cm  (D) 1.857 cm

MCQ 9.3.33  When a particular mode is exited in a waveguide, there appears an extra electric component, in the direction of propagation. In what mode is the wave propagating ?

(A) Transverse electric  (B) Transverse magnetic
(C) Transverse electromagnetic  (D) Longitudinal

MCQ 9.3.34  Consider the following statements : For a square waveguide of cross-section \( 3 \text{ m} \times 3 \text{ m} \) it has been found

1. at 6 GHz dominant mode will propagate.
2. at 4 GHz all the mode are evanescent.
3. at 11 GHz only dominant modes and no higher order mode will propagate.
4. at 7 GHz degenerate modes will propagate.

Which of the above statements are correct ?

(A) 1 and 2  (B) 1, 2 and 4
(C) 2 and 3  (D) 2, 3 and 4
Match List with List II and select the correct answer using the codes given below the lists.

**List I (Mode)**

- a. Evanescent mode
- b. Dominant mode
- c. TM\(_{10}\) and TM\(_{01}\)

**List II (Characteristic)**

1. Rectangular waveguide does not support
2. No wave propagation
3. Lowest cut-off frequency

**Codes**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(B)</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>(C)</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>(D)</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**MCQ 9.3.36**

**Assertion (A):** A z-directed rectangular waveguide with cross-sectional dimensions 3 cm × 1 cm will support propagation at 4 GHz.

**Reason (R):** \(k^2 + \left(\frac{m\pi}{\lambda}\right)^2 + \left(\frac{n\pi}{\lambda}\right)^2 = \left(\frac{2\pi}{\lambda}\right)^2\), where \(\lambda\) is the wavelength.

(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false
(D) A is false but R is true

**MCQ 9.3.37**

Which one of the following is the correct statement?

A rectangular coaxial line can support
(A) only TEM mode of propagation
(B) both TEM and TE modes of propagation
(C) either TE or TM mode of propagation
(D) TEM, TE or TM mode of propagation

A rectangular waveguide (A) is gradually deformed first into a circular wave guide (B) and lack again into a rectangular waveguide (C) which is oriented through 90°
with respect to (A) If the input mode is TE\(_{10}\), which mode is excited in the output waveguide (C) ?
(A) TE\(_{10}\)  (B) TE\(_{01}\)
(C) TE\(_{11}\)  (D) TM\(_{11}\)

**MCQ 9.3.39**
The dominant mode in a circular waveguide is a:
(A) TEM mode  (B) TM\(_{01}\) mode
(C) TE\(_{21}\) mode  (D) TE\(_{11}\) mode

**MCQ 9.3.40**
The cut-off frequency of the dominant mode of a rectangular wave guide having aspect ratio more than 2 is 10 GHz. The inner broad wall dimension is given by:
(A) 3 cm  (B) 2 cm
(C) 1.5 cm  (D) 2.5 cm

**MCQ 9.3.41**
In a waveguide, the evanescent modes are said to occur if:
(A) The propagation constant is real
(B) The propagation constant is imaginary
(C) Only the TEM waves propagate
(D) The signal has a constant frequency

**MCQ 9.3.42**
**Assertion (A):** A microstrip line cannot support pure TEM mode of propagation.
**Reason (R):** A microstrip line suffers from various forms of losses.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

**MCQ 9.3.43**
Consider the following statements relating to the cavity resonators:
1. For over-coupling the cavity terminals are at voltage maximum in the input line at resonance
2. For over-coupling the cavity terminals are at the voltage minimum in the input line at resonance
3. For under-coupling the normalized impedance at the voltage maximum is the standing wave ratio
4. For over-coupling the input terminal impedance is equal to the reciprocal of the standing wave ratio
Which of the statements given above are correct?
(A) 1 and 2  (B) 3 and 4
(C) 1 and 3  (D) 2 and 4
Consider the following statements relating to the microstrip lines:
1. Modes on microstrip lines are purely TEM
2. Microstrip line is also called open strip line
3. Radiation loss in microstrip line can be reduced by using thin high dielectric materials
4. Conformal transformation technique is quite suitable for solving microstrip problems

Which of the statements given above are correct?
(A) 1, 2 and 3
(B) 2, 3 and 4
(C) 1, 3 and 4
(D) 1, 2 and 4

Match List I (Dominant Mode of Propagation) with List II (Type of transmission Structure) and select the correct answer:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Coaxial line</td>
<td>1. TE</td>
</tr>
<tr>
<td>b. Rectangular waveguide</td>
<td>2. Quasi TEM</td>
</tr>
<tr>
<td>c. Microstrip line</td>
<td>3. Hybrid</td>
</tr>
<tr>
<td>d. Coplanar waveguide</td>
<td>4. TEM</td>
</tr>
</tbody>
</table>

Codes:

(a) 1 4 2 3
(b) 4 1 3 2
(c) 1 4 3 2
(d) 4 1 2 3

For TE or TM modes of propagation in bounded media, the phase velocity
(A) is independent of frequency
(B) is a linear function of frequency
(C) is a non-linear function of frequency
(D) can be frequency-dependent or frequency-independent depending on the source

A waveguide operated below cut-off frequency can be used as
(A) A phase shifter
(B) An attenuator
(C) An isolator
(D) None of the above
MCQ 9.3.48
Assertion (A): The quality factor $Q$ of a waveguide is closely related to its attenuation factor $\alpha$.
Reason (R): Normally attenuation factors obtainable in waveguides are much higher than those obtainable in transmission lines.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is

MCQ 9.3.49
Assertion (A): The greater the ‘$Q$’, the smaller the bandwidth of a resonant circuit.
Reason (R): At high frequencies the $Q$ of a coil falls due to skin effect.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 9.3.50
For a wave propagation in an air filled rectangular waveguide.
(A) guided wavelength is never less than free space wavelength
(B) wave impedance is never less than the free space impedance
(C) TEM mode is possible if the dimensions of the waveguide are properly chosen
(D) Propagation constant is always a real quantity

MCQ 9.3.51
When a particular mode is excited in a wave-guide, there appears an extra electric component in the direction of propagation. The resulting mode is
(A) transverse-electric
(B) transverse-magnetic
(C) longitudinal
(D) transverse-electromagnetic

MCQ 9.3.52
For a hollow waveguide, the axial current must necessarily be
(A) a combination of conduction and displacement currents
(B) time-varying conduction current and displacement current
(C) time-varying conduction current and displacement current
(D) displacement current only

MCQ 9.3.53
As a result of reflections from a plane conducting wall, electromagnetic waves acquire an apparent velocity greater than the velocity of light in space. This is called
(A) velocity propagation
(B) normal velocity
(C) group velocity
(D) phase velocity
MCQ 9.3.54
Assertion (A) : A thin sheet of conducting material can act as a low-pass filter for electromagnetic waves.
Reason (R) : The penetration depth is inversely proportional to the square root of the frequency.
(A) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A)
(B) Both Assertion (A) and Reason (R) are individually true but Reason (R) is not the correct explanation of Assertion (A)
(C) Assertion (A) is true but Reason (R) is false
(D) Assertion (A) is false but Reason (R) is true

MCQ 9.3.55
Consider the following statements in connection with cylindrical waveguides:
1. At low frequency the propagation constant is real and wave does not propagate.
2. At intermediate frequency the propagation constant is zero and wave cut off.
3. At high frequency the propagation constant is imaginary and wave propagates.
4. At transition condition the cut-off frequency is inversely proportional to the eigen values of the Bessel function for the respective TE_{nr} mode.
Which of these statements is/are correct?
(A) 1, 2 and 3
(B) 2 only
(C) 2 and 3 only
(D) 2, 3 and 4

MCQ 9.3.56
How is the attenuation factor in parallel plate guides represented?
(A) \( \alpha = \frac{\text{Power lost}}{\text{Power transmitted}} \)
(B) \( \alpha = 2 \times \frac{\text{Power lost}}{\text{Power transmitted}} \)
(C) \( \alpha = \frac{\text{Power lost per unit length}}{2 \times \text{power transmitted}} \)
(D) \( \alpha = \frac{\text{Power lost}}{\text{Power lost + power transmitted}} \)

MCQ 9.3.57
Which one of the following statements is correct? A wave guide can be considered to be analogous to a
(A) low pass filter
(B) high pass filter
(C) band pass filter
(D) band stop filter

************
SOLUTIONS 9.1

SOL 9.1.1  Option (A) is correct.
Given, electric field intensity of the propagating wave is
\[ E_{zs} = 5\sin(20\pi x)\sin(25\pi y)e^{-\beta z} \text{ V/m} \tag{1} \]
So, we conclude that the wave is propagating in \( x \) direction. Since, the wave has it’s component of electric field in the direction of propagation so, the waveguide is operating in \( \text{TM}_{mn} \) (Transverse magnetic) mode.
Now for determining the value of \( m \) and \( n \), we compare the phasor form of electric field to its general equation given as.
\[ E_{zs} = E_0\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\beta z} \tag{2} \]
where \( a \) and \( b \) are the dimensions of waveguide and since, the waveguide has the dimension \( 10 \times 4 \text{ cm} \) so, we get
\[ a = 10 \text{ cm} \] and \( b = 4 \text{ cm} \)
Now, comparing equation (1) and (2) we get
\[ \frac{m\pi x}{a} = 20\pi x \Rightarrow m = 2 \]
\[ \frac{n\pi y}{b} = 25\pi y \Rightarrow n = 1 \]
Thus, the mode of propagation of wave is \( \text{TM}_{21} \).

SOL 9.1.2  Option (A) is correct.
A wave mode propagates in a waveguide only if it’s frequency is greater than cutoff frequency. If there is no any propagating mode inside the waveguide then energy in the propagating mode is zero. So, average power flow down the waveguide below cutoff frequency is zero.
i.e. Both the statements are correct and R is correct explanation of A.

SOL 9.1.3  Option (C) is correct.
The intrinsic impedance of an airfilled waveguide for TM mode is defined as
\[ \eta_{\text{TM}_{mn}} = \eta_0\sqrt{1 - \left(\frac{f_{\text{cut}}}{f}\right)^2} \]
Since, the operating frequency is twice the cutoff frequency
i.e. \( f = 2f_{\text{cut}} \)
So, we get the intrinsic wave impedance as
\[ \eta_{\text{TM}_{mn}} = \frac{377}{\sqrt{1 - \left(\frac{1}{2}\right)^2}} = 32.649 \Omega \approx 327 \Omega \]
SOL 9.1.4 Option (D) is correct.
The dimensions of wave guide, \( a = 10 \text{ cm} = 0.1 \text{ m} \)
and, \( b = 4 \text{ cm} \)
The mode of propagation, \( m = 2, \ n = 1 \)
Operating frequency, \( f = 7.5 \text{ GHz} = 7.5 \times 10^9 \text{ Hz} \),
Unbounded phase velocity, \( v_p = c = 3 \times 10^8 \text{ m/s} \) (air filled)
So, the cut-off frequency of the waveguide is given as
\[
f_c = \frac{c}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} = \frac{3 \times 10^8}{2} \sqrt{(20)^2 + (25)^2} = 4.8 \times 10^9 \text{ Hz}
\]
Therefore, the phase constant of the wave inside the waveguide is defined as
\[
\beta = \sqrt{\frac{\omega}{v_p}} \sqrt{1 - \left( \frac{f_c}{f} \right)^2} = \frac{2\pi f}{c} \sqrt{\frac{f^2 - f_c^2}{f^2}}
\]
\[
= \frac{2\pi}{3 \times 10^8} \times 10^9 \sqrt{(7.5)^2 - (4.8)^2} = 220.7 \text{ rad/m}
\]

SOL 9.1.5 Option (A) is correct.
Since, the electric field component exists in the direction of propagation so it will be operating in TM (Transverse magnetic) mode. So, for the TM mode the electric field components in phasor form are given as
\[
E_{zs} = -\frac{r}{k^2} \frac{\partial E_{r}}{\partial x}
\]
and
\[
E_{ys} = -\frac{r}{k^2} \frac{\partial E_{r}}{\partial y}
\]
Since, the given electric field component is
\[
E_{zs} = E_0 \sin(50\pi x) \sin(40\pi y) e^{-r} \text{ V/m}
\]
So,
\[
E_{zs} = -\frac{r}{k^2} (50\pi) E_0 \cos(50\pi x) \sin(40\pi y) e^{-r^2}
\]
and
\[
E_{ys} = -\frac{r}{k^2} (40\pi) E_0 \sin(50\pi x) \cos(40\pi y) e^{-r^2}
\]
Therefore, the ratio of the components is
\[
\frac{E_{zs}}{E_{ys}} = \frac{50\pi}{40\pi} \cot(50\pi x) \tan(40\pi y)
\]
\[
= 1.25 \cot(50\pi x) \tan(40\pi y)
\]

SOL 9.1.6 Option (D) is correct.
Relative permittivity of dielectric \( \varepsilon_r = 2.25 \)
Relative permeability of the dielectric, \( \mu_r = 1 \)
Operating frequency, \( f = 10 \text{ GHz} = 10^{10} \text{ Hz} \)
Since, the waveguide is operating in TEM mode so, the phase constant is given as
\[
\beta = \omega \sqrt{\mu \varepsilon_r} = \frac{2\pi f}{c} \sqrt{\mu \varepsilon_r} = \frac{2\pi}{3 \times 10^8} \times (1.5) = 314.2 \text{ rad/m}
\]
The group velocity of the wave in TEM mode will be equal to its phase velocity in the unbounded dielectric medium
Option (A) is correct.

In a $a \times b$ rectangular waveguide, cutoff frequency for (TE)$_{mn}$ or (TM)$_{mn}$ mode is defined as

$$(f_c)_{mn} = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Now, for TM$_{11}$ mode

$$f_a = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{b}\right)^2} = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{a}\right)^2} \quad a = 2b$$

Similarly, for TM$_{12}$ mode

$$f_a = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{b}\right)^2} = \sqrt{17}\frac{1}{2a\sqrt{\mu \varepsilon}}$$

For TE$_{10}$ mode

$$f_a = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{1}{a}\right)^2 + 0} = \frac{1}{2a\sqrt{\mu \varepsilon}}$$

For TE$_{20}$ mode

$$f_a = \frac{1}{2}\sqrt{\mu \varepsilon} \sqrt{\left(\frac{2}{a}\right)^2 + 0} = 2\frac{1}{2a\sqrt{\mu \varepsilon}}$$

So comparing cutoff frequencies of all the modes we get the modes in ascending order of cutoff frequencies as

TE$_{10} <$ TE$_{20} <$ TM$_{11} <$ TM$_{12}$

Option (D) is correct.

Dimensions of wave guide $a = 5$ cm $= 5 \times 10^{-2}$ m and $b = 3$ cm $= 3 \times 10^{-2}$ m

Operating frequency, $f = 3.75$ GHz $= 3.75 \times 10^9$ Hz

Since operating mode of the waveguide is TE$_{10}$ (i.e., $m = 1$ and $n = 0$) so, the cutoff frequency of the airfilled waveguide is given as

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{2}\sqrt{\left(\frac{1}{5 \times 10^{-2}}\right)^2} + 0 = 3 \times 10^9$$

The group velocity of the EM wave in the waveguide is given as

$$v_g = \frac{c}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}} = 3 \times 10^8\sqrt{1 - \left(\frac{3 \times 10^9}{3.75 \times 10^9}\right)^2}$$

$$= 1.8 \times 10^8 \text{ m/s}$$

Option (C) is correct.

Dimensions of waveguide, $a = 2.5$ cm $= 2.5 \times 10^{-2}$ m and $b = 5$ cm $= 5 \times 10^{-2}$ m

Operating frequency, $f = 15$ GHz $= 15 \times 10^9$ Hz

Conductivity of medium, $\sigma = 0$ (lossless dielectric)
Relative permittivity, \( \varepsilon_r = 2 \)
Relative permeability, \( \mu_r = 1 \)

The operating mode of the waveguide is TE\(_{20}\) mode (i.e., \( m = 2 \) and \( n = 0 \))

So, the cutoff frequency of the waveguide in the TE\(_{20}\) mode is given as
\[
f_c = \frac{1}{2} \sqrt{\mu \varepsilon} \sqrt{\left(\frac{m^2}{a^2}\right) + \left(\frac{n}{b}\right)^2} = \frac{3 \times 10^8}{2} \sqrt{2} \sqrt{\frac{2}{2.5 \times 10^{-2}}} \]
\[
= 8.5 \times 10^9
\]

The wave impedance for the TE\(_{20}\) mode is given as
\[
\eta_{TE_{20}} = \frac{\eta}{\sqrt{1 - \left(\frac{k}{f}\right)^2}} = \eta_0 \sqrt{\frac{\mu_0}{\varepsilon_r}} \sqrt{\left(\frac{1}{1 - \left(\frac{k}{f}\right)^2}\right)}
\]
\[
= 377 \frac{1}{\sqrt{1 - \left(8.5 \times 10^{-2}\right)^2}}
\]
\[
= 323 \Omega
\]

**SOL 9.1.10**

Option (D) is correct.

Given, the cutoff frequency for TM\(_2\) mode is
\( (\lambda_c)_b = 2 \text{ mm} = 2 \times 10^{-3} \text{ m} \)

Since, the cutoff wavelength for TM\(_n\) or TE\(_n\) mode for a parallel plate waveguide is defined as
\[
(\lambda_c)_n = \frac{2b}{n} \sqrt{\varepsilon_r}
\]

where \( b \) is the separation between parallel plates of the waveguide and \( \varepsilon_r \) is relative permittivity of the medium. So, putting the known values in the expression, we get
\[
2 \times 10^{-3} = \frac{2b}{2} \sqrt{\varepsilon_r} \quad (n = 2)
\]
\[
b = \frac{2 \times 10^{-3}}{\sqrt{\varepsilon_r}}
\]

Now, for any \( n \) mode to propagate the operating wavelength must be less than or equal to the cutoff frequency.

i.e.
\[
\lambda \leq (\lambda_c)_n
\]

So, from equation (1) for the propagation of wavelength \( \lambda = 0.1 \text{ cm} \) we have the relation as
\[
0.1 \times 10^{-2} \leq \frac{2b}{b} \sqrt{\varepsilon_r}
\]
\[
0.1 \times 10^{-2} \leq \frac{2}{n} \times 2 \times 10^{-3} \sqrt{\varepsilon_r}
\]
\[
n \leq \frac{4 \times 10^{-3}}{0.1 \times 10^{-2}}
\]
\[
n \leq 4
\]

Therefore, the possible modes that can propagate in the waveguide are TEM, TE\(_1\), TE\(_2\), TE\(_3\), TE\(_4\), TM\(_1\), TM\(_2\), TM\(_3\) and TM\(_4\)

Thus, there are nine possible modes that can propagate in the waveguide.
SOL 9.1.11 Option (D) is correct.
Plate separation, \( b = 10 \text{ mm} = 10^{-2} \text{ m} \)
Minimum operating frequency, \( f_{\text{min}} = f_c = 15 \text{ GHz} = 15 \times 10^9 \text{ Hz} \)
Since, for TM\(_n\) mode of parallel plate waveguide, cutoff frequency is defined as
\[
(f_c)_n = \frac{n}{2b\sqrt{\mu \varepsilon}}
\]
So, for (TM)_3 mode \((n = 3)\) we have the cutoff frequency as
\[
(f_c)_3 = \frac{3}{2b\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}
\]
\[
15 \times 10^9 = \frac{3 \times (3 \times 10^9)}{2 \times 10^{-2} \sqrt{\varepsilon_r}}
\]
\[
\sqrt{\varepsilon_r} = 3
\]
\[
\varepsilon_r = 9
\]
or
\[
(f_c)_3 = f_{\text{min}}
\]

SOL 9.1.12 Option (D) is correct.
Plate separation, \( b = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \)
Relative permittivity of medium, \( \varepsilon_r = 2.1 \)
Operating frequency, \( f = 16 \text{ GHz} \)
For propagation of wave the operating frequency must be greater than the cutoff frequency of (TE)_n or (TM)_n mode of parallel plate waveguide
i.e.
\[
f > (f_c)_n
\]
\[
f > \frac{n}{2b\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}
\]
\[
n < \frac{2 \times 16 \times 10^9 \times 20 \times 10^{-3}}{3 \times 10^9}
\]
\[
n < 3.09
\]
So, the maximum allowed mode is
\[
n = 3
\]
Since, all the modes given in the option are in the range, therefore, all the three modes will propagate.

SOL 9.1.13 Option (C) is correct.
Cutoff frequency of (TM)_1 mode, \((f_c)_1 = 2.5 \text{ GHz} = 2.5 \times 10^9 \text{ Hz} \)
Operating wavelength, \( \lambda = 3 \text{ cm} = 3 \times 10^{-2} \text{ m} \)
The cutoff frequency of (TE)_n mode of the parallel plate waveguide is given as
\[
(f_c)_n = 3 (f_c)_1 = 3 \times 2.5 \times 10^9 \text{ Hz} = 7.5 \times 10^9 \text{ Hz}
\]
Since, the operating frequency of the waveguide is defined as
\[
f = \frac{c}{\lambda}
\]
where \( \lambda \) is the operating wavelength. So, the operating frequency of the parallel plate waveguide is
\[
f = \frac{3 \times 10^8}{3 \times 10^{-2}} = 10^{10} \text{ Hz}
\]

Therefore, the group velocity of TE₃ mode is given as
\[
(v_g)_3 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \sqrt{1 - \left(\frac{f}{f_3}\right)^2} = 3 \times 10^8 \sqrt{1 - \left(\frac{7.5 \times 10^{-3}}{1.08 \times 10^3}\right)^2} = 2 \times 10^8 \text{ m/s}
\]

SOL 9.1.14 Option (C) is correct.
At cutoff the mode propagates in the slab at the critical angle which means that the phase velocity will be equal to that of a plane wave in upper or lower media of refractive index \(n_2\). So, the phase velocity at cutoff will be
\[
v_p = \frac{c}{n_2} = \frac{3 \times 10^8}{2.5} = 1.2 \times 10^8 \text{ m/s}
\]

SOL 9.1.15 Option (A) is correct.
The phase velocity at cutoff is independent of the mode and equal to the phase velocity of a plane wave in unbounded media. Since, in the given problem the phase velocity of TM₂ mode is to be determined for same waveguide so, the phase velocity of TM₂ mode will be equal to that of TM₁ mode.
\[
v_{p₂} = v_{p₁}
\]

SOL 9.1.16 Option (C) is correct.
Relative permittivity of material, \(\varepsilon_r = 8.8\)
Separation between strip line, \(b = 0.632 \text{ cm}\)
Characteristic impedance, \(Z_0 = 35\)
So, \(\sqrt{\varepsilon_r}Z_0 = \sqrt{8.8}(35) = 103.8\)
Since, \(\sqrt{\varepsilon_r}Z_0 < 120\)
Therefore, the width to separation ratio of strip line transmission line is given as
\[
\frac{w}{b} = \frac{30\pi}{\sqrt{\varepsilon_r}Z_0} - 0.441
\]
\[
\frac{w}{0.632} = \frac{30\pi}{(\sqrt{8.8})(35)} - 0.441
w = 0.295
\]

SOL 9.1.17 Option (C) is correct.
Guide wavelength of a stripline is defined as,
\[
\lambda_g = \frac{c}{\sqrt{\varepsilon_r}f}
\]
where, \(c\) is velocity of wave in free space, \(f\) is the operating frequency and \(\varepsilon_r\) is the relative permittivity of the medium. So, we get
\[
\lambda_g = \frac{3 \times 10^8}{(\sqrt{8.8})(3 \times 10^9)} = 3.37 \text{ cm}
\]

SOL 9.1.18 Option (B) is correct.
Statement 1
Suppose on the contrary the TEM mode existed. In this case the magnetic field must lie solely in the transverse \(xy\)-plane. The magnetic field lines must form closed paths in this transverse plane, since \(\nabla \cdot \mathbf{H} = 0\). From Ampere’s law, the integral
of this transverse magnetic field around these closed paths must yield the axial conduction or displacement current. But \( E_z = 0 \) for the TEM mode so, no axial displacement current can exist. Also, since there is no center conductor so, no axial conduction current can exist. Therefore statement 1 is correct.

**Statement 2**

The dominant mode is the mode that has lowest cutoff frequency. Now, \( f_{c,\text{rms}} \) is clearly minimized when either \( m \) or \( n \) is zero. Since, TM\(_{01}\) or TM\(_{10}\) mode doesn’t exist so, TM mode can’t be the dominant mode of propagation in rectangular waveguide.

It is also correct statement.

**SOL 9.1.19**

Option (C) is correct.

For a TM\(_{mnp}\) mode, neither \( m \) nor \( n \) can be zero otherwise all field components vanish, however \( p \) can be zero. So, the lowest order TM mode is TM\(_{110}\).

**SOL 9.1.20**

Option (D) is correct.

Since, TE\(_{mnp}\) mode of cavity resonator can have either \( m = 0 \) or \( n = 0 \) (but not both at a time) where as \( p \) can’t be zero for TE mode so, the lowest order of TE mode is

\[
\begin{align*}
\text{TE}_{011} & \quad \text{if} \quad a < b \\
\text{TE}_{101} & \quad \text{if} \quad a > b
\end{align*}
\]

As the dimensions of the cavity resonator are equal \((a = b)\) so, both the TE\(_{101}\) and TE\(_{011}\) are lowest order mode.

**SOL 9.1.21**

Option (D) is correct.

Given, the dimensions of cavity resonator are related as

\[
a > b > c
\]

The condition for propagating TE and TM modes in a cavity resonator are as follows :

1. For TM\(_{mnp}\) mode, neither \( m \) nor \( n \) can be zero however \( p \) can be zero.
2. For TE\(_{mnp}\) mode \( p \) can’t be zero but either \( m \) or \( n \) can be zero (but not both at a time)

The resonant frequency of TM\(_{mnp}\) or TE\(_{mnp}\) mode in a cavity resonator is defined as

\[
f_{mnp} = \frac{1}{2 \sqrt{\mu \varepsilon}} \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2 \right]^{1/2}
\]

So, comparing the resonant frequency for the different values of \( m, n \) and \( p \) using the relation defined in equation (1), we get the lowest order mode will be TM\(_{110}\) and the ascending order can be written as below :

TM\(_{110}\); TE\(_{101}\); TE\(_{011}\); TE\(_{111}\) = TM\(_{111}\)

**SOL 9.1.22**

Option (C) is correct.

In an airfilled cavity resonator, resonant frequency is defined as

\[
f_{mnp} = \frac{1}{2 \sqrt{\mu_0 \varepsilon_0}} \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2 \right]^{1/2}
\]
So, for TE_{101} mode (m = p = 1, n = 0) the resonant frequency is

\[ f_{101} = \frac{3 \times 10^8}{2} \left[ \frac{1}{30 \times 10^{-2}} \right]^2 + \left( \frac{1}{20 \times 10^{-2}} \right)^2 \]

\[ = 9.01388 \times 10^8 \text{ Hz} = 901.4 \text{ MHz} \]

**SOL 9.1.23** Option (A) is correct.

The resonant frequency of a cavity resonator is defined as

\[ f_{\text{res}} = \frac{1}{2\sqrt{\mu_0 \varepsilon_0}} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2 \right]^{1/2} \]

Since, \(a = b = c\) so, the dominant modes are TE_{101} or TE_{011} or TM_{110}. Therefore, taking any of \(m, n\) or \(p\) equal to zero, we get the resonant frequency as

\[ f_{\text{res}} = \frac{3 \times 10^8}{2} \left[ \frac{2}{a^2} \right]^{1/2} \]

\[ 15 \times 10^8 = \frac{3 \times 10^8 \sqrt{2}}{a} \]

\[ a = 1.41 \times 10^{-2} \text{ m} = 1.41 \text{ cm} \]

i.e. \(a = b = c = 1.41 \text{ cm}\)

************
Option (B) is correct. 

Since, the waveguide is operating at TM$_{n}$ mode so, the phasor form of magnetic field of the EM wave will be given as 

$$H_{n} = \frac{1}{h} \omega E H_{0} \cos \left( \frac{nx}{b} \right) e^{-j\beta z}$$

Since, the waveguide is operating at TM$_{1}$ mode (i.e. $n = 1$) 

So, 

$$H_{1} = \frac{1}{h} \omega E H_{0} \cos \left( \frac{y}{b} \right) e^{-j\beta z}$$

Therefore, the instantaneous magnetic field intensity of the wave is given as 

$$H_{z} = \text{Re} \left\{ \frac{1}{h} \omega E H_{0} \cos \left( \frac{y}{b} \right) e^{-j\beta z} \right\}$$

$$= -\frac{1}{h} \omega E H_{0} \cos \left( \frac{y}{b} \right) \sin (\omega t - \beta z)$$

at $t = 0$ 

$$H_{z} = \frac{1}{h} \omega E H_{0} \cos \left( \frac{y}{b} \right) \sin (\beta z) \quad (1)$$

As the EM wave is propagating in $y$-$z$ plane so, in TM mode the $y$ and $z$-components of the magnetic field intensity will be zero. 

i.e. 

$$H_{y} = H_{z} = 0$$

Thus, the field will have the component only in $z$-direction for which we sketch the field lines in $y$-$z$ plane. From equation (1), we conclude that the field intensity $H_{z}$ depends on the values cosines and sines of the two variables defined as 

$$\cos \left( \frac{y}{b} \right) = \begin{cases} + \text{ve} & 0 < y < 0.5 \\ - \text{ve} & 0.5 < y < 1 \end{cases}$$

$$\sin \beta z = \begin{cases} + \text{ve} & 0 < \beta z < \pi \\ - \text{ve} & \pi < \beta z < 2\pi \end{cases}$$

Using these values we get the sketch of the field lines in the $yz$-plane as shown in the figure below where $x$-axis directs into the paper.
SOL 9.2.2 Option (A) is correct.
The phase velocity of the EM wave in the guide is defined as
\[ v_p = \frac{\omega}{\beta} \]
where \( \omega \) is the operating angular frequency and \( \beta \) is the phase constant inside the airfilled waveguide given as
\[ \beta = \frac{\omega}{c} \sqrt{1 - (\frac{f_c}{f})^2} \]
So, we get
\[ \frac{\omega}{c} \sqrt{1 - (\frac{f_c}{f})^2} = \frac{\omega}{c} \]
\[ \Rightarrow \left( \frac{c}{v_p} \right)^2 + \left( \frac{f_c}{f} \right)^2 = 1 \]
The above equation is the equation of a circle. So, the graph between \( \left( \frac{c}{v_p} \right) \) and \( \left( \frac{f_c}{f} \right) \) will be as plotted below:

SOL 9.2.3 Option (A) is correct.
Wavelength for a propagating wave inside the waveguide is defined as
\[ \lambda_0 = \frac{2\pi}{\beta} \]
where \( \beta \) is the phase constant of the wave in the waveguide given as
\[ \beta = \frac{\omega}{c} \sqrt{\mu \varepsilon} \sqrt{1 - \left( \frac{f_c}{f} \right)^2} \]
where \( f_c \) is the cutoff frequency of the waveguide and \( f \) is the operating frequency.
of the waveguide. So, we get

\[ \lambda_g = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0\sqrt{1 - \left(\frac{f_f}{f_c}\right)^2}}} \]

\[ \Rightarrow \lambda_g = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0\sqrt{1 - \left(\frac{f_f}{f_c}\right)^2}}} \quad \text{(for airfilled guide } \mu = \mu_0, \varepsilon = \varepsilon_0) \]

\[ \Rightarrow \lambda_g = \left(\frac{c}{f_f}\right)\left(\frac{1}{\sqrt{1 - \left(\frac{f_f}{f_c}\right)^2}}\right) \]

\[ \Rightarrow \lambda_g = \lambda\left(\frac{1}{\sqrt{1 - \left(\frac{f_f}{f_c}\right)^2}}\right) \]

\[ \Rightarrow \frac{\lambda_g}{\lambda} = \frac{(f_f/f_c)^2 - 1}{(f_f/f_c)^2} \]

Thus, the plot between \((f_f/f_c)\) and \((\lambda_g/\lambda)\) is as sketched below:

\[ (\lambda_g/\lambda) \]

1

1

\( (f_f/f_c) \)

\textbf{SOL 9.2.4} Option (C) is correct.

The propagation constant \(\gamma\) in the parallel plate waveguide is defined as

\[ \gamma^2 + \omega^2\mu\varepsilon = \left(\frac{n\pi}{b}\right)^2 \quad (1) \]

Since, for lossless medium propagation constant is given as

\[ \gamma = j\beta \quad \text{(attenuation constant, } \alpha = 0) \]

Putting it in equation (1), we get

\[ -\beta^2 + \omega^2\mu\varepsilon = \left(\frac{n\pi}{b}\right)^2 \]

At the cutoff frequency, \(\omega = \omega_c\) phase constant is zero (i.e., \(\beta = 0\)). So, we get

\[ \omega_c^2\mu\varepsilon = \left(\frac{n\pi}{b}\right)^2 \]

\[ \omega_c = \frac{n\pi}{b\sqrt{\mu\varepsilon}} \]

So, for TE\(_1\) mode

\[ \omega_{c1} = \frac{\pi}{b\sqrt{\mu\varepsilon}} \]

So TE\(_2\) mode

\[ \omega_{c2} = \frac{2\pi}{b\sqrt{\mu\varepsilon}} \]
For TE₃ mode
\[ \omega_{e} = \frac{3\pi}{b\sqrt{\mu\varepsilon}} \]
Comparing the three expressions we get,
\[ \omega_{a} < \omega_{e} < \omega_{e} \]

**SOL 9.2.5** Option (A) is correct.
As calculated in previous question, the expression for the operating frequency of the wave in the waveguide is given as
\[ -\beta^{2} + \omega^{2}\mu\varepsilon = \left( \frac{n\pi}{b} \right)^{2} \]
\[ \Rightarrow \quad \omega^{2}\mu\varepsilon = \beta^{2} + \left( \frac{n\pi}{b} \right)^{2} \]
\[ \Rightarrow \quad f^{2} \left( 4\pi^{2}\mu\varepsilon \right) = \beta^{2} + \left( \frac{n\pi}{b} \right)^{2} \]
So, for TM₂ mode \(( n = 2)\)
\[ f^{2} = \frac{1}{4\pi^{2}\mu\varepsilon} \left[ \beta^{2} + 4\left( \frac{n\pi}{b} \right)^{2} \right] \]
For TM₃ mode \(( n = 3)\)
\[ f^{2} = \frac{1}{4\pi^{2}\mu\varepsilon} \left[ \beta^{2} + 9\left( \frac{n\pi}{b} \right)^{2} \right] \]
and for TM₄ mode \(( n = 4)\)
\[ f^{2} = \frac{1}{4\pi^{2}\mu\varepsilon} \left[ \beta^{2} + 16\left( \frac{n\pi}{b} \right)^{2} \right] \]
Thus, for the above obtained expressions for the frequencies at different modes, we sketch the \( f-\beta \) curve as shown below:

**SOL 9.2.6** Option (C) is correct.
Operating frequency, \( f = 5 \text{ GHz} = 5 \times 10^{9} \text{ Hz} \)
Separation between the plates \( b = 6 \text{ cm} = 6 \times 10^{-2} \text{ m} \)
So, the cut off frequency for TM₁ mode is given as
\[ f_{c} = \frac{1}{2b\sqrt{\mu_{0}\varepsilon_{0}}} = \frac{3}{2 \times 6 \times 10^{-2}} = 2.5 \times 10^{9} \text{ Hz} \]
For a parallel plate waveguide, phasor form of components of electric field and magnetic field intensity of a propagation wave are given as
\[ E_{ys} = E_{0}\cos\left( \frac{n\pi y}{b} \right)e^{-\gamma z} \]
and
\[ H_{xs} = -\frac{E_{0}}{\eta_{0}\sqrt{1 - \left( \frac{f}{f_{c}} \right)^{2}}} \cos\left( \frac{n\pi y}{b} \right)e^{-\gamma z} \]
So, the average power is given as

\[
P_{\text{ave}} = \int \frac{1}{2} \text{Re}\{ \mathbf{E} \times \mathbf{H}^* \} \, d\mathbf{S} = \int_0^\phi \frac{1}{2} \{-E_{\psi}(H_{\psi})\} (w \, dy)
\]

\[
= \frac{w}{2} \int_0^\phi \frac{E_0^2}{\eta_0 \sqrt{1 - \left( \frac{f}{b} \right)^2}} \cos^2 \left( \frac{\pi y}{b} \right) \, dy
\]

\[
= \frac{w}{2} \frac{E_0^2}{\eta_0 \sqrt{1 - \left( \frac{f}{b} \right)^2}} \int_0^\phi \frac{1 + \cos \left( \frac{2\pi y}{b} \right)}{2} \, dy
\]

\[
= \frac{wb}{4} \frac{E_0^2}{\eta_0 \sqrt{1 - \left( \frac{f}{b} \right)^2}}
\]

The maximum power propagation will be due to the maximum electric field in the medium (the dielectric strength of the medium). So, we have the maximum average power as

\[
(P_{\text{ave}})_{\text{max}} = \frac{wb}{4} \frac{(3 \times 10^6)^2}{\eta_0 \sqrt{1 - \left( \frac{f}{b} \right)^2}} \quad ((E_0)_{\text{max}} = 3 \times 10^6 \text{ V/m})
\]

Putting all the values, we get the average power per unit width as

\[
\frac{(P_{\text{ave}})_{\text{max}}}{w} = \frac{6 \times 10^{-2}}{4} \times \frac{(3 \times 10^6)^2}{120\pi \sqrt{1 - \left( \frac{f}{b} \right)^2}}
\]

\[
= 4.135 \times 10^8 = 414 \text{ MW/m}
\]

**SOL 9.2.7**

Option (D) is correct.

Separation between waveguide plates, \( b = 12 \text{ cm} = 0.12 \text{ m} \)

Operating frequency, \( f = 2.5 \text{ GHz} = 2.5 \times 10^9 \text{ Hz} \)

For the TEM mode, phasor form of electric and magnetic field components are given as

\[
E_{\psi} = E_0 e^{-\gamma z}
\]

\[
H_{\psi} = -\frac{E_0}{\eta_0} e^{-\gamma z}
\]

So, the average power propagated in the waveguide is given as

\[
P_{\text{ave}} = \int \text{Re}\left\{ \frac{1}{2} \mathbf{E} \times \mathbf{H}^* \right\} \, d\mathbf{S}
\]

\[
= \int \frac{1}{2} \text{Re}\{-(E_{\psi})(H_{\psi})\} \, ds = -\int_0^\phi \frac{1}{2}(E_0)\left( -\frac{E_0}{\eta_0} \right) w \, dy
\]

\[
= \frac{E_0^2}{2\eta_0} wb
\]

The maximum electric field, without any voltage breakdown is defined as the dielectric strength of the medium as given and as the dielectric strength of air is \((E_0)_{\text{max}} = 3 \times 10^6 \text{ V/m}\)

So, the maximum average power propagated in the waveguide is

\[
(P_{\text{ave}})_{\text{max}} = \frac{(3 \times 10^6)^2}{2 \times (120\pi)} w(0.12)
\]
Therefore, the maximum time average power propagated per unit width in the waveguide is

\[
\frac{(P_{\text{ave}})_{\text{max}}}{w} = 1.432 \times 10^9 = 1.432 \text{ GW/m}
\]

**SOL 9.2.8** Option (B) is correct.

Maximum operating frequency

\[ f_{\text{max}} = 1.5 \times 10^9 \text{ Hz} \]

Relative permittivity of medium,

\[ \varepsilon_r = 8.4 \]

The cutoff frequency in TEM mode is \( f_c = 0 \) and the cutoff frequency in (TE)\(_n\) or (TM)\(_n\) mode is given as

\[ (f_c)_n = \frac{n}{2b\sqrt{\mu_0\varepsilon_r}} \]

So, for TE\(_1\) or TM\(_1\) mode \((n = 1)\) we get

\[ (f_c)_1 = \frac{1}{2b\sqrt{\mu_0\varepsilon_0\varepsilon_r}} \]

Since, the guide is to be operated only in TEM mode. So, the operating frequency must be less than \((f_c)_1\) while it must be greater than 0 (cutoff frequency in TEM mode).

i.e.

\[ 0 < f < (f_c)_1 \]

or,

\[ f < \frac{1}{2b\sqrt{\mu_0\varepsilon_0\varepsilon_r}} \]

\[ b < \frac{2f_{\text{max}}\varepsilon_r}{\mu_0\varepsilon_0} \]

As the frequency inside the waveguide ranges in \(0 < f < 1.5\) GHz, therefore, the maximum allowable separation between the plates is

\[ b_{\text{max}} = \frac{c}{2f_{\text{max}}\varepsilon_r} = \frac{3 \times 10^8}{2 \times 1.5 \times 10^9 \times \sqrt{8.4}} \]

\[ = 0.345 \text{ m} = 3.45 \text{ cm} \quad (f_{\text{max}} = 1.5 \text{ GHz}) \]

**SOL 9.2.9** Option (A) is correct.

The Brewster’s angle for parallel polarized wave is given as

\[ \tan \theta_{||} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \]

\[ \theta_{||} = \tan^{-1}\left(\frac{1.05}{2}\right) = 35.9^\circ \]

The cutoff frequency for TM\(_1\) mode in 1\(^{\text{st}}\) medium (permittivity = \(\varepsilon_1\)) is given as

\[ (f_c)_1 = \frac{1}{2b\sqrt{\mu_0\varepsilon_0\varepsilon_r}} = \frac{3 \times 10^8}{2 \times 14.1 \times 10^9 \times \sqrt{2}} \]

\[ = 7.52 \times 10^9 \text{ Hz} \]

So, the frequency for which there is no any reflective loss is given as

\[ f_0 = \frac{(f_c)_1}{\cos \theta} \]

where \(\theta\) is ray angle that has the value, \(\theta = 90^\circ - \theta_{||}\). So, we get

\[ f_0 = \frac{7.52 \times 10^9}{\cos(90^\circ - 35.9^\circ)} = \frac{7.52 \times 10^9}{\sin 35.9^\circ} = 12.8 \text{ GHz} \]
NOTE:
Brewster’s angle is the incident angle of a plane wave at the interface of two mediums for which there is no any reflection in the medium.

SOL 9.2.10 Option (A) is correct.
As we have determined in the previous question, the value of \( f_0 \) is
\[
f_0 = 12.8 \text{ GHz}
\]
and the cutoff frequency for TM\(_1\) mode is
\[
(f_1) = 7.52 \text{ GHz}
\]
So, the cutoff frequency for TM\(_2\) mode will be
\[
(f_2) = 2(f_1) = 15.04 \text{ GHz}
\]
Since, the operating frequency \( f_0 \) is below the cutoff frequency for TM\(_2\) mode so, TM\(_2\) mode or the higher modes can’t propagate at the frequency \( f_0 \). Therefore, only one mode (TM\(_1\)) can propagate at the frequency \( f_0 \) through the waveguide.

SOL 9.2.11 Option (A) is correct.
Consider the dominant mode of the waveguide is TE\(_{10}\). Since, the cut-off frequency for TE\(_{mn}\) mode is defined as
\[
f_c = \frac{1}{2\sqrt{\mu f}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}
\]
So, the cutoff frequency for the TE\(_{10}\) mode is
\[
(f_{10})_c = \frac{c}{2} \times \frac{1}{a} = \frac{c}{2a}
\]
(for airfilled waveguide \( c = 1/\sqrt{\mu \varepsilon} \))

Now, the next higher order mode of the waveguide will be TE\(_{01}\) so, it’s cutoff frequency is given as
\[
(f_{01})_c = \frac{c}{2b}
\]
For the given condition design frequency will be
\[
f = 1.1(f_{10})_c = 0.9(f_{10})_c
\]
Since, the operating frequency of the waveguide is
\[
f = 5 \text{ GHz} = 5 \times 10^9 \text{ Hz}
\]
So, we get
\[
(f_{10})_c = \frac{5 \times 10^9}{1.1}
\]
\[
c = \frac{5 \times 10^9}{1.1}
\]
\[
a = \frac{(3 \times 10^8) \times 1.1}{2 \times (5 \times 10^9)} = 3.3 \text{ cm}
\]
and
\[
(f_{01})_c = \frac{5 \times 10^9}{0.9}
\]
\[
c = \frac{5 \times 10^9}{0.9}
\]
\[
b = \frac{(3 \times 10^8) \times 0.9}{2 \times (5 \times 10^9)} = 2.7 \text{ cm}
\]

SOL 9.2.12 Option (C) is correct.
For a propagating mode TM\(_m\) or TE\(_n\) the cutoff wavelength of the symmetric
dielectric slab is defined as
\[ \lambda_c = \frac{2d\sqrt{\varepsilon_1 - \varepsilon_2}}{n-1} \]
where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the permittivities of dielectrics and \( d \) is the slab thickness.
So, we get
\[ \lambda_c = \frac{2 \times 20 \times 10^{-6} \sqrt{2.2 - 2.1}}{n-1} = \frac{1.26 \times 10^{-5}}{n-1} \]
Since the operating wavelength must be lower than or equal to the cutoff wavelength i.e.
\[ \lambda \leq \lambda_c \]
Therefore, for the propagation of wavelength \( \lambda = 2.6 \mu m \) in the dielectric slab waveguide, we have the condition as
\[ 2.6 \times 10^{-6} \leq \frac{1.26 \times 10^{-5}}{n-1} \]
\[ n-1 \leq 4.85 \]
\[ n \leq 5.85 \]
So, the possible values of \( n \) for which the wavelength \( \lambda = 2.6 \mu m \) can propagate in the waveguide are \( n = 1, 2, 3, 4, 5 \). Thus, we get the possible modes as follows:
- TE1, TE2, TE3, TE4, TE5
- TM1, TM2, TM3, TM4, TM5
and as TEM doesn’t exist in the dielectric slab waveguide so, total 10 modes can propagate for the operating wavelength.

**SOL 9.2.13** Option (A) is correct.
For a rectangular waveguide operating in TE\(_{10}\) mode the phasor form of electric field is given as
\[ E_{ys} = E_0 \sin(kx)e^{-jkz} \]
\[ H_{zs} = -\frac{\beta}{\omega \mu} E_0 \sin(kx)e^{-jkz} \]
\[ H_{zs} = j \frac{K}{\omega \mu} E_0 \cos(kx)e^{-jkz} \]
Since, the wave is propagating in TE mode so, no any other field component exists in the waveguide.
Now, the average power in an EM wave is defined as
\[ P_{av} = \frac{1}{2} \text{Re} \{ E_x \times H_y^* \} \]
Since, \( H_z \) has a factor \( j \). So it would lead to an imaginary part of the total power when cross product with \( E_y \) is taken. Therefore, the real power in the case is found through the cross product with complex conjugate of \( H_z \) as below :
\[ P_{av} = \frac{1}{2} \text{Re} \{ E_{ys} \times H_{zs}^* \} = \frac{1}{2} \frac{\beta}{\omega \mu} E_0^2 \sin^2(kx)a_z \]
Thus,
\[ P_{av} \propto \beta \]
Option (A) is correct.

Cutoff wavelength for symmetric slab waveguide is defined as,

\[ (\lambda_c)_n = \frac{2d\sqrt{\varepsilon_{r1} - \varepsilon_{r2}}}{n - 1} \]  

(1)

where \( d \) is the thickness of slab \( n \) is the propagating mode, \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \) are the relative permittivities of the mediums.

Now, the refractive indices of the two mediums can be given as

\[ n_1 = \sqrt{\varepsilon_{r1}} \]  

and \[ n_2 = \sqrt{\varepsilon_{r2}} \]

So, the equation can be rewritten as

\[ (\lambda_c)_n = \frac{2d\sqrt{n_1^2 - n_2^2}}{n - 1} \]

Since, the waveguide supports only a single pair of TE and TM modes. i.e. it supports \( n = 1 \) mode and denies all the higher modes. Therefore, the operating wavelength \( \lambda \) must be with in the range.

\[ (\lambda_c)_1 \geq \lambda \geq (\lambda_c)_2 \]

(2)

where \( (\lambda_c)_1 \) and \( (\lambda_c)_2 \) are the wavelengths for mode \( n = 1 \) and \( n = 2 \) respectively.

Putting \( n = 1 \) in equation (1) we get

\[ (\lambda_c)_1 = \infty \]

Therefore, the condition obtained in equation (2) reduces to

\[ \lambda \geq (\lambda_c)_2 \]

\[ 3.1 \times 10^{-6} = \frac{2 \times 10 \times 10^{-6} \sqrt{n_1^2 - (3.3)^2}}{2 - 1} \]

\[ \sqrt{n_1^2 - (3.3)^2} \leq \frac{3.1 \times 10^{-6}}{2 \times 10 \times 10^{-6}} \]

\[ n_1 \leq 3.304 \]

Thus, the maximum value of \( n \) is 3.304.

Option (A) is correct.

The wave angle must be equal to or greater than the critical angle of total reflection at both interfaces. So, the minimum wave angle in the slab is determined for the greater of the two critical angles determined at two interfaces.

Since, \( n_3 > n_2 \)

It means the critical angle will be greater for \( n_3 \) media and given as

\[ \theta_{c3} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = 48.6^\circ \]

Therefore, the minimum possible wave angle will be 48.6°.

Option (D) is correct.

Phase velocity of a guided mode is defined as

\[ v_p = \frac{\omega}{k} \]

So, maximum phase velocity for the guided mode is
\[ v_{\text{max}} = \frac{\omega}{\beta_{\text{min}}} \quad (1) \]

where \( \beta_{\text{min}} \) is the minimum phase constant given as

\[ \beta_{\text{min}} = n_1 k_0 \sin \theta_{\text{min}} \quad (2) \]

where \( \theta_{\text{min}} \) is the minimum possible wave angle, \( k_0 \) is the wave number in free space and \( n_1 \) is the refractive index of propagating media (slab).

Now, from the given relation for refractive index, we have

\[ n_2 > n_3 \]

So, as described in previous question the minimum wave angle will be determined by larger critical angle (i.e. at the interface of \( n_1 \) and \( n_2 \)) which is given as

\[ \sin \theta_{\text{min}} = \sin \theta_{c,12} = \frac{n_2}{n_1} \]

Putting it in equation (2), we get

\[ \beta_{\text{min}} = n_1 k_0 \frac{n_2}{n_1} = n_2 k_0 \]

Again putting the value of \( \beta_{\text{min}} \) in equation (1), we get

\[ v_{\text{max}} = \frac{\omega}{n_2 k_0} = \frac{c}{n_2} \quad (\text{velocity of wave in air, } c = \frac{\omega}{k_0}) \]

**SOL 9.2.17**

Given, the cross-section dimension of the waveguide is

\[ a = b = 4.5 \text{ cm} \]

The cut off frequency of the rectangular waveguide is defined as

\[ f_{\text{cm}} = \frac{1}{2 \sqrt{\mu_0 \varepsilon_0} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]^{1/2}} \]

So, the cutoff frequency for TE\(_{22}\) mode of waveguide of square cross section is

\[ f_{\text{cm}} = \frac{1}{2 \sqrt{\mu_0 \varepsilon_0} \left[ \left( \frac{2}{a} \right)^2 + \left( \frac{2}{b} \right)^2 \right]^{1/2}} = 3 \times 10^8 \times \frac{\sqrt{2}}{\sqrt{0.045}} = 2 \text{ GHz} \]

The phase constant of the wave inside the waveguide is given as

\[ \beta = \omega \sqrt{\mu_0 \varepsilon_0} \left[ \frac{1}{2} \left( \frac{f}{f_{\text{cm}}} \right)^{1/2} \right] = \frac{2\pi \times 6 \times 10^9}{3 \times 10^8} \left[ 1 - \left( \frac{2}{6} \right)^{1/2} \right] \]

Therefore, the wavelength of the TE\(_{22}\) mode wave is

\[ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{118.47} = 5.303 \times 10^{-2} = 5.3 \text{ cm} \]

**SOL 9.2.18**

Option (C) is correct.

Given, the operating frequency of the waveguide is

\[ f = 6 \text{ GHz} = 6 \times 10^9 \text{ Hz} \]

So, the wave number in the waveguide of dimension ‘\( a \)’ is given as

\[ k = \frac{2\pi f}{c} = \frac{2\pi \times 6 \times 10^9}{3 \times 10^8} = 40\pi \]

Now, the attenuation constant of section of waveguide (attenuator) with dimension \( a/2 \) is given as

\[ \alpha = \sqrt{\left( \frac{\pi}{a/2} \right)^2 - k^2} = \sqrt{\left( \frac{2\pi}{0.04575} \right)^2 - (40\pi)^2} \quad (a = 0.04572 \text{ m}) \]
Since, the total required attenuation is 100 dB along the attenuator so, we have 

\(-100 \text{ dB} = 20 \log e^{-al}\)

where \(l\) is length of the attenuator. (length travelled by wave in the small section of waveguide). Therefore, solving the equation we get,

\[10^{-5} = e^{-al}\]

\[l = \frac{11.5}{55.63} = 0.2067 = 20.67 \text{ cm}\]

**SOL 9.2.19**

Option (B) is correct.

Dimension of waveguide, \(a = 1.07 \text{ cm} = 0.0107 \text{ m}\)

Operating frequency, \(f = 10 \text{ GHz} = 10 \times 10^{9} \text{ Hz} = 10^{10} \text{ Hz}\)

Permittivity of dielectric, \(\varepsilon_r = 8.8\)

and \(\tan \delta = 0.002\)

The phase constant of the EM wave inside the waveguide is defined as

\[\beta = \sqrt{k^2 - \left(\frac{\pi}{a}\right)^2}\]

where \(k\) is the wave number in the unbounded medium given as

\[k = \sqrt{\varepsilon_r k_0} = \left(\frac{2\pi f}{c}\right)\]

\[= \sqrt{8.8} \cdot \frac{2\pi f}{c}\]

\[= \frac{2\pi \times 10^{10}}{3 \times 10^{8}}\]

\[= 621.3 \text{ m}^{-1}\]

So, the phase constant of the wave along the waveguide is

\[\beta = \sqrt{(621.3)^2 - (\frac{\pi}{0.0107})^2}\]

\[= 547.5 \text{ m}^{-1}\]

Therefore, the attenuation constant due to dielectric loss is given as

\[\alpha_d = \frac{k \tan \delta}{2 \beta} = \frac{(621.3)^2(0.002)}{2(547.5)}\]

\[= 0.705 \text{ Np/m} = 6.12 \text{ dB/m}\]

**SOL 9.2.20**

Option (D) is correct.

In a circular waveguide, cutoff frequency for \(\text{TE}_{mn}\) mode is given as

\[f_{\text{cmn}} = \frac{P'_{mn}}{2\pi a \sqrt{\varepsilon \mu}}\]

and the cutoff frequency for \(\text{TM}_{mn}\) mode of the waveguide is given as

\[f_{\text{nmn}} = \frac{P_{\text{mn}}}{2\pi a \sqrt{\varepsilon \mu}}\]

where \(a\) is the cross sectional radius of waveguide, \(P'_{mn}\) and \(P_{mn}\) are the roots of the Bessel’s equation. Their values are related as listed below in increasing order

\[P'_{11} < P_{21} < P'_{01} = P_{11} < P'_{31} < P_{21} < P'_{41}\]

and so on.

So, for the corresponding values of \(P'_{mn}\) and \(P_{mn}\), we get the increasing order of the modes with respect to their cutoff frequencies as shown below on the frequency axis.
Thus, the first four propagating modes are respectively
$TE_{11}$, $TM_{01}$, $TE_{21}$, $TE_{01}$ or $TM_{11}$

**SOL 9.2.21** Option (C) is correct.

Given:
- Thickness of substrate, $d = 0.316$ cm
- Relative permittivity of substrate, $\varepsilon_r = 2.2$
- Characteristic impedance of line, $Z_0 = 100\, \Omega$  

The width to thickness ratio $(w/d)$ is defined as
\[
\frac{w}{d} = \frac{8\varepsilon_r^2}{\varepsilon_r^2 - 2} \quad \text{for} \quad \frac{w}{d} < 2
\]

where $A = \frac{Z_0}{60\sqrt{\frac{\varepsilon_r + 1}{2} + \sqrt{\frac{\varepsilon_r - 1}{2}}}} + \left(\frac{2.2 - 1}{2.2 + 1}\right)\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$

Now, we assume $\frac{w}{d} < 2$. So, we get:
\[
A = \frac{100}{60\sqrt{\frac{2.2 + 1}{2}} + \sqrt{\frac{2.2 - 1}{2.2 + 1}}} = 2.21
\]

and therefore, the width to thickness ratio is
\[
\frac{w}{d} = \frac{8\varepsilon_r^2}{\varepsilon_r^2 - 2} = 0.896 < 2
\]

As the obtained value of $(w/d)$ is less than 2 so, our assumption was correct and we have
\[
\frac{w}{d} = 0.896
\]

or,
\[
w = 0.896 \times d = 0.896 \times 0.316 = 0.283 \text{ cm}
\]

**SOL 9.2.22** Option (A) is correct.

As calculated in previous question the width to thickness ratio is
\[
\frac{w}{d} = 0.896
\]

So, the effective value of permittivity is given as
\[
\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \frac{1}{\sqrt{1 + \frac{12}{w^2}}}
\]

\[
= \frac{(2.2 + 1)}{2} + \frac{2.2 - 1}{2} \frac{1}{\sqrt{1 + \frac{12}{0.896}}}
\]

\[
= 1.758
\]

Therefore the guided wavelength of the EM wave is
\[ \lambda_o = \frac{c}{\sqrt{\varepsilon_r f}} \]
where \( f \) is operating frequency and \( c \) is velocity of wave in free space. So, we get
\[ \lambda_o = \frac{3 \times 10^8}{(\sqrt{1.758})(8 \times 10^9)} \]
= 2.83 cm

**SOL 9.2.23** Option (D) is correct.
The resonant frequency for \( \text{TE}_{mnp} \) mode is defined as
\[ f_r = \frac{1}{2\sqrt{\mu \varepsilon}} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{c} \right)^2 \right]^{1/2} \]
So for \( \text{TE}_{101} \) mode the resonant frequency of the cavity resonator is
\[ f_r = \frac{3 \times 10^8}{2\sqrt{3}} \left[ \left( \frac{1}{0.025} \right)^2 + \left( \frac{1}{0.05} \right)^2 \right]^{1/2} \]
= 3.87 \times 10^9 \text{ Hz}
= 3.87 \text{ GHz}

**SOL 9.2.24** Option (A) is correct.
The quality factor of \( \text{TE}_{101} \) mode is defined as
\[ Q_{\text{TE}_{101}} = \frac{(a^2 + c^2)abc}{\delta [2b(a^2 + c^2) + ac(a^2 + c^2)]} \]
where \( \delta \) is skin depth given as
\[ \delta = \frac{1}{\sqrt{\pi f_r \mu_0 \sigma_c}} \]
where \( f_r \to \) resonant frequency for the defined mode.
\( \mu_0 = 4\pi \times 10^{-7} \)
\( \sigma_c = \) Conductivity of copper
So, we get the skin depth as
\[ \delta = \frac{1}{\sqrt{\pi (3.87 \times 10^9)(4\pi \times 10^{-7})(5.8 \times 10^1)}} \]
= 1.06 \times 10^{-6}
Therefore, the quality factor of the resonator is
\[ Q_{\text{TE}_{101}} = \frac{(1.06 \times 10^{-6})\left[(2.5)^2 + (5)^2\right][2(2.5)(2)(5) \times 10^{-2}}{2 \times 2 \times (2.5)^2 + (5)^2 + (2.5)(5)(2.5)^2 + (5)^2} \]
= 7732.7 \approx 7733

**SOL 9.2.25** Option (B) is correct.
Given, the inner radius of the guide is
\( a = 1 \text{ cm} = 0.01 \text{ m} \)
The cutoff frequency for \( \text{TE}_{mn} \) mode of a circular waveguide is defined as
\[ f_{r, mn} = \frac{p_{mn}'}{2\pi a\sqrt{\mu \varepsilon}} \]
where \( p_{mn}' \) is the \( m^{th} \) root of Bessel’s function \( (J'_m = 0) \).
Now, from the given data we have

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
$P'_{11} = 1.841$

So, the cutoff frequency of TE_{11} mode in the circular waveguide is

$$f_{c11} = \frac{1.841}{2\pi(10^{-2})\sqrt{\mu_0\varepsilon_0}} = \frac{3 \times 10^8 \times 1.841}{2\pi \times 10^{-2}}$$

$$= 8.79 \times 10^9 \text{ Hz} = 8.79 \text{ GHz}$$

**SOL 9.2.26** Option (A) is correct.

The resonant frequency of TM_{mnl} mode in cylindrical cavity is defined as

$$f_{c,mnl} = \frac{-1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{p_{mn}}{a}\right) + \left(\frac{\ell n^2}{d}\right)}$$

where $a$ is radius of cylindrical cavity, $d$ is height of the cylindrical cavity and $p_{mn}$ is the root of Bessel’s equation.

Since, the dominant mode in cylindrical cavity is TM_{010} so, the cutoff frequency for dominant mode is

$$f_{010} = \frac{p_{01}}{2\pi a\sqrt{\mu_0\varepsilon_0}} = \frac{p_{01} c}{2\pi a}$$

Therefore, the cutoff wavelength for dominant mode is given as

$$\lambda_{c,010} = \frac{c}{f_{010}}$$

$$2 \times 10^{-2} = \frac{2\pi a}{p_{01}}$$

$$a = \frac{(2.405)(2 \times 10^{-2})}{2\pi}$$

$$= 7.65 \times 10^{-2}$$

$$= 0.765 \text{ cm}$$

$\lambda_{c,010} = 2 \text{ cm}$

***********
**SOLUTIONS 9.3**

**SOL 9.3.1** Option (D) is correct.

Given, the magnetic field component along the z-direction as

\[ H_z = 3 \cos(2.094 \times 10^2 x) \cos(2.618 \times 10^2 y) \cos(6.283 \times 10^9 t - \beta z) \]

So,

\[ \beta_x = 2.094 \times 10^2 \]
\[ \beta_y = 2.618 \times 10^2 \]
\[ \omega = 6.283 \times 10^9 \text{ rad/s} \]

For the wave propagation inside the rectangular waveguide,

\[ \beta = \sqrt{\frac{\omega^2}{c^2} - (\beta_x^2 + \beta_y^2)} \]

Substituting the values, we get

\[ \beta = \sqrt{(6.283 \times 10^9)^2 - (2.094^2 + 2.618^2) \times 10^4} \approx 261 \]

Since, \( \beta \) is imaginary so, mode of operation is non-propagating i.e., \( v_p = 0 \)

**SOL 9.3.2** Option (A) is correct.

TM_{11} is the lowest order mode of all the TM_{mn} modes.

**SOL 9.3.3** Option (D) is correct.

From the given expressions of \( E \) and \( H \), we can write,

\[ \beta = \frac{280\pi}{\lambda} \]

or

\[ \frac{2\pi}{\lambda} = \frac{280\pi}{\lambda} \Rightarrow \lambda = \frac{1}{140} \]

So, the wave impedance is given as

\[ \eta = \frac{|E|}{|H|} = \frac{E_p}{3} = \frac{120\pi}{\sqrt{\varepsilon_r}} \]  \hspace{1cm} (1)

Since, the operating frequency of the wave is

\[ f = 14 \text{ GHz} \]

So, the operating wavelength of the wave can also be given as

\[ \lambda = \frac{c}{\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8}{\sqrt{\varepsilon_r} \times 14 \times 10^9} = \frac{3}{140\sqrt{\varepsilon_r}} \]

or

\[ \frac{1}{140} = \frac{3}{140\sqrt{\varepsilon_r}} \]

or,

\[ \varepsilon_r = 9 \]
From equation (1) we have
\[ \frac{E_p}{3} = \frac{120\pi}{\sqrt{9}} \Rightarrow E_p = 120\pi \]

**SOL 9.3.4** Option (A) is correct.
Rectangular and cylindrical waveguide doesn’t support TEM modes and have cut off frequency.
Coaxial cable supports TEM wave and does not have cut off frequency.

**SOL 9.3.5** Option (A) is correct.
Cut-off Frequency for TE\(_{mn}\) mode of a rectangular waveguide is defined as
\[ f_c = \frac{c}{2} \sqrt{\left(\frac{m^2}{a^2}\right) + \left(\frac{n^2}{b^2}\right)} \]
So, for TE\(_{11}\) mode \((m = 1, n = 1)\) the cutoff frequency is
\[ f_c = \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2} = 6.25 \text{ GHz} \quad (c = 3 \times 10^8 \text{ cm/s}) \]

**SOL 9.3.6** Option (A) is correct.
Given, the electric field intensity of the wave inside rectangular waveguide as
\[ E = \frac{\omega \mu}{\eta} \left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{2\pi x}{a}\right) \sin(\omega t - \beta z) a_y \]
This is TE mode and we know that
\[ E_y \propto \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \]
So, comparing it with the given expression we get \(m = 2\) and \(n = 0\). Therefore, the propagating mode is TE\(_{20}\).

**SOL 9.3.7** Option (B) is correct.
The cut-off frequency for the TE\(_{mn}\) mode of the waveguide is defined as
\[ f_c = \frac{c}{2} \sqrt{\left(\frac{m^2}{a^2}\right) + \left(\frac{n^2}{b^2}\right)} \]
So, the cutoff frequency of the TE\(_{20}\) \((m = 2, n = 0)\) mode is
\[ f_c = \frac{c}{2} \left(\frac{m}{a}\right) = \frac{3 \times 10^8}{2} \times \frac{2}{0.03} = 10 \text{ GHz} \quad (a = 3 \text{ cm}) \]
Therefore, the wave impedance of the TE\(_{20}\) mode is given as
\[ \eta' = \frac{\eta_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{10^{10}}{3 \times 10^8}\right)^2}} = 400\Omega \quad (f = 30 \text{ GHz}) \]

**SOL 9.3.8** Option (B) is correct.
The cut-off frequency of TE\(_{mn}\) mode is defined as
\[ f_c = \frac{c}{2} \sqrt{\left(\frac{m^2}{a^2}\right) + \left(\frac{m^2}{b^2}\right)} \]
So, the cutoff frequency of TE\(_{30}\) \((m = 3, n = 0)\) mode is
\[ f_c = \frac{c}{2} \left(\frac{m}{a}\right) \]
or \[ 18 \times 10^9 = \frac{3 \times 10^8}{2} \frac{3}{a} \quad (f = 18 \text{ GHz}) \]
or \[ a = \frac{1 \times 10^{-9}}{m} = \frac{5}{2} \text{ cm} \]
SOL 9.3.9 Option (D) is correct.

SOL 9.3.10 Option (D) is correct.
For any propagating mode inside a rectangular waveguide the velocities are related as

\[ v_p > c > v_g \]

i.e. the phase velocity of the wave inside the waveguide is greater than the velocity of light in the free space.

SOL 9.3.11 Option (D) is correct.
In a microwave test bench, the microwave signal is modulated at 1 kHz because crystal detector fails at microwave frequencies.

SOL 9.3.12 Option (A) is correct.
The cutoff frequency of TE_{mn} mode in a rectangular waveguide is defined as

\[ f_c = \frac{1}{2 \sqrt{\mu \varepsilon}} \sqrt{\frac{m^2}{a}} + \left( \frac{n^2}{b} \right) \]

Since in the given rectangular waveguide \( a > b \) so, the dominant mode is \( TE_{20} \) and the cutoff frequency for the dominant mode is given as

\[ f_c = \frac{3 \times 10^8}{4} \sqrt{\frac{1}{0.03} + \frac{1}{0.12}} = 3 \times 10^8 \times 2.5 \text{ GHz} \]

SOL 9.3.13 Option (B) is correct.
Phase velocity of an EM wave inside an air-filled rectangular waveguide

\[ v_p = \frac{c}{\sqrt{1 - \left( \frac{f}{f_c} \right)^2}} \]

where \( c \) is velocity of EM wave in free space \( f_c \) is the cutoff frequency of the propagating mode and \( f \) is the operating frequency. Since, for a wave propagation the operating frequency must be greater than the cutoff frequency.

i.e. \( f > f_c \)

Therefore, the phase velocity of the wave will be always greater than the velocity of wave in free space.

i.e. \( v_p > c \)

SOL 9.3.14 Option (A) is correct.
In a hollow metal wave guide \( v_p > c > v_g \)
where \( v_p \to \) Phase velocity
\( c \to \) Velocity of light in free space.
\( v_g \to \) Group velocity
So, the phase velocity of a wave propagating in a hollow metal waveguide is greater than the velocity of light in free space.
SOL 9.3.15 Option (A) is correct.
In a wave guide dominant gives lowest cut-off frequency and hence the highest cut-off wavelength.

SOL 9.3.16 Option (B) is correct.
As the impedance of perfect conductor is zero, electric field is minimum and magnetic field is maximum at the boundary.

SOL 9.3.17 Option (B) is correct.
Cutoff frequency for TE$_{mn}$ mode in a rectangular waveguide is defined as
\[ f_c = \frac{v_p}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \]
Since, for the given rectangular waveguide \(a > b\) so, the dominant mode is TE$_{10}$ and the cutoff frequency of the dominant mode of rectangular waveguide is
\[ f_c = \frac{v_p}{2a} = \frac{3 \times 10^8}{2 \times 10^7} = 15 \times 10^3 \]  \(\text{(For air } v_p = 3 \times 10^8)\)
\[ = 15 \text{ GHz} \]

SOL 9.3.18 Option (D) is correct.
In TE mode \(E_z = 0\), at all points within the wave guide. It implies that electric field vector is always perpendicular to the waveguide axis. This is not possible in semi-infinite parallel plate wave guide.

SOL 9.3.19 Option (A) is correct.
In a rectangular resonant cavity TE$_{mnp}$ mode must have its \(p = 1\). So, the mode TE$_{110}$ doesn’t exist in the rectangular resonant cavity.

SOL 9.3.20 Option (A) is correct.
The transverse electric field and transverse magnetic field inside a waveguide are related as
\[ |E| = \eta |H| \]
where \(\eta\) is intrinsic impedance
or
\[ \eta = \frac{|E|}{|H|} \]
i.e. the ratio of transverse electric field to the transverse magnetic field is called waveguide impedance.

SOL 9.3.21 Option (C) is correct.
Cutoff wavelength for H$_{mn}$ mode of a rectangular waveguide is defined as
\[ \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} \]
where \(a\) and \(b\) are the dimensions of waveguide.
So, for the H$_{10}$ mode \((m = 1, n = 0)\), the cutoff wavelength is
\[ \lambda_c = \frac{2}{\sqrt{\left(\frac{1}{8}\right)^2}} = 16 \text{ cm} \]
\((a = 8 \text{ cm})\)
SOL 9.3.22 Option (D) is correct.
A rectangular waveguide supports TE and TM waves where as it doesn’t support TEM waves.
The propagation constant for TE or TM waves inside a rectangular waveguide is defined as
\[ \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\varepsilon} \]

SOL 9.3.23 Option (C) is correct.
Cut-off frequency for TE$_{mn}$ or TM$_{mn}$ mode inside a rectangular waveguide is defined as
\[ f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \]
Where $a$ and $b$ are the dimensions of rectangular waveguide.

SOL 9.3.24 Option (D) is correct.
Cut-off frequency for TE$_{10}$ mode is
\[ f_{10} = \frac{1}{2\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \]
\[ = \frac{3 \times 10^8}{2} \times \left(\frac{1}{4.755 \times 10^{-7}}\right) \]
\[ = 3.16 \text{ GHz} \]

Cut-off frequency for TE$_{01}$ mode is
\[ f_{01} = \left(\frac{3 \times 10^8}{2}\right) \times \left(\frac{1}{2.215 \times 10^{-7}}\right) \]
\[ = 6.77 \text{ GHz} \]

cut-off frequency for TE$_{11}$ mode is
\[ f_{11} = \frac{3 \times 10^8}{2} \times \sqrt{\left(\frac{1}{4.755 \times 10^{-7}}\right)^2 + \left(\frac{1}{2.215 \times 10^{-7}}\right)^2} \]
\[ = 7.47 \text{ GHz} \]

and the cut-off frequency for TE$_{20}$ mode is
\[ f_{20} = \frac{3 \times 10^8}{2} \times \frac{2}{(4.755 \times 10^{-7})} = 6.3 \text{ GHz} \]

Since the operating frequency $f = 12 \text{ GHz}$ so, we have, $f_{10}, f_{01}, f_{11}, f_{20} > f$.
Therefore, all the modes will propagate.

Note : For avoiding so many calculation we should directly calculate the higher frequency modes first for higher operating frequency. As in this case if we calculates $f_{11}$ first then by getting $f > f_{11}$ it is clear that TE$_{01}$, TE$_{10}$ and TE$_{11}$ all the three modes are propagating and by observing option we directly can say option (D) is correct.

SOL 9.3.25 Option (D) is correct.
Consider a rectangular waveguide has dimensions $a = b$ and the corresponding resonator has the dimensions $a = b = d$. now take and operating point that has frequency $f$ just greater than the cut-off frequency for $m = n = 1$. So we have the
propagating modes in waveguide.

- TE$_{01}$, TE$_{10}$, TE$_{11}$ and TM$_{11}$.

Where as the propagating modes in resonator are

- TE$_{011}$, TE$_{101}$, TM$_{110}$

Therefore the cavity resonator does not possess as many modes as corresponding waveguides.

As the resonating frequency of a TE$_{mnp}$ or TM$_{mnp}$ mode is defined as

$$f_{cmn} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

So for the different modes (different values of $m$, $n$ and $p$) the resonant frequency are very closely spaced and also the resonant frequencies of cavity can be changed by altering its dimensions.

SOL 9.3.26 Option (C) is correct.

Microstrip lines cannot support pure TEM mode but shielded coaxial lines can support pure TEM mode.

SOL 9.3.27 Option (A) is correct.

Given,

- Operating frequency, $f = 3$ GHz $= 3 \times 10^9$ Hz
- Dimensions of waveguide $a = 6$ cm and $b = 4$ cm

The cut-off frequency for TE$_{mnp}$/TM$_{mnp}$ mode is defined as

$$f_{c} = \frac{1}{2\sqrt{\mu\varepsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

So, for TE$_{10}$, $f_{c10} = \frac{3 \times 10^9}{2} \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2}

\quad = 2.5$ GHz

\quad (a \to 1)

for TE$_{01}$, $f_{c01} = \frac{3 \times 10^9}{2} \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2}

\quad = 3.75$ GHz

\quad (b \to 2)

For TE$_{11}$ or TM$_{11}$, $f_{c11} = \frac{3 \times 10^9}{2} \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2}

\quad = 4.506$ GHz

\quad (c \to 3, d \to 4)

SOL 9.3.28 Option (A) is correct.

A TEM wave doesn’t have an electric component in its direction of propagation consequently there is no longitudinal displacement current. The total absence of a longitudinal current inside a waveguide leads to the conclusion that there can be no closed loops of magnetic field lines in any transverse plane. Therefore, TEM waves cannot exist in a hollow waveguide of any shape.

i.e. Both A and R are true and R is correct explanation of A.

SOL 9.3.29 Option (A) is correct.

Phase velocity of a wave propagating in a waveguide is defined as

$$v_p = \frac{c}{\sqrt{\mu\varepsilon}}$$
The group velocity of the wave propagating in waveguide is defined as

\[ v_g = c \sqrt{1 - \left( \frac{f}{f_c} \right)^2} \]

where \( c \) is the velocity of wave in free space, \( f_c \) is the cutoff frequency and \( f \) is the operating frequency. As the operating frequency \( f \) is always greater than cutoff frequency \( f_c \). So, comparing the above two expressions we get

\[ v_p > c > v_g \]

**SOL 9.3.30**
Option (B) is correct.

In a microstrip line operating wavelength is defined as

\[ \lambda = \lambda_0 \sqrt{\varepsilon_r} \]

where, \( \lambda_0 \) is free space wave length and \( \varepsilon_r \) is the effective dielectric constant. So, Statement 1 is correct.

The electromagnetic fields exist partly in air above the dielectric substrate and partly within the substrate. Statement 2 is correct.

The effective dielectric constant of microstrip line is \( \varepsilon_r \) and given as

\[ 1 < \varepsilon_r < \varepsilon_r \]

i.e. greater than dielectric constant of air (1). Statement 3 is correct.

Conductor losses, increase with decreasing characteristic impedance in microstrip line. Statement 4 is correct.

**SOL 9.3.31**
Option (B) is correct.

Stripline carries two conductors and a homogenous dielectric. So, it supports a TEM mode (Pure TEM).

\[ a \rightarrow 2 \]

Hollow rectangular waveguide can propagate TEM and TE modes but not TEM mode.

\[ b \rightarrow 3 \]

Microstripline has some of its field lines in the dielectric region and some fraction in the air region. So it cannot support a pure TEM wave instead the fields are quasi-TEM.

\[ c \rightarrow 1 \]

**SOL 9.3.32**
Option (B) is correct.

Given, the dimension of waveguide is \( a = 2.286 \text{ cm} \), \( b = 1.016 \text{ cm} \).

The cut off wavelength of the guide, for \( \text{TE}_{mn} \) mode is defined as

\[ \lambda_c = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}} \]

So, for \( \text{TE}_{01} \) Mode the cut off wavelength of the guide is

\[ \lambda_c = \frac{2}{\sqrt{(\frac{0}{a})^2 + (\frac{1}{b})^2}} \]

\[ = 2b = 2.032 \text{ cm} \]
SOL 9.3.33  Option (C) is correct.
Since, the electric component is existed in the direction of propagation. So the electric field is not transverse to the propagating wave and therefore the mode is transverse magnetic (TM mode).

SOL 9.3.34  Option (C) is correct.
Given, dimension of waveguide \( a = b = 3 \, \text{cm} \) and so the dominant mode is either \( \text{TE}_{01} \) or \( \text{TE}_{10} \) mode.
So, the cutoff frequency for dominant mode is given as
\[
\frac{1}{2 \sqrt{\mu_0 \varepsilon_0}} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} = \frac{3 \times 10^8}{2} \times \frac{1}{3 \times 10^{-2}} = 5 \, \text{GHz}
\]
So, at 6 GHz dominant mode will propagate.
Statement 1 is correct.
At 4 GHz no modes will propagate so the modes are evanescent at 4 GHz.
Statement 2 is correct.
At 11 GHz along with the dominant mode \( \text{TE}_{11} \) mode \( (f_c = 5\sqrt{2}) \) will also propagate.
Statement 3 is incorrect.
Degenerate modes are the different modes that have the same cut off frequency and at 7 GHz frequency \( \text{TE}_{01} \) and \( \text{TE}_{10} \) propagates that has the same cut off frequency i.e. Degenerate modes propagate at 7 GHz.
Statement 4 is correct.

SOL 9.3.35  Option (C) is correct.
Evanescent mode → No wave propagation dominant mode is the mode that has lowest cutoff frequency.
Rectangular waveguide does not support \( \text{TM}_{01} \) and \( \text{TM}_{10} \) mode.
\( A \to 2, \, B \to 3, \, C \to 1 \)

SOL 9.3.36  Option (D) is correct.
Assertion (A) : Given the dimension of waveguide, \( a = 3 \, \text{cm}, \, b = 1 \, \text{cm} \)
So, the dominant mode (\( \text{TE}_{10} \)) has the cutoff frequency.
\[
f_c = \frac{3 \times 10^8}{2} \times \left( \frac{1}{3 \times 10^{-2}} \right)
= 5 \, \text{GHz}
\]
\[
f = 4 \, \text{GHz} < f_c
\]
So, at 4 GHz there is no propagating mode. i.e Assertion (A) is false.
Reason (R) : The wave equation for the rectangular waveguide is defined as
\[
\sqrt{k_z^2 - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2} = \left( \frac{2\pi}{\lambda} \right)
\]
for \( a = 3, \, b = 1 \) we have
\[
k_z^2 - \left( \frac{m\pi}{3} \right)^2 - \left( \frac{n\pi}{1} \right)^2 = \left( \frac{2\pi}{\lambda} \right)^2
\]
So, Reason (R) is also false.
SOL 9.3.37 Option (D) is correct. A rectangular coaxial line can support all the three modes (TE, TM or TEM).

SOL 9.3.38 Option (C) is correct. Consider the rectangular waveguide (A) has the dimension $a \times b$ after deforming into waveguide (C) the dimension is changed to $b \times a$ and so the input mode $TE_{10}$ is charged to $TE_{01}$. (Since the frequency of mode must remain same for both the waveguide dimensions).

SOL 9.3.39 Option (D) is correct. The dominant mode in a circular waveguide is $TE_{11}$.

SOL 9.3.40 Option (B) is correct. Consider the dimension of inner broad wall of waveguide is $a$ (i.e. $a > b$). So, the dominant mode will be $TE_{10}$.

Since, the cutoff frequency of the $TE_{mn}$ mode is defined as

$$f_c = \frac{1}{2\sqrt{\mu_0 \varepsilon_0}} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]^{1/2}$$

So, for dominant mode ($TE_{10}$) we have

$$10 \times 10^9 = \frac{1}{2\sqrt{\mu_0 \varepsilon_0}} \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{0}{b} \right)^2 \right]^{1/2}$$

$$10 \times 10^9 = \frac{3 \times 10^8 \times 1}{a}$$

$$a = \frac{3 \times 10^8}{2 \times 10^9} = 1.5 \text{ cm}$$

SOL 9.3.41 Option (A) is correct. Propagation constant in a waveguide is defined as

$$\gamma = 2\pi f \sqrt{\mu_0 \varepsilon_0} \sqrt{1 - \left( \frac{f_c}{f} \right)^2}$$

Since, for the evanescent mode of waveguide the operating frequency is less than the cutoff frequency.

i.e. $f < f_c$

or $f \times f < 1$

So, for this condition the propagation constant $\gamma$ is purely real.

SOL 9.3.42 Option (C) is correct. Microstrip lines consist no ground plate and so the electric field lines remain partially in air and partially in the lower dielectric substrate. This makes the mode of propagation quasi TEM (not pure TEM)

Due to the open structure and presence of discontinuity in microstrip line, it radiates electromagnetic energy and therefore radiation losses take place.

SOL 9.3.43 Option (B) is correct. Statements 1 and 3 are correct.

SOL 9.3.44 Option (C) is correct.
Modes on microstrip lines are quasi TEM (not purely TEM). So the 1st statement is incorrect while rest of the statements are correct.

**SOL 9.3.45** Option (D) is correct.

Coaxial line → The dominant mode of propagation is TEM. (a → 4)

Rectangular waveguide → propagating mode is TE or TM. (b → 1)

Microstrip line → The mode of propagation is Quasi TEM. (c → 2)

Coplanar waveguide → The propagation mode is hybrid of propagation (TE<sub>mm</sub> + TM<sub>mn</sub>) (d → 3)

**SOL 9.3.46** Option (B) is correct.

The phase velocity of TE or TM mode is defined as

\[ v_p = \frac{c}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}} \]

where

- \( c \) → Velocity of wave in free space
- \( f_c \) → cutoff frequency
- \( f \) → operating frequency

So, \( v_p \) is a nonlinear function of frequency.

**SOL 9.3.47** Option (C) is correct.

A waveguide operated below cut off frequency can be used as an attenuator.

**SOL 9.3.48** Option (C) is correct.

Quality factor of a waveguide is defined as

\[ Q = \frac{\omega}{2\alpha v_p} \]

i.e. \( Q \) closely related to \( \alpha \).

Also the attenuation factors obtained in waveguides are much higher than that in transmission lines.

So, both statements are true but R is not the correct explanation of A.

**SOL 9.3.49** Option (C) is correct.

Quality factor (\( Q \)) of a resonator is defined as

\[ Q = \frac{\text{Resonant frequency (} f_r \text{)}}{\text{Bandwidth}} \]

or,

\[ \text{Bandwidth} = \frac{f_r}{Q} \propto \frac{1}{Q} \]

Therefore, the greater the ‘\( Q \)’, the smaller the bandwidth of resonator.

\( Q \) is also defined for a resonator as

\[ Q = \frac{\beta}{2\alpha} \]

where \( \beta \) is phase constant and \( \alpha \) is attenuation constant of a resonator given as

\[ \alpha = \sqrt{\frac{\omega \mu \sigma}{2}} \propto \omega \]

So

\[ Q \propto \frac{1}{\omega} \]

So, at higher frequency the \( Q \) of coil falls due to skin effect.
SOL 9.3.50  Option (A) is correct.
Guided wavelength of a propagating wave in rectangular waveguide is
\[ \lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{c_l}{\lambda}\right)^2}} \]
where \( \lambda \) is free space wavelength and \( c_l \) is cutoff frequency.
Since, for propagation the operating wavelength must be less than cut off frequency
i.e.
\[ \lambda \leq c_l \]
So, we get \( \lambda_g \geq \lambda \)
So, for a wave propagation in an air filled rectangular waveguide, guided wavelength
is never less than free space wavelength.

SOL 9.3.51  Option (A) is correct.
Transverse magnetic mode (TM mode) consists of magnetic field intensity
perpendicular to the direction of propagation where as the electric field intensity
may be in the direction of propagation.

SOL 9.3.52  Option (D) is correct.
Since the conduction current requires conductor along the axis and a hollow
waveguide doesn’t have a conductor along its axis. So, the axial current is due to
displacement current only.

SOL 9.3.53  Option (D) is correct.
Electromagnetic waves propagating in a medium (bounded that has the velocity
greater than the velocity in free space (velocity of light in space) is given as
\[ v_p = \frac{C}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}} \]
or \[ v_p > C \]  The velocity \( v_p \) is called phase velocity of the wave.

SOL 9.3.54  Option (A) is correct.
A and R both true and R is correct explanation of A.

SOL 9.3.55  Option (A) is correct.
Statement 1, 2 and 3 are correct.

SOL 9.3.56  Option (B) is correct.
Attenuation factor in a parallel plate waveguide is defined as
\[ \alpha = \frac{P_l}{2P_0} = \frac{(\text{Power lost per unit length})}{2 \times (\text{Power transmitted})} \]

SOL 9.3.57  Option (C) is correct.
Since the waveguide has a cutoff frequency \( f_c \) below which no wave propagates
while above \( f_c \) all the waves propagates so it can be considered as high pass filter.

***********
EXERCISE 10.1

MCQ 10.1.1 A Hertzian dipole of length $\lambda/25$ is located at the origin. If a point $P$ is located at a distance $r$ from the origin then for what value of ‘$r$’ the point will be in radiation zone.

(A) $r = \frac{2\lambda}{5}$

(B) $r = \frac{\lambda}{5}$

(C) Both (A) and (B)

(D) none of these

MCQ 10.1.2 A quarter wave monopole antenna is operating at a frequency, $f = 25\text{ MHz}$. The length of antenna will be

(A) 48 m

(B) 3 m

(C) 6 m

(D) 12 m

MCQ 10.1.3 A half wave dipole antenna is located at origin as shown in figure below. The antenna is fed by a current $i(t) = 83.3\cos\omega t\ mA$. What will be the electric field strength at point $P$

(A) 25 mV/m

(B) 50 mV/m

(C) 50 $\mu$V/m

(D) 2.5 $\mu$V/m

MCQ 10.1.4 The transmitting antenna of a radio navigation system is a vertical metal mast 25 m in height inducted from the earth. A source current is supplied to it’s base such that the current amplitude in antenna decreases linearly toward zero at the top of the mast. The effective length of antenna will be

(A) 50 m

(B) 20 m

(C) 12.5 m

(D) 25 m
MCQ 10.1.5 A vertical antenna of length 8.5 m is operating at a frequency, \( f = 2 \text{ MHz} \). The radiation resistance of the antenna is

(A) 1.97 \( \Omega \)  
(B) 0.51 \( \Omega \)  
(C) 39.4 \( \Omega \)  
(D) 26.3 \( \Omega \)

MCQ 10.1.6 The current in a short circuit element of length \( l = 0.03\lambda \) is given by

\[
I(z) = \begin{cases} 
\frac{I_0}{2} & \text{for } 0 < |z| \leq \frac{l}{4} \\
I_0 & \text{for } \frac{l}{4} < |z| \leq \frac{l}{2} 
\end{cases}
\]

What will be the radiation resistance of the element?

(A) 0.71 \( \Omega \)  
(B) 0.6 \( \Omega \)  
(C) 0.05 \( \Omega \)  
(D) 0.4 \( \Omega \)

MCQ 10.1.7 A dipole antenna radiating at 200 MHz is fed from a 60 \( \Omega \) transmission line matched to the source. What will be the length of the dipole that matches the line impedance at the signal frequency?

(A) 0.83 m  
(B) 0.41 m  
(C) 0.49 m  
(D) 0.24 m

MCQ 10.1.8 A certain antenna is used to radiate a 0.2 GHz signal to a satellite in space. Given the radiation resistance of the antenna is 31.6 \( \Omega \). The antenna is

(A) half wave dipole  
(B) quarter wave dipole  
(C) one-fifth wave dipole  
(D) none of these

MCQ 10.1.9 A time harmonic uniform current \( I_0 \cos(2\pi \times 10^7 t) \) flows in a small circular loop antenna of radius 30 cm. Radiation resistance of the antenna is

(A) 92.3 m\( \Omega \)  
(B) 325.05 m\( \Omega \)  
(C) 10.83 \( \Omega \)  
(D) 3.076 m\( \Omega \)

Statement for Linked Question 10 - 11:

An antenna is a center fed rod having cross sectional radius 4 cm and conductivity \( \sigma = 2.9 \times 10^7 \text{ S/m} \). The length of the antenna is 30 m.

MCQ 10.1.10 If a 0.25 MHz current flows in the antenna then the loss resistance of the antenna is

(A) 1.93 \( \Omega \)  
(B) 1.97 \( \Omega \)  
(C) 0.022 \( \Omega \)  
(D) 0.031 \( \Omega \)

MCQ 10.1.11 The radiation efficiency of the antenna is

(A) 95.4\%  
(B) 96.8\%  
(C) 98.6\%  
(D) 93.5\%
MCQ 10.1.12 A 200 MHz uniform current flows in a small circular loop of radius 20 cm. If the loop is made of copper wire of radius 5 mm then its loss resistance will be (conductivity of copper, \( \sigma = 5.8 \times 10^7 \text{ S/m} \))

(A) 0.104 \( \Omega \)  
(B) 6.52 \( \times \) 10\(^{-5} \) \( \Omega \)  
(C) 9.57 \( \Omega \)  
(D) 1.53 \( \Omega \)

MCQ 10.1.13 A quarter wave monopole antenna is connected to a transmission line of characteristic impedance \( Z_0 = 75 \Omega \). The standing wave ratio will be

\( Z_m = (36.5 + j21.25) \Omega \)

(A) 1.3874  
(B) 1.265  
(C) 2.265  
(D) 2.583

MCQ 10.1.14 Radiated power of a vertical antenna is 0.2 kW. What will be the maximum electric field intensity at a distance of 10 km from the antenna ?

(A) 3.8 mV/m  
(B) 1.9 mV/m  
(C) 19 mV/m  
(D) 3.6 mV/m

MCQ 10.1.15 A quarter wave monopole antenna is fed by a current \( i(t) = 41.7 \cos \omega t \) mA. The average power radiated by antenna is

(A) 254 mW  
(B) 127 mW  
(C) 63.5 mW  
(D) 31.7 mW

MCQ 10.1.16 A dipole antenna in free space has a linear current distribution. If the length of the dipole is 0.01\( \lambda \) then the value of current \( I_0 \) required to radiate a total power of 250 mW is

(A) 5.03 A  
(B) 2.53 A  
(C) 7.56 A  
(D) 50.3 A

MCQ 10.1.17 A monopole antenna in free space has the length of the antenna 0.02\( \lambda \). The antenna is extending vertically over a perfectly conducting plane and has a linear current distribution. What value of \( I_0 \) is required to radiate a total power of 2 W ?

(A) 11.4 A  
(B) 7.1 A  
(C) 14.2 A  
(D) 3.6 A

Statement for Linked Question 18 - 19:
A Hertzian dipole is operating at a frequency, \( f = 0.2 \text{ GHz} \).

MCQ 10.1.18 What will be the maximum effective area of the dipole ?

(A) 0.54 m\(^2\)  
(B) 1.07 m\(^2\)  
(C) 0.18 m\(^2\)  
(D) 0.27 m\(^2\)
If the antenna receives 1.5 μW of power then what is the power density of the incident wave?

(A) 8.33 μW/m²  
(B) 5.56 μW/m²  
(C) 1.40 μW/m²  
(D) 2.793 μW/m²

Directivity of quarter wave monopole is

(A) 1.64  
(B) 1.22  
(C) 3.28  
(D) 0.609

An antenna has a uniform radiation intensity in all directions. The directivity of the antenna is

(A) 5  
(B) 0.25  
(C) 0.2  
(D) 4

The input power of a certain antenna with an efficiency of 90% is 0.8 Watt. If the antenna has maximum radiation intensity of 1 W/Sr then it’s directivity will be

(A) 5.26  
(B) 16.53  
(C) 0.76  
(D) 9.55

An antenna has maximum radiation intensity of 1.5 W/Sr. If the directivity of the antenna is $D = 20.94$ then radiated power of antenna will be

(A) 1.11 W  
(B) 0.30 W  
(C) 0.26 W  
(D) 0.90 W

Normalized radiation intensity of an antenna is given by

$$U(\theta, \phi) = \begin{cases} 
\sin \theta & 0 \leq \theta \leq \pi/2, \ 0 \leq \phi \leq 2\pi \\
0 & \text{otherwise}
\end{cases}$$

The directivity of antenna will be

(A) 2.55  
(B) 8.0  
(C) 0.81  
(D) 1.27

An antenna has the uniform field pattern given by

$$U(\theta) = \begin{cases} 
4 & 0 < \theta < \pi/3 \\
0 & \pi/3 < \theta < \pi
\end{cases}$$

where $U(\theta)$ is independent of $\phi$. The directivity of the antenna is

(A) 1/4  
(B) 4  
(C) 16  
(D) 1

Three element array that has the current ratios 1:2:1 as shown in figure
The resultant group pattern of this array will be same as the two element antenna array with

(A) $\alpha = 0, \ d = \lambda/4$

(B) $\alpha = 180^\circ, \ d = \lambda/2$

(C) $\alpha = 0, \ d = \lambda/2$

(D) $\alpha = 180^\circ, \ d = 2\lambda$

MCQ 10.1.27 When the two three-element arrays with current ratio $1:2:1$ are displaced by $\lambda/2$ then it forms

(A) Four element array with current ratio $1 : 3 : 3 : 1$

(B) Three element array with current ratio $2 : 4 : 2$

(C) Four element array with current ratio $3 : 1 : 1 : 3$

(D) Three element array with current ratio $1 : 3 : 1$
MCQ 10.2.1 A Hertzian dipole of length $\lambda/100$ is located at the origin and fed with a current of $i(t) = 2\sin 10^8 t$ A. A point $P$ is located at a distance $r$ from the dipole as shown in figure. What will be the magnetic field at $P$?

(A) $1.15\sin(10^8 t + 90^\circ)$
(B) $1.15\cos(10^8 t + 90^\circ)$
(C) $1.15\sin(10^8 t - 90^\circ)$
(D) $2.30\sin(10^8 t + 90^\circ)$

MCQ 10.2.2 Directivity of Hertzian monopole antenna is
(A) 5  
(B) 3 
(C) 1/2  
(D) 9

MCQ 10.2.3 Directive gain of Hertzian dipole antenna is
(A) $1.5\sin^2\theta$  
(B) $3\sin^2\theta$ 
(C) $\frac{\sin^2\theta}{3}$  
(D) $\frac{2}{3}\sin^2\theta$

MCQ 10.2.4 Two Hertzian dipole antennas are placed at a separation of $d = \lambda/2$ on $z$-axis to form an antenna array as shown in figure below:

---

GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia
If the 1st antenna carries a current $I_1 = I_0 / 0^\circ$ and the 2nd antenna carries a current $I_2 = I_0 / 180^\circ$ then the resultant field pattern of the antenna array will be

(A) (B) (C) (D)

MCQ 10.2.5 An antenna array is formed by two Hertzian dipoles placed at a separation of $\lambda/4$ as shown in figure. The current fed to the two antennas are $I_1$, and $I_2$, respectively.

If $I_2$ is lagging $I_1$ by an angle $\pi/2$ then the resultant field pattern of antenna array will be

(A) (B)
MCQ 10.2.6 The group pattern function of a linear binomial array of $N$-elements as shown in figure is

\[
\begin{align*}
(A) \left[ \cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) \right]^{N-1} & \quad \text{(B) } \left[ \cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) \right]^N \\
(C) \left[ \cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) \right]^{N+1} & \quad \text{(D) } \left[ \cos \left( \beta d \cos \theta + \alpha \right) \right]^{N-1}
\end{align*}
\]

Statement for Linked Question 7 - 8 :

Maximum electric field strength radiated by an antenna is $6 \text{ mV/m}$ measured at $40 \text{ km}$ from the antenna.

MCQ 10.2.7 If the antenna radiates a total power of $100 \text{ kW}$ then the directivity of antenna is

(A) $-2.02 \text{ dB}$
(B) $9.6 \text{ dB}$
(C) $0.0096 \text{ dB}$
(D) $-20.18 \text{ dB}$

MCQ 10.2.8 If the efficiency of the radiation is $95\%$ then it’s maximum power gain is

(A) $9.12 \times 10^{-3}$
(B) $9.4 \times 10^{-3}$
(C) $0.11 \times 10^{-3}$
(D) $9.6 \times 10^{-3}$
Statement for Linked Question 9 - 10:
A radar with an antenna of 2.8 m in radius transmits 30 kW at a frequency 3 GHz. The effective area of the antenna is 70% of its actual area.

MCQ 10.2.9 If the minimum detectable power is 0.13 mW for a target of cross section 1.25 m² then the maximum range of the radar is
(A) 584.3 m
(B) 1270 m
(C) 292.1 m
(D) 977.8 m

MCQ 10.2.10 The average signal power density at half of the range of radar will be
(A) 350.25 W/m²
(B) 69.80 W/m²
(C) 80.69 W/m²
(D) 250.35 W/m²

Statement for Linked Question 11 - 13:
A transmitting antenna is being fed by a current source of amplitude $I_0 = 50$ A and frequency $f = 180$ kHz. The effective length of antenna is 20 m.

MCQ 10.2.11 What will be the maximum field intensity at a distance 80 km from the antenna?
(A) 3.39 mV/m
(B) 1.41 mV/m
(C) 2.83 mV/m
(D) 0.71 mV/m

MCQ 10.2.12 The time average radiated power of the antenna is
(A) 0.43 kW
(B) 0.29 kW
(C) 2.33 kW
(D) 1.14 kW

MCQ 10.2.13 What will be the radiation resistance of the antenna?
(A) 0.23 Ω
(B) 2.91 Ω
(C) 0.34 Ω
(D) 1.7 Ω

Statement for Linked Question 14 - 15:
A metallic wire of cross sectional radius 6 mm is wound to form a small circular loop of radius 2 m with 10 turns. Conductivity of metallic wire is $\sigma = 2.9 \times 10^7$ S/m.

MCQ 10.2.14 If a 0.5 MHz uniform current flows in the loop then its radiation resistance will be
(A) $2.37 \times 10^{-6}$ Ω
(B) $1.42 \times 10^{-3}$ Ω
(C) $2.37 \times 10^{-4}$ Ω
(D) $4.53 \times 10^{-4}$ Ω

MCQ 10.2.15 Radiation efficiency of the antenna will be
(A) 18.36%
(B) 0.101%
(C) 10.89%
(D) 0.055%
MCQ 10.2.16  The polar radiation pattern of a $\lambda/8$ thin dipole antenna is

(A)  
(B)  
(C)  
(D)  

Statement for Linked Question 17 - 18 :
Two short antennas at the origin in free space carry identical currents $4\cos\omega t\ A$, one in the $a_x$ direction and other in the $a_z$ direction.

MCQ 10.2.17  If both the antennas are of length 0.1 m and wavelength is $\lambda = 2\pi$ m then the electric field $E$ at the distant points $P(0,0,1000)$ and $Q(1000,0,0)$ will be at point $P$ and at point $Q$

(A)  
(B)  
(C)  
(D)  

MCQ 10.2.18  $E$ at point $(0,1000,0)$ at $t = 0$ will be

(A) $9.92(a_x + a_z)$ mV/m  
(B) $-9.92(a_x + a_z)$ mV/m  
(C) $1.2(a_x + a_z)$ mV/m  
(D) $-12(a_x + a_z)$ mV/m
Statement for Linked Question 19 - 20

In a free space short circuit vertical current element is located at the origin in free space. The radiation field due to the element at any point is given as

\[ E_{\theta s} = \frac{10}{\pi} \sin \theta e^{-j\theta_{1000}} \text{V/m} \]

MCQ 10.2.19 \( E_{\theta s} \) at point \( P \) \((r = 100, \theta = \pi/2, \phi = \pi/6)\) is

(A) \( 0.2e^{j1000\pi} \text{V/m} \)
(B) \( 0.2e^{-j1000\pi} \text{V/m} \)
(C) \( 0.1e^{-j1000\pi} \text{V/m} \)
(D) \( 0.1e^{j1000\pi} \text{V/m} \)

MCQ 10.2.20 If the vertical element is shifted to a point \((0.1, \frac{\pi}{2}, \frac{\pi}{6})\) then, \( E_{\theta s} \) at point \( P(100, \frac{\pi}{2}, \frac{\pi}{6}) \) changes to

(A) \( 0.1e^{-j1000\pi} \text{V/m} \)
(B) \( 0.1e^{j1000\pi} e^{0.5\pi} \text{V/m} \)
(C) \( 0.1e^{-0.5\pi} \text{V/m} \)
(D) \( 0.1e^{j1000\pi} e^{-0.5\pi} \text{V/m} \)

MCQ 10.2.21 A short circuit current element of length \( l = 0.06\lambda \) carries the current distributed as

\[ I(z) = I_0 \left| \frac{1-2|z|}{l} \right| \quad \text{for} \ -\frac{l}{2} \leq z \leq \frac{l}{2} \]

The radiation resistance of the antenna will be

(A) \( 0.71 \Omega \)
(B) \( 2.84 \Omega \)
(C) \( 2.13 \Omega \)
(D) \( 0.18 \Omega \)

MCQ 10.2.22 An antenna is made of straight copper wire of length 1 cm carrying current of frequency 0.3 GHz. If the wire has a cylindrical cross section of radius 1 mm then the ratio of the radiation resistance to the ohmic resistance of wire will be \( \frac{R_{rad}}{R_i} \approx \)

(A) 11
(B) 6
(C) 17
(D) 5

MCQ 10.2.23 A 2 cm long Hertzian dipole antenna radiates 2 W of power at a frequency of 0.6 GHz. The rms current in the antenna is

(A) \( 1.78 \text{A} \)
(B) \( 3.56 \text{A} \)
(C) \( 1.26 \text{A} \)
(D) \( 0.89 \text{A} \)

************
The radiation pattern of an antenna in spherical co-ordinates is given by
\[ U(\theta) = \cos^4 \theta; \quad 0 \leq \theta \leq \pi/2 \]

The directivity of the antenna is
(A) 10 dB  \hspace{1cm} (B) 12.6 dB  \hspace{1cm} (C) 11.5 dB  \hspace{1cm} (D) 18 dB

For a Hertz dipole antenna, the half power beam width (HPBW) in the \(E\)-plane is
(A) 360°  \hspace{1cm} (B) 180°  \hspace{1cm} (C) 90°  \hspace{1cm} (D) 45°

At 20 GHz, the gain of a parabolic dish antenna of 1 meter and 70% efficiency is
(A) 15 dB  \hspace{1cm} (B) 25 dB  \hspace{1cm} (C) 35 dB  \hspace{1cm} (D) 45 dB

A \(\lambda/2\) dipole is kept horizontally at a height of \(\frac{\lambda}{2}\) above a perfectly conducting infinite ground plane. The radiation pattern in the lane of the dipole (\(E\) plane) looks approximately as

(A) \hspace{1cm} (B)

(C) \hspace{1cm} (D)

A mast antenna consisting of a 50 meter long vertical conductor operates over a perfectly conducting ground plane. It is base-fed at a frequency of 600 kHz. The radiation resistance of the antenna in Ohms is
(A) \(\frac{2\pi^2}{5}\)  \hspace{1cm} (B) \(\frac{\pi^2}{5}\)  \hspace{1cm} (C) \(\frac{4\pi^2}{5}\)  \hspace{1cm} (D) \(20\pi^2\)
Two identical and parallel dipole antennas are kept apart by a distance of \( \lambda/4 \) in the H-plane. They are fed with equal currents but the right most antenna has a phase shift of +90°. The radiation pattern is given as.

(A) ![Pattern A](image1)

(B) ![Pattern B](image2)

(C) ![Pattern C](image3)

(D) ![Pattern D](image4)

Consider a lossless antenna with a directive gain of +6 dB. If 1 mW of power is fed to it the total power radiated by the antenna will be

(A) 4 mW

(B) 1 mW

(C) 7 mW

(D) 1/4 mW

Two identical antennas are placed in the \( \theta = \pi/2 \) plane as shown in Fig. The elements have equal amplitude excitation with 180° polarity difference, operating at wavelength \( \lambda \). The correct value of the magnitude of the far-zone resultant electric field strength normalized with that of a single element, both computed for \( \phi = 0 \), is

(A) \( 2 \cos \left( \frac{2\pi s}{\lambda} \right) \)

(B) \( 2 \sin \left( \frac{2\pi s}{\lambda} \right) \)

(C) \( 2 \cos \left( \frac{\pi s}{\lambda} \right) \)

(D) \( 2 \sin \left( \frac{\pi s}{\lambda} \right) \)

A person with receiver is 5 km away from the transmitter. What is the distance that this person must move further to detect a 3-dB decrease in signal strength

(A) 942 m

(B) 2070 m

(C) 4978 m

(D) 5320 m

A medium wave radio transmitter operating at a wavelength of 492 m has a tower antenna of height 124. What is the radiation resistance of the antenna?

(A) 25 Ω

(B) 36.5 Ω

(C) 50 Ω

(D) 73 Ω
### MCQ 10.3.11
**GATE 2001**
In uniform linear array, four isotropic radiating elements are spaced $\lambda/4$ apart. The progressive phase shift between the elements required for forming the main beam at 60° off the end-fire is:

- (A) $-\pi$
- (B) $-\frac{\pi}{2}$ radians
- (C) $-\frac{\pi}{4}$ radians
- (D) $-\frac{\pi}{8}$ radians

### MCQ 10.3.12
**GATE 2000**
If the diameter of a $\lambda/2$ dipole antenna is increased from $\lambda/100$ to $\lambda/50$, then its
- (A) bandwidth increases
- (B) bandwidth decreases
- (C) gain increases
- (D) gain decreases

### MCQ 10.3.13
**GATE 2000**
For an 8 feet (2.4m) parabolic dish antenna operating at 4 GHz, the minimum distance required for far field measurement is closest to
- (A) 7.5 cm
- (B) 15 cm
- (C) 15 m
- (D) 150 m

### MCQ 10.3.14
**GATE 1999**
An electric field on a place is described by its potential $V = 20(r^{-1} + r^{-2})$ where $r$ is the distance from the source. The field is due to
- (A) a monopole
- (B) a dipole
- (C) both a monopole and a dipole
- (D) a quadruple

### MCQ 10.3.15
**GATE 1999**
A transmitting antenna radiates 251 W isotropically. A receiving antenna located 100 m away from the transmitting antenna, has an effective aperture of 500 cm². The total received by the antenna is
- (A) 10 $\mu$W
- (B) 1 $\mu$W
- (C) 20 $\mu$W
- (D) 100 $\mu$W

### MCQ 10.3.16
**GATE 1998**
The vector $H$ in the far field of an antenna satisfies
- (A) $\nabla \cdot H = 0$ and $\nabla \times H = 0$
- (B) $\nabla \cdot H \neq 0$ and $\nabla \times H \neq 0$
- (C) $\nabla \cdot H = 0$ and $\nabla \times H \neq 0$
- (D) $\nabla \cdot H \neq 0$ and $\nabla \times H = 0$

### MCQ 10.3.17
**GATE 1998**
The radiation resistance of a circular loop of one turn is 0.01 $\Omega$. The radiation resistance of five turns of such a loop will be
- (A) 0.002 $\Omega$
- (B) 0.01 $\Omega$
- (C) 0.05 $\Omega$
- (D) 0.25 $\Omega$

### MCQ 10.3.18
**GATE 1998**
An antenna in free space receives 2 $\mu$W of power when the incident electric field is 20 mV/m rms. The effective aperture of the antenna is
- (A) 0.005 m²
- (B) 0.05 m²
- (C) 1.885 m²
- (D) 3.77 m²
MCQ 10.3.19  The maximum usable frequency of an ionospheric layer at 60° incidence and with 8 MHz critical frequency is
(A) 16 MHz  (B) $\frac{16}{\sqrt{3}}$ MHz
(C) 8 MHz  (D) 6.93 MHz

MCQ 10.3.20  The far field of an antenna varies with distance $r$ as
(A) $\frac{1}{r}$  (B) $\frac{1}{r^2}$
(C) $\frac{1}{r^3}$  (D) $\frac{1}{\sqrt{r}}$

MCQ 10.3.21  The critical frequency of an ionospheric layer is 10 MHz. What is the maximum launching angle from the horizon for which 20 MHz wave will be reflected by the layer?
(A) 0°  (B) 30°
(C) 45°  (D) 90°

MCQ 10.3.22  The directivity of a $\lambda/2$ long wire antenna is
(A) 1.5  (B) 1.66
(C) 2  (D) $\sqrt{2}$

MCQ 10.3.23  The characteristic impedance of TV receiving antenna cable is 300 $\Omega$. If the conductors are made of copper separated by air and are 1 mm thick, what is the phase velocity and phase constant, when receiving VHF channel 3 (63 MHz) and VHF 69 (803 MHz)?
(A) 1.32 rad/m and 17.82 rad/m
(B) 1.52 rad/m and 16.82 rad/m
(C) 1.52 rad/m and 17.82 rad/m
(D) 1.32 rad/m and 16.82 rad/m

MCQ 10.3.24  An antenna located on the surface of a flat earth transmits an average power of 200 kW. Assuming that all the power is radiated uniformly over the surface of a hemisphere with the antenna at the center, the time average poynting vector at 50 km is
(A) Zero  (B) $\frac{2}{\pi} a$, W/m²
(C) $\frac{40}{\pi} \mu$, W/m²  (D) $\frac{40}{\pi} a \cdot \mu$, W/m²

MCQ 10.3.25  An antenna can be modeled as an electric dipole of length 5 m at 3 MHz. Find the reduction resistance of the antenna assuming uniform current over the length.
(A) 2 $\Omega$  (B) 1 $\Omega$
(C) 4 $\Omega$  (D) 0.5 $\Omega$
A dipole with a length of 1.5 m operates at 100 MHz while the other has a length of 15 m and operates at 10 MHz. The dipoles are fed with same current. The power radiated by the two antennas will be

(A) the longer antenna will radiate 10 times more power than the shorter one.

(B) both antennas radiate same power.

(C) shorter antenna will radiate 10 times more power than the longer antenna

(D) longer antenna will radiate \( \sqrt{10} \) times more power than the shorter antenna

A short current element has length \( l = 0.03 \lambda \), where \( \lambda \) is the wavelength. The radiation resistance for uniform current distribution is

(A) \( 0.072\pi^2 \Omega \)

(B) \( 80\pi^2 \Omega \)

(C) \( 72 \Omega \)

(D) \( 80 \Omega \)

In a three element Yagi antenna

(A) All the three elements are of equal length

(B) The driven element and the director are of equal length but the reflector is longer than both of them

(C) The reflector is longer than the driven element which in turn is longer than the director

(D) The reflector is longer than the driven element which in turn is longer than the reflector

Multiple member of antennas are arranged in arrays in order to enhance what property ?

(A) Both directivity and bandwidth

(B) Only directivity

(C) Only bandwidth

(D) Neither directivity nor bandwidth

If the total input power to an antenna is \( W_i \), the radiated power is \( W_r \), and the radiation intensity is \( \phi \), then match List-I with List-II and select the correct answer using the code given below the lists:

<table>
<thead>
<tr>
<th>List-I</th>
<th>List-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Power gain</td>
<td>1. ( W_r/W_i )</td>
</tr>
<tr>
<td>b. Directive gain</td>
<td>2. ( W_r/4\pi )</td>
</tr>
<tr>
<td>c. Average power radiated</td>
<td>3. ( 4\pi \phi/W_i )</td>
</tr>
<tr>
<td>d. Efficiency of the antenna</td>
<td>4. ( 4\pi \phi/W_r )</td>
</tr>
</tbody>
</table>

Codes : 

(A) 3 4 2 1

(B) 4 3 2 1

(C) 3 4 1 2

(D) 4 3 1 2
MCQ 10.3.31  Where does the maximum radiation for an end-fire array occur?
(A) Perpendicular to the line of the array only
(B) Along the line of the array
(C) AT 45° to the line of the array
(D) Both perpendicular to and along the line of the array

MCQ 10.3.32  As the aperture area of an antenna increases, its gain
(A) increases
(B) decreases
(C) remains steady
(D) behaves unpredictably

MCQ 10.3.33  Which one of the following is correct? Normal mode helical antenna has
(A) low radiation efficiency and high directive gain
(B) high radiation efficiency and low directive gain
(C) low radiation efficiency and low directive gain
(D) high radiation efficiency and high directive gain

MCQ 10.3.34  For taking antenna far field pattern, what must be the distance \( R \), between transmitting and receiving antennas?
(A) \( R > \frac{2D^2}{\lambda} \)
(B) \( R > \frac{4D^2\lambda^2}{3} \)
(C) \( R > \frac{D^2}{2\lambda^2} \)
(D) \( R > \frac{2D^2}{\lambda^2} \)

MCQ 10.3.35  A transmitting antenna has a gain of 10. It is fed with a signal power of 1 W. Assuming free-space propagation, what power would be captured by a receiving antenna of effective area 1 m² in the bore sight direction at a distance of 1 m?
(A) 10 W
(B) 1 W
(C) 2 W
(D) 0.8 W

MCQ 10.3.36  The Fraunhofer region where the pattern measurement of transmitting antenna has to be taken from a distance of \( \frac{2D^2}{\lambda} \), where \( D \) is the maximum aperture dimension and \( \lambda \) is the free-space wavelength. What is the region generally known as?
(A) The near field
(B) The far field
(C) Quiet zone
(D) Induction field

MCQ 10.3.37  Match List I (Type of Antenna) with List II (Example) and select the correct answer using the code given below the lists:
**List-I**

a. Aperture antenna  
b. Circularly polarized  
c. Frequency independent  
d. Isotropic antenna

**List-II**

1. Helical antenna  
2. Point source antenna  
3. Log periodic antenna  
4. Microstrip antenna

**Codes:**

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(B)</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(C)</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>(D)</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**MCQ 10.3.38**  
A TEM wave impinges obliquely on a dielectric-dielectric boundary ($\varepsilon_r_1 = 2$, $\varepsilon_r_2 = 1$). The angle of incidence for total reflection is

(A) $30^\circ$  
(B) $45^\circ$  
(C) $60^\circ$  
(D) $75^\circ$

**MCQ 10.3.39**  
In a four element Yagi-Uda antenna

(A) There is one driven element, one director and two reflectors  
(B) There is one driven element, two directors and one reflector  
(C) There are two driven elements, one director and two reflectors  
(D) All the four elements are driven elements

**MCQ 10.3.40**  
**Assertion (A):** For extremely high frequency ranges or above, compared to linear antennas, aperture antennas are more useful.  
**Reason (R):** The larger the effective area of an antenna, the sharper is the radiated beam.

(A) Both A and R are true and R is the correct explanation of A  
(B) Both A and R are true but R is NOT the correct explanation of A  
(C) A is true but R is false  
(D) A is false but R is true

**MCQ 10.3.41**  
The current distribution along a travelling wave antenna can be written in the form

(A) $|Z| = |e^{j\beta z}$  
(B) $|(Z) = \sin \beta z$  
(C) $|(Z) = |$  
(D) $|(Z) = \cos (\omega t - \beta z)$

**MCQ 10.3.42**  
Following antenna is frequently used for local area transmission at UHF/VHF

(A) Ground monopole  
(B) Turnstile antenna  
(C) Slot antenna  
(D) Loop antenna
MCQ 10.3.43  For frequencies up to 1650 kHz, the transmitting antenna used is a
(A) parabolic dish  (B) vertical antenna
(C) Yagi antenna  (D) turnstile antenna

MCQ 10.3.44  The radiation field of an antenna at a distance \( r \) varies as
(A) \( 1/r \)  (B) \( 1/r^2 \)
(C) \( 1/r^3 \)  (D) \( 1/r^4 \)

MCQ 10.3.45  The wave radiated by a helical antenna is
(A) linearly polarized  (B) right circularly polarized
(C) left circularly polarized  (D) elliptically polarized

MCQ 10.3.46  In a certain microstrip patch antenna, the unexcited patch is of length \( L \), width \( W \), thickness of the substrate being \( h \) and its relative permittivity \( \varepsilon_r \). Then, the capacitance of the unexcited patch is
(A) \( LW/\varepsilon_r h \)  (B) \( LW/\varepsilon_0 \varepsilon_r h \)
(C) \( \varepsilon_r LW/h \)  (D) \( \varepsilon_0 \varepsilon_r LW/h \)

MCQ 10.3.47  A radio communication link is to be established via the ionosphere. The virtual height at the mid-point of the path is 300 km and the critical frequency is 9 MHz. The maximum usable frequency for the link between the stations of distance 800 km assuming flat earth is
(A) 11.25 MHz  (B) 12 MHz
(C) 15 MHz  (D) 25.5 MHz

MCQ 10.3.48  Assertion (A) : Programmes broadcast by radio stations operating in the medium wave band of 550 to 1650 kHz situated at long distance in excess of 500 km cannot be heard during day-time but may be heard during night-time.
Reason (R) : In the night-time, radio waves reflected from the \( F \)-layer suffer negligible attenuation since \( D \)-and \( E \)-layers are absent during the night-time.
(A) Both A and B are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 10.3.49  Assertion (A) : For an end-fire array, the current in successive antennas must lag in phase.
Reason (R) : Radiation of successive antennas will cancel along the axis.
(A) Both A and R are true and R is the correct explanation of A
MCQ 10.3.50

**Assertion (A):** The radio horizon for space wave is more than the optical horizon.

**Reason (R):** The atmosphere has varying density.

(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

MCQ 10.3.51

**IES EC 2001**

What is the radiation resistance of a dipole antenna $\lambda/20$ long approximately equal to?

(A) 2 $\Omega$
(B) 40 $\Omega$
(C) 0.6 $\Omega$
(D) 20 $\Omega$

MCQ 10.3.52

**IES EE 2008**

Consider the following statements about the effective length of a half wave dipole

(Elevation angle $\theta$ is measured from the dipole axis):

1. Effective length is a function of $\theta$
2. Effective length is maximum for $\theta = \pi/2$
3. Maximum effective length is larger than physical length
4. Effective length is the same for the antenna in transmitting and receiving modes.

Which of the statements given above are correct?

(A) 1, 2 and 4
(B) 2, 3 and 4
(C) 1, 2 and 3
(D) 1, 3 and 4

***********
SOLUTIONS 10.1

SOL 10.1.1  Option (D) is correct.
The boundary between near and far zone is defined by \( r = r_0 \) (distance from the antenna) as
\[
\frac{2d^2}{\lambda} = r_0
\]
where \( d \) is the length of dipole.

So, the near and far zones of the field are as following:
- Near zone for \( r > r_0 \)
- Far zone for \( r > r_0 \)

Now, for the Hertzian dipole of length \( \frac{\lambda}{50} \), we have
\[
r_0 = \frac{2(\lambda/50)^2}{\lambda} = \frac{\lambda}{1250}
\]

Since \( r = \frac{2\lambda}{5} > r_0 \)
and \( r = \frac{\lambda}{5} > r_0 \)

So, both the positions are at far zone (radiation zone).

SOL 10.1.2  Option (C) is correct.
Given, the operating frequency of the antenna is
\[
f = 25 \text{ MHz}
\]
Since, the antenna is quarter wave monopole so, the length of the monopole antenna will be given as
\[
l = \frac{\lambda}{4}
\]
where \( \lambda \) is the operating wavelength of the antenna given as
\[
\frac{c}{f} = \frac{3 \times 10^8}{25 \times 10^6} = 12 \text{ m}
\]
Thus, we get the length of antenna as
\[
l = \frac{12}{4} = 3 \text{ m}
\]

SOL 10.1.3  Option (D) is correct.
Given, the current fed to the antenna is
\[
i(t) = 83.3 \cos \omega t \text{ mA}
\]
So, the magnitude of the current flowing in the antenna is
\[
I_0 = 83.3 \times 10^{-3} \text{ A}
\]
and from the figure we get the location of point \( P \) as
\[
r = 100 \text{ Km} = 10^5 \text{ m}
\]
and \( \theta = \pi/2 \)

Therefore, the electric field strength at point \( P \) is given as

\[
|E_{os}| = \frac{\eta_0 I_0 \cos\left(\frac{\pi}{2} \cos \theta\right)}{2\pi \nu \sin \theta} \]

\[
= \frac{(120\pi)(83.3 \times 10^{-3}) \cos\left(\frac{\pi}{2} \cos 90^\circ\right)}{2\pi(10^5)\sin \frac{\pi}{2}} \]

\[
= \frac{(120\pi)(83.3 \times 10^{-3})(1)}{2\pi(10^5)(1)} = 5 \times 10^{-5} = 50 \mu \text{V/m} \]

**SOL 10.1.4** Option (A) is correct.

Since, the amplitude of current decreases linearly toward zero at the top so, the current amplitude at a height \( z \) above the plane is given as

\[
I(z) = I_0 \left(1 - \frac{z}{h}\right) \]

where \( I_0 \) is amplitude of source current and \( h \) is the height of the antenna.

Therefore, the effective length of the antenna is

\[
l_e = \int_0^h \left(1 - \frac{z}{h}\right)dz \]

\[
= \left[z - \frac{z^2}{2h}\right]_0^h = h - \frac{h^2}{2} = \frac{h}{2} = 25 \text{ m} \quad \text{(given } h = 50 \text{ m}) \]

**SOL 10.1.5** Option (B) is correct.

Length of antenna, \( dl = 7.5 \text{ m} \)

Operating frequency, \( f = 2 \text{ MHz} = 2 \times 10^6 \text{ Hz} \)

So, the operating wavelength of the antenna is

\[
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^6} = 1.5 \times 10^2 \]

Therefore, the radiation resistance of the antenna is given as

\[
R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 = 80\pi^2 \left(\frac{7.5}{1.5 \times 10^2}\right)^2 = 1.97 \Omega \]

**SOL 10.1.6** Option (A) is correct.

Since, the current has the step distribution and both the current levels are distributed for equal intervals so, the average current will be given as

\[
I_{avg} = \frac{I_0 + I_0}{2} = 0.75I_0 \]

Since, the average current flowing in the antenna is 0.75 times the uniform current \( I_0 \), therefore, the radiated power will be \((0.75)^2\) times of the value obtained for \( I_0 \) and due to the same reason the radiation resistance will down to \((0.75)^2\) times the value for a uniform current.

i.e., \[
R_{rad} = (0.75)^2 \left[80\pi^2 \left(\frac{dl}{\lambda}\right)^2\right] = 0.5625[80\pi^2(0.03)^2] = 0.4 \Omega \]
SOL 10.1.7  Option (B) is correct.
Since, the dipole must match the line impedance.
i.e. 
\[ R_{\text{rad}} = Z_0 \]
where \( Z_0 \) is characteristic impedance so, we get
\[ R_{\text{rad}} = 60 \, \Omega \]
\[ 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 = 60 \]
\[ 80\pi^2 \left( \frac{dl}{c/f} \right)^2 = 60 \]
\[ dl = \left[ \frac{60}{80\pi} \times \left( \frac{3 \times 10^8}{100 \times 10^8} \right)^{1/2} \right] \]
\[ (f = 100 \, \text{MHz}) \]
\[ = 0.827 \, \text{m} \]

SOL 10.1.8  Option (D) is correct.
Operating frequency, \( f = 0.2 \, \text{GHz} = 0.2 \times 10^9 \, \text{Hz} \)
Radiation resistance, \( R_{\text{rad}} = 31.6 \, \Omega \)
So, the operating wavelength of antenna is
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{0.2 \times 10^9} = 1.5 \, \text{m} \]
Now, the radiation resistance of the antenna is defined as
\[ R_{\text{rad}} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 \]
So, putting all values we get
\[ 31.6 = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 \]
\[ \frac{dl}{\lambda} \approx 0.2 \]
\[ dl \approx \frac{\lambda}{5} \]
i.e. Antenna is one fifth wave dipole.

SOL 10.1.9  Option (A) is correct.
Current flowing in the antenna, \( i(t) = I_0 \cos(2\pi \times 10^7 t) \)
Radius of the circular loop, \( b = 30 \, \text{cm} = 30 \times 10^{-2} \, \text{m} \)
So, we get the operating frequency of the antenna as
\[ f = \frac{2\pi \times 10^7}{2\pi} = 10^7 \, \text{Hz} \]
The operating wavelength of the antenna is
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^7} = 30 \, \text{m} \]
Since, \( \lambda >> b \) so, the radiation resistance of the antenna is given as
\[ R_{\text{rad}} = \frac{320\pi^4 S^2}{\lambda^4} \quad \text{where} \ S \ \text{is area of the circular loop.} \]
\[ = \frac{320\pi^4 \times (\pi b^2)^2}{\lambda^4} \quad (S = \pi b^2) \]
Option (A) is correct.

Cross sectional radius of antenna, \( a = 4 \text{ cm} = 4 \times 10^{-2} \text{ m} \)
Conductivity of the antenna, \( \sigma = 2.9 \times 10^7 \text{ S/m} \)
Length of antenna, \( dl = 30 \text{ m} \)
Operating frequency \( f = 0.5 \text{ MHz} = 0.5 \times 10^6 \text{ Hz} \)

So, the surface resistance of the antenna is

\[
R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = \sqrt{\frac{\pi \times 0.5 \times 10^6 \times 4\pi \times 10^{-7}}{2.9 \times 10^7}} = 2.61 \times 10^{-4} \Omega
\]

Therefore, the loss resistance of the antenna is given as

\[
R_l = R_s \left( \frac{dl}{2\pi a} \right) = (2.61 \times 10^{-4}) \left( \frac{30}{2\pi \times 4 \times 10^{-2}} \right) = 0.031 \Omega
\]

Option (D) is correct.

The radiation resistance of the antenna is defined as

\[
R_{rad} = \frac{80\pi^2}{\lambda^2} \left( \frac{dl}{\lambda} \right)^2
\]

where \( dl \) is the length of the antenna and \( \lambda \) is the operating wavelength. So, we get

\[
R_{rad} = 80\pi^2 \times \left( \frac{30}{c/f} \right)^2 = 80\pi^2 \times \left( \frac{3 \times 10^8}{3 \times 10^8} \right)^2 = 1.97
\]

Therefore, the radiation efficiency of the antenna is

\[
\eta_r = \frac{R_{rad}}{R_{rad} + R_l} = \frac{1.97}{1.97 + 0.031} = 98.6\%
\]

Option (B) is correct.

Given,

Operating frequency, \( f = 100 \text{ MHz} = 10^8 \text{ Hz} \)
Radius of circular loop, \( b = 20 \text{ cm} = 20 \times 10^{-2} \text{ m} \)
Cross sectional radius of wire, \( a = 5 \text{ mm} = 5 \times 10^{-3} \text{ m} \)
Conductivity of copper, \( \sigma = 5.8 \times 10^7 \text{ S/m} \)

The surface resistance of antenna is given as

\[
R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}} = \sqrt{\frac{\pi \times 10^8 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}} = 2.61 \times 10^{-3} \Omega
\]

So, the loss resistance of the antenna is

\[
R_l = \left( \frac{b}{a} \right) R_s = \frac{20 \times 10^{-2}}{5 \times 10^{-3}} \times 2.61 \times 10^{-3} = 0.104 \Omega
\]
SOL 10.1.13 Option (D) is correct.

Given that the quarter wave monopole antenna is connected to transmission line. So, the load impedance of transmission line will be the input impedance of monopole antenna.

\[ Z_L = Z_{in} \]

Since, the input impedance of quarter wave monopole antenna is

\[ Z_{in} = (36.5 + j21.25) \, \Omega \]

So the reflection coefficient of transmission line is given as

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(36.5 + j21.25) - 75}{(36.5 + j21.5) + 75} = 0.3874 < 140.3^\circ \]

Therefore, the standing wave ratio along the transmission line is

\[ S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.265 \]

SOL 10.1.14 Option (D) is correct.

Radiated power of an antenna is defined as

\[ P_{rad} = \frac{I^2 (dl)^2}{12\pi} \eta h \beta^2 \]  \hspace{1cm} (1)

where \( I \) is the current in the antenna, \( dl \) is the length of the antenna and \( \beta \) is the phase constant.

Now, the maximum electric field intensity at a distance \( R \) from the antenna is defined as

\[ |E|_{\text{max}} = \frac{I dl}{4\pi R} \eta h \beta \]  \hspace{1cm} (2)

So, comparing equation (1) and (2), we get

\[ |E|_{\text{max}} = \frac{1}{R} \sqrt{90 P_{rad}} \]

\[ = \frac{1}{10 \times 10^3} \sqrt{90 \times 0.4 \times 10^3} \]

\[ = 19 \text{ mV/m} \] \hspace{1cm} \( P_{rad} = 0.4 \text{ kW} \)

SOL 10.1.15 Option (D) is correct.

Given, current flowing in the antenna is

\[ i(t) = 41.7 \cos \omega t \text{ mA} \]

So, the magnitude of the current flowing in the antenna is

\[ I_0 = 41.7 \text{ mA} \]

Now, for a quarter wave monopole antenna, radiation resistance is

\[ R_{rad} \approx 73 \Omega \]

So, the average power radiated by the antenna is given as

\[ P_{rad} = \frac{1}{2} I_0^2 R_{rad} = \frac{1}{2} \times (41.7 \times 10^{-3})^2 \times 73 \]

\[ = 63.5 \text{ mW} \]
SOL 10.1.16  Option (B) is correct.

Total radiated power,  \( P_{\text{rad}} = 250 \text{ mW} = 0.25 \text{ W} \)
Length of antenna,  \( dl = 0.01\lambda \)

Now, the radiated power of an antenna in terms of current \( I_0 \) flowing in the antenna is defined as

\[
P_{\text{rad}} = \frac{1}{2} (I_0)^2 R_{\text{rad}}
\]

where \( R_{\text{rad}} \) is the radiation resistance of the antenna. Since, the current is linearly distributed over the antenna So, we get the average current in the antenna as

\[
I_{\text{avg}} = \frac{I_0}{2}
\]

Since, the average current flowing in the antenna is half of the uniform current \( I_0 \) therefore, the radiated power will be \( \frac{1}{4} \)th of the value obtained for uniform current in equation (1)

\[
P_{\text{rad}} = \frac{1}{4} \left( \frac{1}{2} I_0^2 R_{\text{rad}} \right)
\]

\[
0.25 = \frac{1}{8} \times I_0^2 \times 80\pi^2 \left( \frac{dl}{\lambda} \right)^2
\]

\[
0.25 = I_0^2 (10\pi^2)(0.01)^2
\]

\[
I_0 = 25.33
\]

or,

\[
I_0 = 5.03 \text{ A}
\]

SOL 10.1.17  Option (D) is correct.

Length of antenna,  \( dl = 0.02\lambda \)
Total radiated power,  \( P_{\text{rad}} = 4 \text{ W} \)

Since, the monopole antenna is extending over the conducting plane so, the power will be radiated only over the upper half space and therefore, the radiation resistance of the antenna will reduces to half of its value

\[
R_{\text{rad}} = \frac{1}{2} \left[ 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 \right] = 40\pi^2 \left( \frac{dl}{\lambda} \right)^2
\]

As the current is distributed linearly. So, the average current in the antenna is

\[
I_{\text{avg}} = \frac{I_0}{2}
\]

Since, the average current flowing in the antenna is half of the uniform current \( I_0 \) therefore, the radiated power will be \( \frac{1}{4} \)th of the value obtained for \( I_0 \).

\[
P_{\text{rad}} = \frac{1}{4} \left( \frac{1}{2} I_0^2 R_{\text{rad}} \right)
\]

\[
0.25 = \frac{1}{8} \times I_0^2 \times 80\pi^2 \left( \frac{dl}{\lambda} \right)^2
\]

\[
0.25 = I_0^2 (10\pi^2)(0.02)^2
\]

\[
I_0 = 50.33
\]

or,

\[
I_0 = 14.2 \text{ A}
\]

SOL 10.1.18  Option (A) is correct.

Operating frequency,  \( f = 0.2 \text{ GHz} \)
So, the operating wavelength of the Hertzian dipole is
Now, the effective area of the dipole is defined as
\[ A_e = \frac{\lambda^2}{4\pi} G_d \]
where \( G_d \) is the directive gain and since the directive gain of Hertzian dipole is \( 1.5\sin^2\theta \) so, putting this value, we get
\[ A_e = \frac{(1.5)^2}{4\pi}(1.5\sin^2\theta) = 0.27\sin^2\theta \]
Therefore, the maximum effective area of the dipole is
\[ A_{e,\text{max}} = 0.27 \, \text{m}^2 \] (maximum value of \( \sin\theta \) is 1)

SOL 10.1.19 Option (C) is correct.

The time average power density of the incident wave is defined in terms of received power as
\[ P_{\text{ave}} = \frac{P_r}{A_e} \]
where, \( P_r \) is the received power and \( A_e \) is the effective aperture area and as calculated in the previous question, the maximum effective area of the Hertzian dipole is
\[ A_e = 0.27 \, \text{m}^2 \]
So, we get the average power density of the incident wave as
\[ P_{\text{ave}} = \frac{1.5 \times 10^{-6}}{0.27} = 5.56 \, \mu\text{W/m}^2 \]

SOL 10.1.20 Option (D) is correct.

For a quarter-wave monopole antenna pattern function is
\[ f(\theta) = \cos\left[\frac{\pi}{2}\cos\theta\right] \]
So, the normalized radiation intensity of the quarter wave monopole antenna is given as
\[ U(\theta,\phi) = f^2(\theta) \frac{\cos^2[\pi/2\cos\theta]}{\sin^2\theta} \]
Therefore, the maximum radiation intensity is
\[ U_{\text{max}} = 1 \]
Now, the power radiated by the quarter wave monopole antenna is evaluated as
\[ P_{\text{rad}} = \int_{0}^{\pi/2} \int_{0}^{2\pi} U(\theta,\phi) \sin\theta d\theta d\phi \]
\[ = \frac{\pi}{2} \int_{0}^{\pi/2} \cos^2\left[\frac{\pi}{2}\cos\theta\right] \sin\theta d\theta d\phi \]
\[ = (2\pi)(0.609) \]
Therefore, the directivity of quarter wave monopole antenna is
\[ D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \]
SOL 10.1.21 Option (B) is correct.
As the radiation intensity in all directions are same so,
\[ U(\theta, \phi) = U_{\text{ave}} \]
where, \( U(\theta, \phi) \) is radiation intensity in a particular direction and \( U_{\text{ave}} \) is the average radiation intensity. So, the directive gain in a particular direction is
\[ G_d(\theta, \phi) = \frac{U(\theta, \phi)}{U_{\text{ave}}} = \frac{U_{\text{ave}}}{U_{\text{ave}}} = 1 \]
Therefore the directivity of the antenna is
\[ D = G_{d,\text{max}} = 1 \]

SOL 10.1.22 Option (C) is correct.
Maximum radiation intensity, \( U_{\text{max}} = 1 \text{ W/Sr} \)
Efficiency of antenna, \( \eta = 95\% \)
Input power of antenna, \( P_{\text{in}} = 0.8 \text{ Watt} \)
So, the output radiated power is given as
\[ P_{\text{rad}} = \eta P_{\text{in}} = (0.95) \times (0.8) = 0.76 \text{ Watt} \]
Therefore, the directivity of antenna is evaluated as
\[ D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi \times 1}{0.76} = 16.53 \]

SOL 10.1.23 Option (A) is correct.
Maximum radiation intensity, \( U_{\text{max}} = 1.5 \text{ W/Sr} \)
Directivity of antenna, \( D = 20.94 \)
Since, the directivity of antenna is defined as
\[ D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \]
So, radiated power of the antenna is given as
\[ P_{\text{rad}} = \frac{4\pi (1.5)}{20.94} = 0.9 \text{ Watt} \]

SOL 10.1.24 Option (B) is correct.
From the given value of radiation intensity, we get maximum radiation intensity of the antenna as
\[ U_{\text{max}} = 1 \]
So, the radiated power of the antenna is evaluated as
\[ P_{\text{rad}} = \int_0^{\pi/2} U(\theta, \phi) \sin \theta d\theta d\phi = \int_0^{\pi/2} \int_0^{2\pi} (\sin \theta)(\sin \theta d\theta d\phi) = \frac{\pi^2}{2} \]
Therefore, the directivity of antenna is
\[ D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi (1)}{\pi^2/2} = 2.546 \]
SOL 10.1.25 Option (C) is correct.

Given, the field pattern of antenna,

\[ U(\theta) = \begin{cases} 
4 & 0 < \theta < \pi/3 \\
0 & \pi/3 < \theta < \pi 
\end{cases} \]

So, the total radiated power of the antenna is given as

\[ P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/3} U(\theta) \sin \theta d\theta d\phi \]
\[ = 4 \times 2\pi \left[ -\cos \theta \right]_{0}^{\pi/3} \]
\[ = 4\pi \]

Therefore, the directivity of the antenna is

\[ D = \frac{4\pi U_{\text{max}}}{P_{rad}} = \frac{4\pi(4)}{4\pi} = 4 \]

SOL 10.1.26 Option (D) is correct.

The three element antenna array has the current ratio 1:2:1

\[ I_{1} < 0^\circ \quad 2I_{2} < 0^\circ \quad I_{3} < 0^\circ \]

We can split the middle element to two elements each of them carrying current \( I_{2} / 0^\circ \) as shown below.

\[ \lambda/2 \quad \lambda/2 \quad \lambda/2 \quad \lambda/2 \quad \lambda/2 \]

Now all the four elements are carrying current \( I_{1} / 0^\circ \) and separation between them are \( d = \lambda/2 \). So, this array can be replaced by two array antenna with two elements as shown below:

\[ 1 \quad 2 \quad 3 \quad 4 \]

Since the currents are in same phase, so the phase difference between the currents will be zero.

i.e. \( \alpha = 0 \)

and separation between the antennas as obtained from the above shown figure is \( d = \lambda/2 \)

SOL 10.1.27 Option (B) is correct.

As shown below the three element array displaced by \( \lambda/2 \).

\[ 1:2:1 \quad 1:2:1 \]

Now we split all the elements with current \( I_{0} \) as shown below:
The three current elements \( I_0 \) located at the same position can be treated as the single element carrying current \( 3I_0 \) as shown below:

\[
\begin{align*}
I_1 & \quad \lambda/2 \quad 3I_2 \quad \lambda/2 \quad 3I_2 \quad \lambda/2 \quad I_0 \\
\end{align*}
\]

Thus, the current ratio will be \( 1 : 3 : 3 : 1 \) of the four element array.
SOLUTIONS 10.2

SOL 10.2.1 Option (B) is correct.  
Current in the dipole, \( i(t) = 0.5 \sin 10^8 t \) A  
Length of the dipole, \( dl = \lambda/100 \)  
So, the magnitude of the current flowing in dipole is \( I_0 = 0.5 \)  
and from the shown figure, we get \( r = 100\lambda \) and \( \theta = 60^\circ \)  
Now, the magnetic field components at any point \((r, \theta, \phi)\) due to hertzian dipole located at origin are defined as  
\[
H_{qs} = H_{rs} = 0
\]
and  
\[
H_{qs} = \frac{jI_0 \beta dl}{4\pi r} \sin \theta e^{-j\beta r}
\]
where \( I_0 \) is the magnitude of current flowing in Hertzian dipole, \( dl \) is the length of dipole and \( \beta \) is phase constant. So, putting all the given values, we get  
\[
H_{qs} = \frac{j(0.5) \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda}{100} \right)}{4\pi (100\lambda)} \sin 60^\circ e^{-j\frac{2\pi}{\lambda}(000\lambda)}
\]
\[
= \frac{j}{4 \times 10^4 \lambda} \times \sqrt{3} \frac{2}{2}
\]
As,  
\[
\lambda = \frac{2\pi c}{\omega} = \frac{2\pi \times 3 \times 10^8}{10^8} = 6\pi \quad (\omega = 10^8 \text{ rad/s})
\]
Therefore,  
\[
H_{qs} = \frac{j}{24\pi \times 10^7} \times \frac{\sqrt{3}}{2}
\]
\[
= 1.1486 \times 10^{-6} e^{j0^\circ} \text{ A/m}
\]
Thus, the net magnetic field intensity at point \( P \) will be  
\[
H = \text{Im}(H_{qs} e^{j\omega t} a_\phi) = 1.1486 \times 10^{-6} \sin(\omega t + 90^\circ)
\]
\[
= 1.15 \sin(10^8 t + 90^\circ) \mu\text{A/m}
\]

SOL 10.2.2 Option (B) is correct.  
The field intensities of the Hertzian monopole are defined as  
\[
E_{qs} = \frac{j I_0 \beta dl}{4\pi r} \sin \theta e^{-j\beta r}
\]
and  
\[
H_{qs} = \frac{j I_0 \beta dl}{4\pi r} \sin \theta e^{-j\beta r}
\]
So, the time average power of the Hertzian monopole is
\[ P_{\text{ave}} = \frac{1}{2} \text{Re}\{E \times H^*\} = \frac{1}{2} \left( \frac{I_0 \, dl}{4\pi} \right)^2 \frac{\eta_0 \beta^2}{r^2} \sin^2 \theta \]

Now, the radiation intensity of the antenna is given as
\[ U(\theta, \phi) = r^2 P_{\text{ave}} = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \sin^2 \theta \]

So, the maximum radiation intensity is
\[ U_{\text{max}} = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \]

(maximum value of \( \sin \theta = 1 \))

As the radiated power of an antenna is given as
\[ P_{\text{rad}} = \int U(\theta, \phi)(\sin \theta \, d\theta \, d\phi) \]

where, the integral is taken in the range \( 0 < \theta < \frac{\pi}{2} \), \( 0 < \phi < 2\pi \) for Hertzian monopole. So, we get
\[ P_{\text{rad}} = \int_0^{\pi/2} \int_0^{2\pi} \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \sin^2 \theta \sin \theta \, d\theta \, d\phi \]
\[ = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \left( \int_0^{\pi/2} \sin^2 \theta \, d\theta \right) \left( \int_0^{2\pi} d\phi \right) \]
\[ = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \left( \frac{4\pi}{3} \right) \]

Since, the directivity of an antenna is defined as
\[ D = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} \]

So, putting the values obtained above we get the directivity of Hertzian monopole antenna as
\[ D = \frac{4\pi (1)}{(4\pi/3)} = 3 \]

SOL 10.2.3 Option (B) is correct.

The field intensities of Hertzian dipole antenna are defined as
\[ H_{\phi \theta} = \frac{3I_0 \beta dl}{4\pi} \sin \theta e^{-j\phi} \]
\[ E_{\phi \theta} = \eta H_{\phi \theta} \]

So, average radiated power of the antenna is given as
\[ P_{\text{ave}} = \frac{1}{2} \text{Re}\{E \times H^*\} = \frac{1}{2} \left( \frac{I_0 \, dl}{4\pi} \right)^2 \frac{\eta_0 \beta^2}{r^2} \sin^2 \theta \]

The radiation intensity of the antenna is defined as
\[ U(\theta, \phi) = r^2 P_{\text{ave}} \]
\[ = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \sin^2 \theta \]

So, the total radiated power of the antenna is
\[ P_{\text{rad}} = \int U(\theta, \phi)(\sin \theta \, d\theta \, d\phi) \]
\[ = \int_0^{\pi/2} \int_0^{2\pi} \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \sin^3 \theta \, d\theta \, d\phi \]
\[ = \frac{(I_0 \, dl)^2}{32\pi^2} \eta_0 \beta^2 \left( \frac{8\pi}{3} \right) \]

Since, the directive gain of the antenna is defined as
Therefore, we get the directive gain of the Hertzian dipole antenna as

\[ G_d = \frac{4\pi U(\theta, \phi)}{P_{\text{rad}}} \]

So, the phase difference between the two currents is

\[ \alpha = 180^\circ = \pi \text{ rad} \]

The unit pattern function of a Hertzian dipole antenna (i.e., the unit pattern function of both the antenna) is

\[ f_1(\theta) = |\cos \theta| \]

where \( \theta \) is angle with \( z \)-axis.

The field pattern of \( f_1(\theta) \) has been plotted below:

Now, the group pattern function of the two antenna is defined as

\[ f_2(\theta) = \cos \left[ \frac{1}{2} (2\pi \lambda \cos \theta / \lambda + \alpha) \right] \]

where \( \alpha \) is the phase difference, \( \beta \) is phase constant and \( d \) is the separation between two antennas. So, we get

\[ f_2(\theta) = \cos \left[ \frac{1}{2} \left( \left( \frac{2\pi \lambda}{\lambda} \frac{\lambda}{2} \cos \theta + \pi \right) \right) \right] = \cos \left[ \frac{1}{2} \left( \pi \cos \theta + \pi \right) \right] \]

This field pattern is plotted as below:
Therefore, the resultant pattern \( f(\theta) \) of the antenna array will be drawn by just multiplying these two patterns
i.e. \[ f(\theta) = [f_1(\theta)] \times [f_2(\theta)] \]
Thus, the obtained plot for the antenna array has been shown below:

\[ \text{SOL 10.2.5} \]
Option (D) is correct.
Separation between the two antennas, \( d = \lambda/4 \)
Phase difference between the currents, \( \alpha = -\pi/2 \)
The unit pattern function of a Hertzian dipole antenna (i.e., the unit pattern function of both the antenna) is
\[ f_1(\theta) = |\cos \theta| \]
where \( \theta \) is angle with \( z \)-axis
This field pattern has been plotted below:

Now, the group pattern function of the two antenna is defined as
\[ f_2(\theta) = \cos \left[ \frac{1}{2} \left( \beta d \cos \theta + \alpha \right) \right] \]
where \( \alpha \) is the phase difference between the currents in the dipole, \( \beta \) is phase constant and \( d \) is the separation between two antennas. So, we get
\[ f_2(\theta) = \cos \left[ \frac{1}{2} \left( \frac{2\pi \lambda}{4} \cos \theta - \frac{\pi}{2} \right) \right] \]
\[ = \cos \left[ \frac{1}{2} \left( \frac{\pi}{2} \cos \theta - \frac{\pi}{2} \right) \right] \]
It’s null (zero) will be at \( \theta = \pi \) and maxima will be at \( \theta = 0^\circ \). So, the field pattern \( f_2(\theta) \) is as plotted below
Therefore, the resultant pattern \( f(\theta) \) of the antenna array will be drawn by just multiplying these two patterns i.e.

\[ f(\theta) = [f_i(\theta)] \times [f_i(\theta)] \]

Thus, the obtained pattern for the antenna array has been shown below:

\[ \text{SOL 10.2.6} \quad \text{Option (B) is correct.} \]

The normalized array factor for the antenna is given as

\[ (AF)_n = \frac{1}{\sum} \left[ 1 + N e^{j\psi} + \frac{N(N-1)}{2!} e^{2j\psi} + \ldots e^{(N-1)j\psi} \right] \]

where

\[ \psi = \left( \beta d \cos \theta + \alpha \right) \]

and

\[ \sum = 1 + N + \frac{N(N-1)}{2!} + \frac{N(N-1)(N-2)}{3!} + \ldots \]

\[ = (1 + 1)^{N-1} = 2^{N-1} \]

So,

\[ (AF)_n = \left[ \frac{1}{2^{N-1}} \right] ^{N-1} \left[ 1 + e^{j\psi} \right] ^{N-1} = \frac{1}{2^{N-1}} \left[ e^{j\psi/2} \right] ^{N-1} \left[ e^{-j\psi/2} + e^{j\psi/2} \right] ^{N-1} \]

Therefore, the group pattern function of the array is

\[ f(\theta) = \left| \cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) \right| ^{N-1} \]

\[ \text{SOL 10.2.7} \quad \text{Option (A) is correct.} \]

Maximum electric field, \( E_{\text{max}} = 6 \text{ mV/m} = 6 \times 10^{-3} \text{ V/m} \)

Location of point of field maxima, \( r = 40 \text{ km} = 40 \times 10^3 \text{ m} \)

Total radiated power is \( P_{\text{rad}} = 100 \text{ kW} = 10^5 \text{ W} \)

The average radiated power of an antenna is defined as

\[ P_{\text{ave}} = \frac{1}{2} \text{Re} \{ E_s \times H_s^* \} \]
So, the radiation intensity of the antenna is given as
\[ U(\theta, \phi) = r^2 F_{ave} \]
\[ = \frac{r^2}{2} \Re \{ \mathbf{E} \times \mathbf{H}' \} \]

Therefore, the maximum radiation intensity of the antenna is
\[ U_{max} = \left| \frac{r^2}{2} \Re \{ \mathbf{E} \times \mathbf{H}' \} \right| = \frac{r^2}{2} \frac{E^2}{\eta} \]
\[ = \frac{r^2}{2\eta} (E_{max})^2 = (40 \times 10^3)^2 \times (6 \times 10^{-3})^2 \] \( (\eta_0 = 120\pi) \)

Since, the directivity of an antenna is defined as
\[ D = \frac{4\pi U_{max}}{P_{rad}} \]
So, we get
\[ D = \frac{4\pi \times (40 \times 10^3)^2 \times (6 \times 10^{-3})^2}{2 \times 120\pi \times 10^4} = 0.0096 \]

Therefore, in decibel the directivity is given as
\[ 10 \log_{10} D = -20.18 \text{ dB} \]

**SOL 10.2.8** Option (B) is correct.
Consider the maximum power gain is \( G_p \) and directive gain is \( G_d \) so, the radiation efficiency is defined as
\[ \eta_r = \frac{G_p}{G_d} \]
or,
\[ G_p = \eta_r G_d = (0.95) G_d \] \( (\eta_r = 95\%) \)
Therefore, the maximum power gain is
\[ G_{p,max} = (0.95) G_{d,max} = (0.95) D \]
\[ = 0.95 \times 0.0096 = 0.00912 = 9.12 \times 10^{-3} \]

**SOL 10.2.9** Option (B) is correct.
Minimum detectable power, \( P_{min} = 0.13 \text{ mW} \)
Transmitted power, \( P_{rad} = 30 \text{ kW} = 30 \times 10^3 \text{ W} \)
Operating frequency, \( f = 3 \text{ GHz} = 3 \times 10^9 \text{ Hz} \)
Target cross section, \( \sigma = 1.25 \text{ m}^2 \)
Radius of antenna, \( a = 1.8 \text{ m} \)

Since, the effective area of the antenna is 70% of it’s actual area so, the effective area of the antenna is
\[ A_e = \frac{70}{100} \times (\pi a^2) = (0.7) \times (\pi \times (1.8)^2) = 7.125 \text{ m}^2 \]

As the maximum range is the point where the received power is equal to the minimum detectable power. So, the received power by the target located at its maximum range is
\[ P_r = P_{min} = 0.13 \text{ mW} = 0.13 \times 10^{-3} \text{ W} \]

Now, the operating wavelength of the antenna is
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m} \]
So, the directive gain of the antenna is given as

$$G_d = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times (7.125)}{(0.1)^2}$$

$$= 2850\pi$$

Since, the maximum detectable range of the antenna is defined as

$$r_{max} = \left[ \frac{\lambda^2 G_d^2 \sigma P_{rad}}{(4\pi)^2 \sigma P_r} \right]^{1/4}$$

where \(P_r\) is the received power by the target located at its maximum range. So, putting all the values in the above expression, we get

$$r_{max} = \left[ \frac{(0.1)^2 (2850\pi)^2 (1.25)}{(4\pi)^2} \frac{30 \times 10^3}{0.13 \times 10^{-3}} \right]^{1/4}$$

$$= 584.27 \text{ m}$$

SOL 10.2.10 Option (A) is correct.

As calculated in previous question, the maximum detectable range of radar is

$$r_{max} = 584.3 \text{ m}$$

So, half of the range will be at the position

$$r = \frac{1}{2} r_{max} = 292.2 \text{ m}$$

Therefore, the time average power density at half of the range of the radar is

$$P_{ave} = \frac{G_d P_{rad}}{4\pi r^2} = \frac{(2850\pi) \times 30 \times 10^3}{4\pi (292.2)^2}$$

$$= 250.35 \text{ W/m}^2$$

SOL 10.2.11 Option (D) is correct.

Current amplitude \(I_0 = 50 \text{ A}\)

Operating frequency, \(f = 180 \text{ kHz} = 180 \times 10^3 \text{ Hz}\)

Effective length, \(l = 20 \text{ m}\)

Location of the observation point, \(R = 80 \text{ km} = 8 \times 10^4 \text{ m}\)

So, the maximum field intensity at the observation point is given as

$$E_{max} = \frac{I_0 dl}{4\pi R \eta \beta} = \frac{I_0}{4\pi R \eta \beta} \times 2l$$

As, the operating wavelength is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{180 \times 10^3} = \frac{10^4}{6}$$

and so the phase constant is

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{10^4} \times 6 = 12\pi \times 10^{-4}$$

Therefore, the maximum field intensity at the observation point is

$$E_{max} = \frac{50}{4\pi \times (8 \times 10^{10})} \times (120\pi) \times 12\pi \times 10^{-4} \times 2 \times 20$$

$$= 0.002827 = 2.83 \text{ mV/m}$$

SOL 10.2.12 Option (B) is correct.

The time average power density of antenna is defined as
So, the time average radiated power is given as

\[ P_{rad} = \frac{1}{2} \Re \{ E_x \times H_x^* \} \]

So, the time average radiated power is given as

\[ P_{rad} = \frac{1}{2} \int \mathbf{P}_{ave} \cdot d\mathbf{S} = \frac{1}{2} \int_{0}^{\pi/2} \int_{0}^{2\pi} R^2 \frac{E_0}{\eta_0} \sin \theta \, d\theta \, d\phi \]

\[ = \frac{1}{2} \int_{0}^{\pi/2} \int_{0}^{2\pi} \frac{R^2}{\eta_0} \left( \frac{I_0}{3\pi R} \eta_0 \times 2L \right)^2 \sin \theta \, d\theta \, d\phi \]

\[ = \frac{1}{2} \times 2\pi \times \eta_0 \left( \frac{I_0 \beta \times 2L}{4\pi} \right)^2 \int_{0}^{\pi/2} \sin \theta \, d\theta \]

\[ = \pi \times 120\pi \times \left( \frac{50 \times 12\pi \times 10^{-4} \times 2 \times 20}{4\pi} \right)^2 \times 1 \]

\[ = 426.37 \text{ W} = 0.43 \text{ kW} \]

**SOL 10.2.13** Option (D) is correct.

As calculated in previous question the time average radiated power is

\[ P_{rad} = 0.43 \text{ kW} \]

Amplitude of the current in the antenna is

\[ I_0 = 50 \text{ A} \]

So, the radiation resistance of the antenna is given as

\[ R_{rad} = \frac{2P_{rad}}{I_0^2} = 2 \times \frac{0.43 \times 10^3}{(50)^2} \]

\[ = 0.34 \Omega \]

**SOL 10.2.14** Option (D) is correct.

Cross sectional radius of wire \( a = 6 \text{ mm} = 6 \times 10^{-3} \text{ m} \)

Radius of the circular loop, \( b = 1 \text{ m} \)

Operating frequency, \( f = 0.5 \text{ MHz} = 0.5 \times 10^6 \text{ Hz} \)

No. of turns, \( N = 10 \)

So, the operating wavelength of the antenna is

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{0.5 \times 10^6} = 6 \times 10^2 \text{ m} \]

Therefore, the radiation resistance of the antenna is given as

\[ R_{rad} = N^2 \times 320\pi \left( \frac{b}{\lambda} \right)^4 = 10^2 \times 320 \times \pi^6 \left( \frac{1}{6 \times 10^2} \right)^4 = 2.37 \times 10^{-4} \Omega \]

**SOL 10.2.15** Option (A) is correct.

As calculated in the previous question, radiation resistance of the antenna is

\[ R_{rad} = 2.37 \times 10^{-4} \Omega \]

So, the surface resistance of the antenna is given as

\[ R_s = \sqrt{\frac{\pi f_0}{\sigma}} = \sqrt{\frac{\pi \times 0.5 \times 10^6 \times 4\pi \times 10^{-7}}{2.9 \times 10^7}} \]

\[ = 2.61 \times 10^{-4} \Omega \]
Therefore, the loss resistance of the antenna is
\[ R_l = N \times \left( \frac{h}{d} \right) R_s = 10 \times \left( \frac{1}{6 \times 10^3} \right) \times 2.61 \times 10^{-4} \]
\[ = 0.435 \Omega \]
Thus, the radiation efficiency of the antenna is
\[ \eta_{rad} = \frac{R_{rad}}{R_l + R_{rad}} = 0.055\% \]

**SOL 10.2.16** Option (B) is correct.

Radiation function of the dipole antenna of height \( h \) is defined as
\[ F(\theta) = \frac{\cos(\beta h \cos \theta) - \cos \beta h}{\sin \theta} \]

Since, the height of dipole antenna is \( h = \lambda/8 \). So, we get
\[ \left| F(\theta) \right| = \left| \frac{\cos(1.25 \cos \theta) - \cos(1.25 \pi)}{\sin \theta} \right| \]
This function has been drawn as to obtain the pattern shown below:

\[ \text{Diagram of radiation pattern} \]

**SOL 10.2.17** Option (D) is correct.

Since, the point \( P(0,0,1000) \) lies along the axial direction of antenna carrying current in \( \mathbf{a}_z \) direction, so it’s contribution to the field will be zero. Now for the antenna carrying current along \( \mathbf{a}_x \) direction, we have
- Amplitude of the current in antenna, \( I_0 = 4 \text{ A} \)
- \( (i(t) = 4 \cos \omega t \text{ A}) \)
- Length of the antenna, \( dl = 0.1 \text{ m} \)
- The position of point \( P \) is \( r = 1000 \) and \( \theta = 90^\circ \) as shown in the figure below:

\[ \text{Diagram of current and field} \]

So, the electric field component in free space is defined as
\[ E_{\phi} = \eta_{rad} H_{\phi} \]
Since, the antenna are carrying current along $a_x$ and $a_z$ while the point is located at $y$-axis so, both the antenna will contribute to the field. Therefore, summing the fields obtained due to the two antennas in previous question, we get,

$$E_x = -j(1.2 \times 10^{-2}) e^{-j10000} a_x V/m$$

So, in the time domain

$$E(t) = Re(E e^{j\omega t})$$

$$= (1.2 \times 10^{-2}) \sin(\omega t + 1000)(a_x + a_z) V/m$$

Thus, the field at $t = 0$ at point $(0, 1000, 0)$ is

$$E = -(9.92 \times 10^{-3})(a_x + a_z) V/m$$

**SOL 10.2.19** Option (D) is correct.

The field component due to the current element is given as

$$E_{\theta_y} = \frac{10}{\pi} \sin \theta e^{-j100r}$$

So, at point $P (r = 100, \theta = \pi/2, \phi = \pi/6)$

$$E_{\theta_y} = \frac{10}{100} \sin \left(\frac{\pi}{2}\right) e^{-j100r(100)}$$

$$= 0.1 e^{-j10000r} V/m$$

**SOL 10.2.20** Option (C) is correct.

Since, the vertical element is shifted from origin to a point $y = 0.1$ on the $y$-axis the distance of point $P$ from the two locations of antenna is approximately same and therefore the magnitude of field component, $|E_{\theta_y}|$ will be same in both cases but the phase angle will change due to the change in location of current element. So, the field intensity at point $P$ due to the new location of vertical element is given as

$$E_{\theta_{\Delta y}} = |E_{\theta_y}| e^{-j100r(l-1)}$$  \hspace{1cm} (1)$$

where $l$ is the difference between the length of point $P$ from two locations as shown in figure below :
Now, using geometry we get the length $l$ as
\[ l = 0.1 \cos \left( \frac{\pi}{3} \right) = 0.05 \]
Putting the value in equation (1), we get the field component as
\[ E_{\phi l} = 0.1 e^{-j1000\pi} e^{j005(l)} = 0.1 e^{-j1000\pi} e^{0.5\pi} \text{ V/m} \]

**SOL 10.2.21** Option (B) is correct.
Radiation resistance of a short circuit current element is determined as
\[ R_{\text{rad}} = 80\pi^2 \left( \frac{l}{\lambda} \right)^2 \]
where $l$ is the length of dipole
But, as the current is not uniform so, we determine the average current through the element. Now, from the given expression of current in the element, we get
\[ I_1(z) = I_0 \left( \frac{l + 2z}{l} \right) \quad \text{for} \quad \frac{l}{2} \leq z \leq 0 \]
and
\[ I_2(z) = I_0 \left( \frac{l - 2z}{l} \right) \quad \text{for} \quad 0 \leq z \leq \frac{l}{2} \]
Therefore, the average current in the element is given as
\[ I_{\text{avg}} = \frac{I_1(z) + I_2(z)}{2} = \frac{I_0 \left( \frac{l + 2z}{l} \right) + I_0 \left( \frac{l - 2z}{l} \right)}{2} = \frac{I_0}{2} \]
Since, the average current flowing in the antenna is half of the uniform current $I_0$ therefore, the radiated power will be $\frac{1}{4}$th of the value obtained for $I_0$ and due to the same reason the radiation resistance will down to $\frac{1}{4}$th of its value.
i.e.
\[ [R_{\text{rad}}]_{\text{net}} = \frac{1}{4} R_{\text{rad}} = \frac{1}{4} \times 2.84 = 0.71 \Omega \]

**SOL 10.2.22** Option (B) is correct.
Length of wire, \( dl = 1 \text{ cm} = 0.01 \text{ m} \)
Operating frequency, \( f = 0.3 \text{ GHz} = 0.3 \times 10^9 \text{ Hz} \)
Cross section radius, \( r = 1 \text{ mm} = 10^{-3} \text{ m} \)
So, radiation resistance is given as
\[ R_{\text{rad}} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 \]
Now, the ohmic resistance of the wire is defined as

\[ R_l = \frac{L}{\sigma \pi a \delta} \]

where

- \( \sigma \) → Conductivity
- \( a \) → Radius of the cross section
- \( \delta \) → Skin depth
- \( L \) → Length of the wire

Since, the skin depth of the wire is given as

\[ \delta = \frac{1}{\sqrt{\pi \mu_0 \sigma}} = \frac{1}{3.82 \times 10^7 \pi (0.3 \times 10^9)(4\pi \times 10^{-7})(5.8 \times 10^{-9})} \]

So, we get

\[ R_l = \frac{0.01}{(5.8 \times 10^7)(2\pi \times 10^{-7})(3.82 \times 10^{-7})} = 0.0072 \Omega \]

Therefore, the ratio of the radiation resistance to the ohmic resistance of wire will be

\[ \frac{R_{rad}}{R_l} = 10.977 \approx 11 \]

**SOL 10.2.23** Option (D) is correct.

Length of antenna, \( dl = 2 \text{ cm} = 0.02 \text{ m} \)

Radiated power, \( P_{rad} = 2 \text{ W} \)

Operating frequency, \( f = 0.6 \text{ GHz} = 0.6 \times 10^9 \text{ Hz} \)

So, operating wavelength of antenna is

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{0.6 \times 10^9} = 0.5 \text{ m} \]

Therefore, the radiation resistance of the antenna is given as

\[ R_{rad} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 = 80\pi^2 \left( \frac{0.02}{0.5} \right)^2 = \frac{16\pi^2}{125} \]

As the radiated power of the antenna is defined as

\[ P_{rad} = \frac{1}{2}(I_0)^2 R_{rad} = (I_{r.m.s})^2 R_{rad} \quad (I_{r.m.s} = I_0/\sqrt{2}) \]

So, the rms current in the antenna is

\[ I_{r.m.s} = \sqrt{\frac{P_{rad}}{R_{rad}}} = \sqrt{\frac{2}{(16\pi^2/125)}} = 1.26 \text{ A} \]

************
SOL 10.3.1  Option (B) is correct.
The directivity of an antenna is defined as

\[ D = \frac{U_{\text{max}}}{U_{\text{ave}}} \]

where \( U_{\text{max}} \) is the maximum radiation intensity of the antenna and \( U_{\text{ave}} \) is the average radiation intensity. Since, the given antenna has the radiation pattern

\[ U(\theta) = \cos^4 \theta \quad (0 \leq \theta \leq \pi/2) \]

So, the maximum radiation intensity is

\[ U_{\text{max}} = 1 \]

The average radiation intensity is

\[ U_{\text{ave}} = \frac{1}{4\pi} \int F(\theta, \phi) d\Omega = \frac{1}{4\pi} \left[ \int_0^{2\pi} \int_0^{\pi/2} F(\theta, \phi) \sin \theta d\theta d\phi \right] \]
\[ = \frac{1}{4\pi} \left[ \int_0^{2\pi} \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta d\phi \right] = \frac{1}{4\pi} \left[ 2\pi \left( -\cos^5 \theta \right) \right]_0^{\pi/2} \]
\[ = \frac{1}{4\pi} \times 2\pi \left[ -0 + \frac{1}{5} \right] = \frac{1}{4\pi} \times \frac{2\pi}{5} = \frac{1}{10} \]

Therefore, the directivity of the antenna is

\[ D = \frac{1}{10} = 10 \]

or, \[ D(\text{in dB}) = 10 \log 10 = 10 \text{ dB} \]

SOL 10.3.2  Option (D) is correct.
The beam-width of Hertzian dipole is 180° so, its half power beam-width is 90°.

SOL 10.3.3  Option (A) is correct.
The operating wavelength of the antenna is

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{20 \times 10^9} = \frac{3}{200} \quad (f = 20 \text{ GHz}) \]

Therefore, the gain of parabolic antenna is given as

\[ G_p = \eta \pi^2 \left( \frac{D}{\lambda} \right)^2 \]
\[ = 0.7 \times \pi^2 \left( \frac{1}{\lambda} \right)^2 = 30705.4 \quad \text{(efficiency, } \eta = 70\%) \]

or, \[ 10 \log_{10} G_p = 44.87 \text{ dB} \]

SOL 10.3.4  Option (C) is correct.
Using the method of images, the configuration is as shown below

**GATE CLOUD Electromagnetics By RK Kanodia & Ashish Murolia**
Here $d = \lambda$, $\alpha = \pi$, thus, $\beta d = 2\pi$

So, the array factor of the antenna is given as

$$A.F. = \cos \left( \frac{\beta d \cos \psi + \alpha}{2} \right) = \cos \left( \frac{2\pi \cos \psi + \pi}{2} \right) = \sin (\pi \cos \psi)$$

**SOL 10.3.5** Option (B) is correct.

Since, the antenna is installed at conducting ground. So, the power will be radiated only on the half side of the antenna and therefore, the radiation resistance of the antenna will be half of its actual value and given as

$$R_{rad} = \frac{1}{2} \left[ \frac{80\pi^2}{(d/\lambda)^2} \right] = 40\pi^2 \left( \frac{50}{0.5 \times 10^2} \right)^2 = \frac{2\pi^2}{5} \Omega$$

**SOL 10.3.6** Option (B) is correct.

The array factor of the antenna is defined as

$$A.F. = \cos \left( \frac{3d \sin \theta + \alpha}{2} \right)$$

Here, $d = \frac{\lambda}{4}$

and $\alpha = 90^\circ$

Thus, $A.F. = \cos \left( \frac{\frac{\pi}{4} + \sin \theta + \frac{\pi}{2}}{2} \right) = \cos \left( \frac{\pi}{4} \sin \theta + \frac{\pi}{2} \right)$  \hspace{1cm} ($\beta = \frac{2\pi}{\lambda}$)

The option (A) satisfy this equation.

**SOL 10.3.7** Option (B) is correct.

The directive gain of an antenna at a particular direction $(\theta, \phi)$ is defined as

$$G_d(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{rad}} \hspace{1cm} (1)$$

Since, for lossless antenna

$$P_{rad} = P_{in}$$

So, we get

$$P_{rad} = P_{in} = 1 \text{ mW}$$

Again the directive gain of the antenna is given

$$10 \log G_d(\theta, \phi) = 6 \text{ dB}$$

So,

$$G_d(\theta, \phi) = 3.98$$

Putting it in equation (1) we get the total power radiated by antenna as

$$4\pi U(\theta, \phi) = P_{rad} G_d(\theta, \phi) = 1 \text{ m} \times 3.98 = 3.98 \text{ mW}$$

**SOL 10.3.8** Option (A) is correct.

Normalized array factor is given as
A.F. = \[ 2 \left| \cos \frac{\psi}{2} \right| \]

where,
\[ \psi = \beta d \sin \theta \cos \phi + \delta \]
\[ \theta = 90^\circ \]
\[ d = \sqrt{2} s \]
\[ \phi = 45^\circ \]
\[ \delta = 180^\circ \]

So,
\[ \text{A.F.} = 2 \left| \cos \frac{\psi}{2} \right| = 2 \cos \left[ \frac{\beta d \sin \theta \cos \phi + \delta}{2} \right] \]
\[ = 2 \cos \left[ \frac{2\pi}{\lambda} \sqrt{2} s \cos 45^\circ + \frac{180^\circ}{2} \right] \]
\[ = 2 \cos \left[ \frac{\pi s}{\lambda} + 90^\circ \right] = 2 \sin \left( \frac{\pi s}{\lambda} \right) \]

SOL 10.3.9 Option (C) is correct.
The signal strength (power) at a distance \( r \) from an antenna is inversely proportional to the distance \( r \).
i.e.
\[ P \propto \frac{1}{r^2} \]

So,
\[ \frac{P_1}{P_2} = \frac{r_2^2}{r_1^2} \] (1)
Since, 3 dB decrease \( \rightarrow \) Strength is halved \( (10^{3/10} = 10^{0.3} = 2) \)
Therefore,
\[ \frac{P_1}{P_2} = 2 \]
Substituting it in equation (1), we get
\[ 2 = \frac{r_2^2}{5^2} \]
\[ r_2 = 5\sqrt{2} \text{ km} = 7071 \text{ m} \]
Thus, the required distance to move is
\[ d = r_2 - \eta = 7071 - 5000 = 2071 \text{ m} \]

SOL 10.3.10 Option (C) is correct.
We have
\[ \lambda = 492 \text{ m} \]
and height of antenna, \( dl = 124 \text{ m} \approx \frac{\lambda}{4} \)
So, it is a quarter wave monopole antenna and radiation resistance of a quarter wave monopole antenna is 36.5 \( \Omega \).

SOL 10.3.11 Option (D) is correct.
We have
\[ \psi = \beta d \cos \theta + \delta \] (1)
where
\[ d = \frac{\lambda}{4} \]
Distance between elements
\[ \psi = 0 \]
Because of end fire
\[ \theta = 60^\circ \]

Putting all the values in equation (1) we get
\[ 0 = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \cos 60^\circ + \delta = \frac{\pi}{2} \times \frac{1}{2} + \delta \]
or \[ \delta = -\frac{\pi}{4} \]

**SOL 10.3.12** Option (C) is correct.

For a dipole antenna we have

\[ \text{BW} \propto \frac{1}{(\text{Diameter})} \]

So, as diameter increases Bandwidth decreases.

**SOL 10.3.13** Option (A) is correct.

Far field region for an antenna is defined for the distance \( r \) from the antenna as

\[ r > \frac{2d^2}{\lambda} \]

where \( d \) is the largest dimension of the antenna and \( \lambda \) is the operating wavelength.

Now, the operating wavelength of the antenna is given as

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{4 \times 10^9} = \frac{3}{40} \text{ m} \]

So, for the closest far field we have

\[ r = \frac{2d^2}{\lambda} = \left( \frac{2 \times (2.4)^2}{\frac{3}{40}} \right) = \frac{80 \times (2.4)^2}{3} \approx 150 \text{ m} \]

**SOL 10.3.14** Option (D) is correct.

We know that for a monopole its electric field varies inversely with \( r^2 \) while its potential varies inversely with \( r \). Similarly, for a dipole its electric field varies inversely as \( r^3 \) and potential varies inversely as \( r^2 \).

In the given expression both the terms \( \left( \frac{1}{r^2} + \frac{1}{r} \right) \) are present, so, this potential is due to both monopole and dipole.

**SOL 10.3.15** Option (A) is correct.

Power received by an antenna is defined as

\[ P_r = \frac{P_t}{4\pi r^2} \times A_e \]

where \( P_t \) is the power radiated by the transmitting antenna, \( r \) is the distance between transmitter and receiver and \( A_e \) is the effective aperture area of the receiving antenna. So, we get

\[ P_t = \frac{251 \times 500 \times 10^{-4}}{4 \times \pi \times (100)^2} \quad (A_e = 500 \text{ cm}^2, r = 100 \text{ m}, P_t = 251 \text{ W}) \]

\[ = 100 \mu \text{W} \]

**SOL 10.3.16** Option (D) is correct.

Magnetic field intensity in terms of vector potential is defined as

\[ \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \]

where \( \mathbf{A} \) is auxiliary potential function.

So,

\[ \nabla \cdot \mathbf{H} = \nabla \cdot (\nabla \times \mathbf{A}) = 0 \]

and

\[ \nabla \times \mathbf{H} = \nabla \times (\nabla \times \mathbf{A}) \neq 0 \]
SOL 10.3.17 Option (A) is correct.
Radiation resistance of a circular loop is given as
\[ R_{\text{rad}} = \frac{8}{3} \eta \pi^3 \left[ \frac{N \Delta S}{\lambda^2} \right] \]
where \( N \) is number of turns. Since, the radiation resistance of a circular loop is 0.01\( \Omega \).
i.e. \( R_{\text{cl}} = 0.01 \Omega \)
So, we get the net radiation resistance of the five turns of such loop as
\[ R_{r2} = N^2 \times R_{\text{cl}} = (5)^2 \times 0.01 = 0.25 \Omega \quad (N = 5) \]

SOL 10.3.18 Option (D) is correct.
Aperture area of a receiving antenna is defined in terms of received power as
\[ A_e = \frac{P_r}{\text{Poynting vector of incident wave}} \]
Since,
\[ P_t = \frac{|E|^2}{\eta_0} \quad (\eta_0 = 120\pi \text{ is intrinsic impedance of space}) \]
So,
\[ A_e = \frac{2 \times 10^{-6}}{\left(\frac{E^2}{\eta_0}\right)} = \frac{2 \times 10^{-6}}{\left(20 \times 10^{-3}\right)^2} \times 120 \times 3.14 = \frac{2 \times 10^{-6} \times 12 \times 3.14}{400 \times 10^{-6}} = 1.884 \text{ m}^2 \]

SOL 10.3.19 Option (C) is correct.
Maximum usable frequency \( f_{\text{max}} \) in terms of incidence angle \( i \) is defined as
\[ f_{\text{max}} = \frac{f_0}{\sin i} \]
where \( f_0 \) is critical frequency. So, we get
\[ f_{\text{max}} = \frac{8 \text{ MHz}}{\sin 60^\circ} = \frac{8}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{16}{\sqrt{3}} \text{ MHz} \]

SOL 10.3.20 Option (B) is correct.
Far field \( \alpha \frac{1}{r} \)

SOL 10.3.21 Option (C) is correct.
The maximum usable frequency is given as
\[ f_{\text{m}} = \frac{f_0}{\sin i} \]
where \( i \) is launching angle and \( f_0 \) is critical frequency so, we get
\[ 20 \times 10^6 = \frac{10 \times 10^6}{\sin i} \]
or, \( \sin i = \frac{1}{2} \)
or, \( i = 30^\circ \)

SOL 10.3.22 Option (C) is correct.
The directive gain of half wave ($\lambda/2$) dipole antenna is given as

$$G_d = 1.66 \frac{\cos^2\left(\frac{\pi}{2}\cos \theta\right)}{\sin^2 \theta}$$

So, the directivity of the antenna is

$$D = G_{d, max}$$

Since, the maximum value of the function $\frac{\cos^2\left(\frac{\pi}{2}\cos \theta\right)}{\sin^2 \theta}$ is 1. So, the directivity of $\lambda/2$ long wire antenna is

$$D = 1.66(1) = 1.66$$

**SOL 10.3.23** Option (A) is correct.

Since, the EM waves are travelling in free space, so the phase velocity of the wave will be equal to the velocity of light in free space.

i.e. $v_p = c$

So, at frequency, $f = 63$ MHz (Channel 3)

wavelength,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{63 \times 10^6} = 4.76 \text{ m}$$

So, phase constant, $\beta = \frac{2\pi}{\lambda} = 1.32 \text{ rad/m}$

and at frequency, $f = 803$ MHz (channel 69)

wavelength,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{803 \times 10^6} = 0.374 \text{ m}$$

So, phase constant, $\beta = \frac{2\pi}{\lambda} = 16.82 \text{ rad/m}$

**SOL 10.3.24** Option (A) is correct.

Since, the antenna is located at earth so, power radiated to the hemisphere will be half of the transmitted value.

i.e. $P_r = \frac{P}{2} = \frac{200 \text{ kW}}{2} = 100 \text{ kW}$

Now, the average poynting vector (power radiated per unit area) at a distance $r$ from the antenna is given as

$$\mathbf{P}_{\text{avg}} = \frac{P}{\pi r^2} \mathbf{a}_r$$

where $\mathbf{a}_r$ denotes the direction of Poynting vector. So, for $r = 50 \text{ km}$, we have

$$\mathbf{P}_{\text{avg}} = \frac{100 \times 10^3}{\pi (50 \times 10^3)^2} \mathbf{a}_r = \frac{40}{\pi} \mathbf{a}_r \mu\text{W/m}^2$$

**SOL 10.3.25** Option (B) is correct.

Radiation resistance of a dipole antenna is defined as

$$R_{\text{rad}} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2$$

... (1)

Given,

The length of dipole, $dl = 5 \text{ m}$

operating frequency, $f = 3$ MHz $= 3 \times 10^6$ Hz

So, the operating wavelength of the antenna is given as
\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^7} = 100 \text{ m} \]

Putting these values in equation (1) we get

\[ R_{rad} = 80\pi^2 \left( \frac{5}{100} \right)^2 = \frac{80\pi^2}{400} = 1.97 \approx 2 \Omega \]

**SOL 10.3.26** Option (C) is correct.

The radiated power of an antenna is defined as

\[ P_{rad} = \frac{1}{2} I_0^2 R_{rad} \]

i.e.

\[ P_{rad} \propto R_{rad} \]  ... (i)

Now, the radiation resistance of the antenna is given as

\[ R_{rad} = 80\pi^2 \left( \frac{dl}{\lambda} \right)^2 \]

i.e.

\[ R_{rad} \propto \frac{(dl)^2}{\lambda^2} \]

Since

\[ \lambda = \frac{c}{f} \]

So, we get

\[ R_{rad} \propto f^2 (dl)^2 \]  ... (ii)

Combining eq(1) and (2) we conclude that

\[ P_{rad} \propto (dl)^2 (f)^2 \]

Now, for the 1st antenna we have

\[ (dl)(f) = (1.5)(100 \times 10^9) = 1.5 \times 10^8 \]

for 2nd antenna

\[ (dl)(f) = (15)(10 \times 10^9) = 1.5 \times 10^9 \]

Since, the product of length and frequency are same for both the antenna So, the power radiated by both the antennas will be same.

**SOL 10.3.27** Option (B) is correct.

Given the length of current element, \( l = 0.03\lambda \).

So, the radiation resistance of the system is given as

\[ R_{rad} = 80\pi^2 \left( \frac{1}{\lambda} \right)^2 = 80\pi^2 \left( \frac{0.03\lambda}{\lambda} \right)^2 = 0.072\lambda^2 \Omega \]

**SOL 10.3.28** Option (D) is correct.

In a three element Yagi antenna there are one reflector, one folded dipole (driven element) and one director. The length of reflector is greater than driven element which in turn is longer than the director.

**SOL 10.3.29** Option (C) is correct.

Antenna arrays are formed to produce a greater directivity i.e. more energy radiated in some particular direction and less in other directions.

**SOL 10.3.30** Option (B) is correct.

\[
\text{Input power} = W_t \\
\text{Radiated power} = W_r
\]
Radiation intensity \( f \) is given by:

So, the power gain of the antenna is

\[ G_p = \frac{4\pi f}{W_r} \quad (a \rightarrow 3) \]

Directive gain of antenna is

\[ G_d = \frac{4\pi f}{W_r} \quad (b \rightarrow 4) \]

Average radiated power of the antenna is

\[ P_r = \frac{W_r}{4\pi} \quad (c \rightarrow 2) \]

The efficiency of antenna is

\[ \eta = \frac{W_r}{W_t} \quad (d \rightarrow 1) \]

**SOL 10.3.31** Option (C) is correct.

Maximum radiation for an end fire array occurs along the line of the array.

**SOL 10.3.32** Option (B) is correct.

The gain of antenna is directly proportional to the aperture area. So, with increase of aperture area, received power increases and therefore the gain increases.

**SOL 10.3.33** Option (B) is correct.

In the helical antenna, normal mode of operation is very narrow in bandwidth and therefore the directivity is high. While the radiation efficiency is low.

**SOL 10.3.34** Option (B) is correct.

For an antenna near and far zone are specified by a boundary defined as

\[ R = \frac{2d^2}{\lambda} \]

where, \( R \) is the distance from antenna, \( d \) is the largest dimension of antenna and \( \lambda \) is the operating wavelength of antenna. So, any target located at a distance \( R > \frac{2d^2}{\lambda} \) from antenna is in the far zone for the antenna and any target located at a distance \( R < \frac{2d^2}{\lambda} \) is in the near zone.

**SOL 10.3.35** Option (B) is correct.

Gain of transmitting antenna, \( G_o = 10 \)

Transmitted power, \( P_t = 1W \)

Effective area of receiving antenna, \( A_r = 1 \text{ m}^2 \)

Distance between transmitter and receiver, \( r = 1 \text{ m} \)

So, total received power by the receiving antenna is

\[ P_r = \frac{P_t}{4\pi r^2} G_o A_r = \frac{1}{4\pi (1)^2} \times (10) \times (1) = 0.79 \text{ W} \]

**SOL 10.3.36** Option (C) is correct.

Since, the region \( r > \frac{2D^2}{\lambda} \) is called far zone for the antenna and as it is given that in the Fraunhofer region measurement to be taken from a distance of \( \frac{2D^2}{\lambda} \) from antenna so, the defined region is far zone or far field.
SOL 10.3.37 Option (C) is correct.

Helical antenna is used to provide circularly polarized wave and the log periodic antenna is frequency independent.

SOL 10.3.38 Option (C) is correct.

For a wave travelling from medium 1 to medium 2, the incidence angle $\theta_ı$ of the wave for which it is totally reflected by medium 2 is given as

$$n_ı \sin \theta_ı = n_2 \sin 90^\circ$$

where $n_ı$ and $n_2$ are the refractive index of medium 1 and medium 2 respectively. Since, refracting index of a medium having permittivity $\varepsilon$ and permeability $\mu$ is defined as

$$n = \sqrt{\frac{\varepsilon \mu}{\mu_0}}$$

So, putting it in equation (1), we get

$$\sqrt{\frac{\varepsilon_ı \mu_0}{\varepsilon_2 \mu_0}} \sin \theta_ı = \sqrt{\frac{\varepsilon_2}{\varepsilon_ı}}$$

$$\sin \theta_ı = \sqrt{\frac{\varepsilon_ı}{\varepsilon_2}}$$

$$\theta_ı = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

SOL 10.3.39 Option (C) is correct.

Yagi-Uda antenna must have one reflector and one driven element while it can have any number of directors. So, the four element Yagi-Uda antenna will have 2 directors, one reflector, and one driven element.

SOL 10.3.40 Option (B) is correct.

Directivity of an antenna is directly proportional to the effective area and therefore larger the effective area, sharper the radiated beam. This is the reason for using an aperture antenna instead a linear antenna for extremely high frequency ranges.

SOL 10.3.41 Option (B) is correct.

Current distortion along a travelling wave antenna in general is defined as

$$I(z,t) = I_0 \cos(\cot - \beta z)$$

but when we eliminate $t$ by taking it’s phasor form, the current can be written as

$$I(z) = I_0 e^{\frac{j\beta z}{\lambda}}$$
SOL 10.3.42  Option (C) is correct.
Turnstile antenna is generally used at UHF/VHF for local area transmission.

SOL 10.3.43  Option (C) is correct.
The frequencies up to 1650 kHz is in the range of medium frequency. Vertical radiators ranging from $\lambda/6$ to $\lambda/5$ used for broadcasting the medium frequencies as the operating conditions and economic consideration.

SOL 10.3.44  Option (B) is correct.
The radiation field intensity of an antenna at a distance $r$ is defined as
$$E_{\theta r} = \frac{\eta I_0 \beta dl}{4\pi r} \sin \theta e^{-j\omega r}$$
or $$|E_{\theta r}| = \frac{\eta I_0 \beta dl \sin \theta}{4\pi r} \propto \frac{1}{r}$$

SOL 10.3.45  Option (A) is correct.
The resultant field in a helical antenna is either circularly polarized or elliptically polarized depending on the pitch angle $\alpha$.
The radiated wave by a helical antenna is circularly polarized only when,
$$\alpha = \tan^{-1} \left( \frac{C}{2\lambda} \right)$$
else it is elliptically polarized. In conclusion for a general term we can say the wave radiated by a helical antenna is elliptically polarized.

SOL 10.3.46  Option (A) is correct.
The unexcited patch is shown below

The capacitance between the plates is given as
$$C = \frac{\varepsilon (\text{Area of plates})}{(\text{separation between plates})} = \frac{\varepsilon_0 \varepsilon_r LW}{h}$$

SOL 10.3.47  Option (D) is correct.
Maximum usable frequency between two stations of distance $D$ is defined as
$$f_{\text{MHF}} = f_c \sqrt{1 + \left( \frac{D}{2h} \right)^2}$$
where $h$ vertical height at the mid point of path and $f_c$ is critical frequency we put all the values to get,
\[ f_{250} = 9 \times 10^6 \sqrt{1 + \left(\frac{800}{\frac{1}{2} \times 300}\right)^2} = 1.5 \times 10^7 \text{ Hz} = 15 \text{ MHz} \]

**SOL 10.3.48** Option (B) is correct.

\( D \)-layer is the lower most region of ionosphere which is present only during the day light hours and disappears at night because recombination rate is highest and also \( E \)-region is weekly ionised during night hour hence radiowave suffer negligible attenuation in night hour. This is the reason that the wave band which can’t be heard during day time but may be heard during night time.

**SOL 10.3.49** Option (D) is correct.

**SOL 10.3.50** Option (B) is correct.

The atmosphere has varying density (refractive index) with the height from earth given as \( \frac{dh}{d\mu} \). Radius of curvature of the wave path is

\[ R = -\frac{dh}{d\mu} \]

Solving it, we get the effective earth radius (Radio horizon)

\[ = \frac{4}{3} \text{ actual earth radius (optical horizon)} \]

**SOL 10.3.51** Option (B) is correct.

The radiation resistance of a dipole antenna is defined as

\[ R_{rad} = 80 \pi^2 \frac{dl^2}{\lambda} \]

Since,

\[ dl = \frac{\lambda}{20} \]

So,

\[ R_{rad} = 80 \pi^2 \left(\frac{\lambda/20}{\lambda}\right)^2 = 2 \Omega \]

**SOL 10.3.52** Option (B) is correct.

Effective length of a half wave dipole antenna is

\[ l_e(\theta) = 2 \frac{1}{\beta} \left[ \cos \left(\frac{\pi}{2} \cos \theta\right) \right] \sin \theta \]

i.e. \( l_e \) is function of \( \theta \).

The maximum value of \( l_e \) is at \( \theta = \pi/2 \).

\[ l_e \left(\frac{\pi}{2}\right) = \frac{2}{\beta} = \frac{\lambda}{\pi} < \frac{\lambda}{2} \]

i.e. maximum value of \( l_e \) is less than its actual value \( \frac{\lambda}{2} \).

The effective length is the same for the antenna in transmitting and receiving modes.

So, statements, 1, 2 and 4 and correct while statement 3 is incorrect.

***********